# "You can divide the thing into two parts" <br> Analyzing Referential, Mathematical and Technological Practice in the VMT Environment 

Timothy Koschmann<br>Southern Illinois University<br>Gerry Stahl<br>Drexel University<br>Alan Zemel<br>Drexel University

(Discussion draft - do not cite or distribute without express permission
from the authors.)


#### Abstract

In keeping with the theme of this year's conference, we direct our attention to the analytic practices through which participants, when interacting via computers make sense of their own and others' actions. Participants' endogenous work of analysis has received little attention in prior research on collaborative learning. We would argue, however, that these are the very practices of greatest relevance for study in CSCL. The materials to be presented here come from the Virtual Math Teams (VMT) Project conducted under the auspices of the Math Forum at Drexel University. In this project, students situated at geographically diverse sites solve math problems together using text-based, synchronous chat communication and a shared graphical whiteboard. We examine the interaction of three students and a faculty moderator in their initial period of problem solving. We find evidence of manifold competencies related to discourse production, mathematics and technology use. We focus on the presentation of a prospective problem solution by one particular student and describe in detail how his practices provide for the analyzability of his actions.


## I. The 'Practice Turn' in CSCL Research

The theme of this year's conference is research on practice. CSCL researchers have displayed an interest in practice from the very inception of the field. This interest might be seen as part of a more general "practice turn" (Schatzki, 2001) that has occurred in the human sciences over the last few decades. Human practice, of course, is a very broad topic. If CSCL research is to take a practice turn, what kind of practice should we be studying? Lynch (2001), in a paper on the logic of practice, posited a form of "analytic work that is endogenous to the social production of coordinated talk" (p. 132). He wrote:

For conversation analysts, 'analysis' is a pivotal term that identifies their own methodological activity with the objective domain they investigate. The concerted production of intelligible lines of talk is both the subject and the source of such analysis. (p. 132)

Conversational participants are already engaged in a form of analysis making sense of their own unfolding talk-in-interaction. Conversation Analysis (CA) seeks to study the practices whereby this form of analysis is done and document its underlying logic and methods (see, for example, Sacks, 1992). Our interests here are similar, but instead of studying methods of analysis endogenous to F2F conversation, we direct our attention to computer-mediated communication (CMC). The resources available to participants when interacting through computers are quite different from those in F2F exchanges in which intonation and other features of vocal delivery, gaze, gesture, etc. are so crucially important to sense making. Though these features are absent in CMC, as we will see in the case examined here, it is not without resources for sense making.

## II. The Virtual Math Teams Project

The materials to be discussed come from a corpus assembled at the Math Forum at Drexel University. The Virtual Math Teams (VMT) Project, established in 2003, is one of a variety of programs conducted under the auspices of the Math Forum. In this project, teams of geographically dispersed students use an integrated suite of web-based software tools to explore proposed mathematics topics (Stahl, forthcoming). VMT sessions are run as an enrichment activity conducted outside of the regular school curriculum. Students are recruited through their math teachers at their home schools. Here we study the interaction between three particular students self-identified as Aznx,

Quicksilver and bwang8 (hereafter just 'Bwang'), and a Math Forum facilitator ("Gerry"). The three students represented one team (Team B) in the 2006 VMT Spring Fest. Their collaboration continued for four online sessions, each of approximately one hour in length, and spaced out over a two-week period (see Medina, Suthers \& Vatrapu [forthcoming] for an overview of Sessions I-III and Stahl [forthcoming] for a description of Session IV).

The VMT software environment supports collaboration at a distance using textbased, synchronous chat communication as well as a shared graphical whiteboard and an asynchronous community-wide wiki (Stahl, forthcoming). A screen image of the VMT user interface can be seen in Figure 1. Their moment-to-moment interaction was recorded by the system and can be replayed in real time using the VMT Replayer application. Unlike a video recording of a F2F encounter, in which we see what the camera operator chose to show us, here we see precisely what was made available to the participants themselves to see (i.e., a correspondence of the participants' and the observers' perspective). These recordings, therefore, provide a rich and comprehensive set of materials for examining practices of collaboration within computer-mediated interaction. ${ }^{1}$

We will examine Team B's initial period of joint activity in Session I. In a message posted early in the session, Gerry, the facilitator, provides instructions for where the worksheet for the first session might be found on the 'View Topics' page, establishing the task for the day. Approximately 10 minutes later, Aznx asks, "So how do we submit this?" It would appear that in the intervening period something representing a solution to the posed task had been produced. Our analysis will focus on what that something might be and how it was developed interactionally.

## III. "You can divide the thing into two parts"

Given the constraints of time, we offer here just a sketch of how an analysis might proceed. Let us first begin by examining the task description provided to the team in Session I (see Appendix A). It contains three panels: a series of match-stick figures demonstrating a series graphically, a table representing the same series showing the

[^0]number of match sticks and squares at each stage, and, finally, a list of instructions laying out the task itself. The instructions specify a sequence of actions designed to achieve a curricular goal. They are designed such that when the parties following the instructions reach the end of the directed steps, the instruction followers will have been led to a new understanding of some curricular matter. Such is the work of instruction (c.f., Lynch, 2000).

The curricular matter in this case is made visible in the two numeric series labeled in the table, "sticks" and "squares." The progression in both cases is based upon a simple summation function ( $\left.\Sigma_{i=1 \text { to }}(\mathrm{i})=1+2+3+\ldots+N=(N+1) N / 2\right)$, one employed ubiquitously in probability theory and statistics. The worksheet instructions are artfully designed to build not only toward an understanding of how this function arises in a variety of series, but also to familiarize the participants with the affordances of the VMT interface. The first task instruction asks the students to graphically represent, as match-stick figures, the next three elements in the series. This presumably provides a resource for then satisfying the second task step-filling in the next three rows in the table. The third step builds on the previous two and asks the students to articulate a "pattern of growth" for the series representing the number of sticks and squares.

A record of the team's interaction can be found in Appendix B. As in most chat interfaces, text, in the VMT environment, is composed in a "message entry box" (Garcia \& Jacobs, 1999). When a carriage return is entered, the message is dispatched to the chat server and displayed in a serial list of postings visible to all in the "posting box" (Garcia \& Jacobs, 1999). Prior to dispatching a chat post to the server, its content is only visible to the person typing it, but the fact that that person is preparing a message is made available to the others (e.g., "bwang8 is typing"). ${ }^{2}$ Though the participants are situated at different sites, therefore, their projected actions are available for the others to monitor.

Having located the worksheet, Bwang types, "are we supposed to solve it now?" (post 42). Posed as a procedural inquiry, his post addresses the interactional problem of how one might initiate concerted activity under circumstances in which one's collaborators are not co-present. His note not only displays his readiness to begin, but also characterizes the nature of the team's work as finding a solution. Bwang's query is nominally directed to the moderator who does not respond, but continues to provide

[^1]instructions that will allow all team members to access the problem statement. Aznx (post 53) subsequently announces his own readiness to begin ("Let's start this thing.") But Bwang has already started.

He begins with an assertion that we have appropriated as the title for this talk ("you can divide the thing into two parts"). It is one that would appear to be riddled with referential puzzles. The referent of "the thing" is ambiguous. Perhaps it co-references the same matter as "it" in his earlier post (\#42). But if that is so, and "the thing" references the problem that they are to solve, what exactly are they taking that to be? Is it one of determining how to perform the first assigned instruction? Is it related to computing the number of sticks and/or squares? Or does it have to do with the more general problem of seeing "a pattern of growth"? Given this uncertainty with regard to what "the thing" might be, we are even less secure in our grasp of what "dividing it into two parts" might signify. Rather than seeking clarification, however, Bwang's correspondents "trust" (Garfinkel, 1963) that all these matters will be made clear in time.

Bwang wastes no time in making plain just what "this thing" might be. The VMT interface is a "dual-interaction space" (Çakir, Zemel, \& Stahl, 2009), including not only a chat facility, but also a whiteboard panel. Actions preformed on the whiteboard (e.g., creation of a text or graphic object) are persistently available for all to see. Immediately after his post, Bwang turns to the whiteboard and scribes a series of lines. The resulting gestalt resembles a reconstruction of the third figure from the worksheet, opened like a book and isolating its vertical and horizontal elements (see Fig. 1). Though we now have a visual resource to help us resolve what "dividing the thing into two parts" might mean, we are still left unclear about how restructuring the third figure from the worksheet in this particular way is connected to the task at hand. It does not seem to be an action authorized by any of the worksheet instructions.

On completing the last line on the whiteboard, Bwang returns to the chat panel and types, "so you can see we only need to figure one out to get the total stick" (post 58). Chat posts are often not constructed grammatically as complete sentences, but consist instead of clausal units that must be re-composed by the reader to produce coherent utterances. This practice of building up utterances in installments allows readers to more closely monitor utterances in construction and increases interactivity (Garcia \& Jacobs, 1999). Bwang's post, therefore, is read as part of an utterance in progress. The
concatenated message, therefore, reads, "so you can see we only need to figure one out to get the total stick $1+2+3+\ldots . . . .+N+N$ times that by 2 ".

Bwang, through his actions, has cast the group's task as one of producing certain general formulas related to the generated patterns. Aznx's response, "Can we collaborate this answer even more? To make it even simpler?" (posts $63 \& 64$ ) is not clear. It is odd to find collaborate used as a transitive verb and there are ambiguities of meaning. Is he referring to the problem, which could potentially be further decomposed and clarified, or to Bwang's algebraic formulation? By labeling Bwang's contributions as "this answer", Aznx (post 63) implicitly endorses it as a candidate solution to the task at hand. It is, in fact, the first place in which Bwang's presentation is treated as such.

Bwang responds to Aznx's query by providing an algebraically restructured version of the right side of the formula (" $(1+\mathrm{N}) * \mathrm{~N} / 2+\mathrm{N}) * 2$ ", post 67 ). The rapidity with which this was produced would seem to allow little or no time for derivation, suggesting that the revised formula might have already been known before Aznx asked for it. In the posts that followed (69 to 85), the team continued to discuss the components of the developed formula. Bwang introduced a second formula for computing the number of squares (post 82). It is, in fact, just the simple summation function. It is possible, though we have no way to know, that Bwang first recognized that the 'squares' series was based on a simple summation function and then extended this insight to produce the more elaborate formula for generating the 'sticks' series. Aznx's "so how do we submit this" (post 85) is closure implicative. His this casts a broad net over the whole approach developed by Bwang.

## IV. Referential, Mathematical and Technological Practices

We would now like to make certain general observations about the practices on display here. They evidence manifold competencies with regard to discourse production, mathematics and technology use. Bwang's elegant presentation of a prospective solution begins with a proleptic reference to "dividing this thing into two parts." It is not, however, until we get to the end of the presentation that we discover that "the thing" is not just the third figure in the worksheet, but a general formula for describing the patterns seen both in the set of figures and in the summarizing table. It
is in the ways in which the functional description itself is revealed that we see the most profound evidence of mathematical practice.

Bwang's presentation of a prospective solution exhibits the properties of a derviation of sorts. It proceeds in logical steps that lead eventually to a known conclusion. The presentation had three parts: a graphical derivation, an informal formulaic presentation and, finally, a more conventional algebraic formulation.

The graphic presentation proceeded in four stages: [1] drawing 6 vertical lines, [2] drawing 6 horizontal lines, [3] drawing 3 vertical lines and, [4] producing 3 horizontal lines (see Fig. 2). The first six lines represent an application of the summation function, and the second six, a second application of the same function. But, the two subfigures are plainly incomplete-they are both missing an outer wall. The number of sticks needed to compete the subfigures is, in both cases, 3, which happens to be N. The final line count is: $(6+3)+(6+3)=18$. Like a mathematician's boardwork (c.f., Greiffenhagen, 2008), Bwang's presentation makes visible just how the 'sticks' series is generated. Had he chosen to present the case for $\mathrm{N}=4$, as the first instruction step required, he and his audience would not have had a way to confirm the result. A feature of his demonstration was that the total number of lines produced could be checked against the value provided in the table. ${ }^{3}$

Bwang's "So you can see" (post 58) announces a derivation complete. Raymond (2004) described how "Speakers regularly use 'so' prefaced turn constructional units that articulate the upshot of prior talk to mark the completion of complex turns or activities" (p. 186). In this case, Bwang's opening bridges back, not to prior talk, but to what he had done on the whiteboard. The upshot is presented as already visible for all to see, but just what are we to see?

Like his previous "you can divide the thing into two parts," Bwang's "we only need to figur one out" is is rife with referential puzzles. One what? If the "thing" mentioned in his earlier post has now been divided in two, then each subfigure might be a

[^2]candidate, but just what are we 'figuring out'? This is clarified when we get to the end of this long post and are informed that the object is "to get the total stick."

With this information in hand, the viewer can turn to the "stick" column in the table from the "View Topics" page, extract the entry for $\mathrm{N}=3$ and check it against the count of sticks drawn on the whiteboard. This last part of the post, when concatenated with the two subsequent posts can be read as an informally presented equation (i.e., sticks $(N)=$ $(1+2+3+\ldots+N+N) 2)$. The first post informally presents the left-hand side of a functional equation; the second summarizes the two-stage production of each of the subfigures (i.e., $\Sigma_{i=1 \text { to }}(i)+N=6+3$ ); the last doubles the resultant obtained from the post before. It functions as if one had taken the previous post, wrapped it in parentheses and then multiplied it by two, in effect summing the two subfigures. Note that in presenting the formula in just this way, it recapitulates the demonstration from the whiteboard. One might envision how a mathematician might produce these two representations at a physical blackboard. Here Bwang's demonstration had to be adapted to fit the circumstances, but this was done seamlessly using the affordances of the VMT environment. Note, for example, the ways in which he was able to animate his graphical derivation on the whiteboard and was subsequently able to exploit the conventions of chat interaction to sequentially build his functional description.

We indicated earlier that Bwang's presentation exhibited the properties of an informal derivation in that it leads stepwise to a given conclusion. The known conclusion in this case is a stick (line) count of 18 . The force of the demonstration, therefore, rests crucially on his audience recognizing that he is working with the third case from the worksheet for which the number of sticks and squares are known. He never indicates this in as many words, but he makes it clear in the way that he begins his drawing. He began his illustration with a figural quote of the third example from the worksheet (see Figure 4).

Derivations only provide a gloss for the steps needed to reach the conclusion, each of the steps being incompletely specified. Decisions must be made at every turn as to how much specification is required. Bwang, in his three presentations of the formula, leaves certain aspects of the respective formulations to be worked out by his audience. This is a way in which the discourse is organized to display mathematical competenceBwang treats his teammates as mathematically competent in his choices of what to
make explicit and what to make implicit. It is also seen in his concluding tag line when he asks, "that's the formula, right?" (posts 68) in which he presents the formula as understood and his audience as competent to evaluate it.

We don't need a post-test to understand how the formula was to be understoodthe understanding was made concrete in the participants' actions. In the chat log in Appendix B one can study the ways in which the participants present matters for understanding to each other, build collaboratively on each others' actions and analyze each others' references. We observe them working to make sense of the scene before them populated with chat postings, whiteboard objects, and wiki entries. They can be viewed throughout to engage in a form of analysis. Our analysis here has focused upon the ways in which Bwang made his formula intelligible for the other participants. His practices for doing so provided for the analyzability of his actions. Participants' work of endogenous analysis has received little attention in prior studies of collaborative learning. We would argue, however, that these are the practices of greatest relevance for study in CSCL.

## References

Çakir, M. P., Zemel, A., \& Stahl, G. (2009). The joint organization of interaction within a multimodal CSCL medium. International Journal of Computer-Supported Colalborative Learning, 4, 115-149.
Garcia, A., \& Jacobs, J. (1999). The eyes of the beholder: Understanding the turn-taking system in quasi-synchronous computer-mediated communication. Research on Language and Social Interaction, 32, 337-368.
Garfinkel, H. (1963). A conception of, and experiments with, 'Trust' as a condition of stable concerted actions. In O. J. Harvey (Ed.), Motivation and social interaction (pp. 187-238). New York: Ronald Press.

Greiffenhagen, C. (2008). Video analysis of mathematical practice? Different attempts to 'open up' mathematics for sociological investigation. Forum Qualitative Sozialforschung, 9(3).
Macbeth, D. (2000). On an actual apparatus for conceptual change. Science Education, 84, 228-264.
Sacks, H. (1992). Lectures on conversation. Oxford, U.K.: Blackwell.
Livingston, E. (2006). The context of proving. Social Studies of Science, 36, 39-68.
Lynch, M. (2001). Ethnomethodology and the logic of practice. In T. R. Schatzki, K. K. Cetina \& E. von Savigny (Eds.), The practice turn in contemporary theory (pp. 131148). London: Routledge.

Medina, R., Suthers, D., \& Vatrapu, R. (forthcoming). Representational practices in VMT. To appear in G. Stahl (Ed.), Studying virtual math teams. New York: Springer.

Raymond, G. (2004). Prompting action: The stand-alone "So" In ordinary conversation. Research on Language and Social Interaction, 37, 185-218.

Schatzki, T. R. (2001). Practice theory. In T. R. Schatzki, K. K. Cetina \& E. von Savigny (Eds.), The practice turn in contemporary theory (pp. 1-14). London: Routledge.

Stahl, G. (forthcoming). Meaning making in VMT. To appear in G. Stahl (Ed.), Studying virtual math teams. New York: Springer.

Zemel, A., \& Çakir, M. P. (forthcoming). Reading's work in VMT. To appear in G. Stahl (Ed.), Studying virtual math teams. New York: Springer.

## Figure Captions

Figure 1. Screen image of the VMT interface.

Figure 2. Bwang's re-construction of the third figure from the worksheet.

Figure 3. Bwang's informal equation composed in three posts.

Figure 4. Bwang's figural quote of the $\mathrm{N}=3$ case from the worksheet.


Figure 1

1. $2 . \quad 3$.

2. 


5.
8.

11.

14.

16.



17. | 1 | - |  |
| :--- | :--- | :--- |
| 1 | - | - |
| 1 | 1 | - |
18. through 6.
Construction of the
first object 1. through 6.
Construction of the
first object 1. through 6.
Construction of the
first object
19. 
20. 111
21. 

$\qquad$
12.

15. $\left.\left.\right|_{1}\right|_{1} \left\lvert\, \begin{aligned} & \text { ————} \\ & 1\end{aligned}\right.$
18.

13. through 18. Completion of the first and second objects

Figure 2

```
post 58
```

bwang8: to get the total stick
$\operatorname{stick}(N)=$

Figure 3(a)

## post 60

bwang8: $1+2+3+\ldots .+N+N$

$$
\operatorname{stick}(N)=1+2+3+\ldots+N+N
$$

Figure 3(b)

```
post 61
bwang8: times that by 2
```

$$
\operatorname{stick}(N)=(1+2+3+\ldots+N+N) 2
$$

Figure 3(c)


Figure 4

## VMT Spring Fest

Here are the first few examples of a particular pattern or sequence, which is made using sticks to form connected squares:


## Session I

1. Draw the pattern for $\mathrm{N}=4, \mathrm{~N}=5$, and $\mathrm{N}=6$ in the whiteboard. Discuss as a group: How does the graphic pattern grow?
2. Fill in the cells of the table for sticks and squares in rows $\mathrm{N}=4, \mathrm{~N}=5$, and $\mathrm{N}=6$. Once you agree on these results, post them on the VMT Wiki
3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

Appendix B: Chat Log

| \# | chat handle | chat posting or whiteboard action | initiate | complete |
| :---: | :---: | :---: | :---: | :---: |
| 40 | Gerry | You can click on the button at the top that says "View Topic" to see the math problem | 18:28:45 | 18:29:12 |
| 41 | bwang8 | ok | 18:29:32 | 18:29:32 |
| 42 | bwang8 | are we suppose to solve it now? | 18:29:33 | 18:29:50 |
| 43 | Gerry | Then you can click on the button in the little window that appears to open the topic in another big growser window | 18:29:29 | 18:30:13 |
| 44 | Gerry | browser* | 18:30:32 | 18:30:36 |
| 45 | Aznx | It didn't open. | 18:30:33 | 18:30:40 |
| 46 | Aznx | Now it did. | 18:30:50 | 18:30:52 |
| 47 | Aznx | So, are we supposed to work together? | 18:31:25 | 18:31:32 |
|  | bwang8 | ((Initiates a chat message but deletes without posting)) | 18:31:25 | 18:31:45 |
| 48 | bwang8 | yeah | 18:31:48 | 18:31:49 |
| 49 | bwang8 | ok | 18:31:50 | 18:31:50 |
| 50 | Gerry | Exactly! | 18:31:51 | 18:31:54 |
|  | Quicksilver | ((Initiates a chat message but deletes without posting)) | 18:31:51 | 18:31:57 |
| 51 | Aznx | ((Quicksilver's given name)), you there? | 18:31:59 | 18:32:04 |
| 52 | bwang8 | you can divide the thing into two parts | 18:31:52 | 18:32:05 |
|  | bwang8 | ((Created a line on whiteboard )) |  | 18:32:09 |
| 53 | Aznx | Let's start this thing. | 18:32:06 | 18:32:10 |
|  | bwang8 | ((Creates line objects on whiteboard )) | 18:32:11 | 18:32:38 |
| 54 | Quicksilver | my computer was lagging...What are we doing? | 18:32:30 | 18:32:38 |
| 55 | Aznx | http://home.old.mathforum.org/SFest.html | 18:32:48 | 18:32:49 |
| 56 | Quicksilver | what are the lines for? | 18:32:52 | 18:32:58 |
| 57 | Aznx | go to view topic | 18:32:57 | 18:33:01 |
| 58 | bwang8 | so you can see we only need to figur one out to get the total stick | 18:32:42 | 18:33:05 |
| 59 | Aznx | read the problem | 18:33:04 | 18:33:09 |
| 60 | bwang8 | 1+2+3+....... $+\mathrm{N}+\mathrm{N}$ | 18:33:08 | 18:33:32 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:33:33 | 18:33:36 |

Appendix B: Chat Log

| 61 | bwang8 | times that by 2 | 18:33:34 | 18:33:38 |
| :---: | :---: | :---: | :---: | :---: |
| 62 | Quicksilver | Never mind I figured it out.. | 18:33:35 | 18:33:40 |
|  | bwang8 | ((Initiates a chat message but deletes without posting)) | 18:33:41 | 18:33:43 |
| 63 | Aznx | Can we collaborate this answer even more? | 18:33:51 | 18:34:01 |
| 64 | Aznx | To make it even simpler? | 18:34:02 | 18:34:05 |
| 65 | bwang8 | ok | 18:34:14 | 18:34:15 |
| 66 | Aznx | Because I think we can. | 18:34:07 | 18:34:16 |
| 67 | bwang8 | $((1+\mathrm{N}) * \mathrm{~N} / 2+\mathrm{N})^{*} 2$ | 18:34:25 | 18:34:50 |
| 68 | bwang8 | that's the formula, right? | 18:34:52 | 18:34:58 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:34:43 | 18:35:05 |
| 69 | Aznx | How did you come up with it? | 18:35:09 | 18:35:15 |
| 70 | bwang8 | for total sticks | 18:35:12 | 18:35:16 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:35:17 | 18:35:19 |
|  | bwang8 | ((Initiates a chat message but deletes without posting)) | 18:35:25 | 18:35:26 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:35:20 | 18:35:28 |
| 71 | bwang8 | is a common formual | 18:35:27 | 18:35:34 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:35:29 | 18:35:35 |
| 72 | bwang8 | formula | 18:35:38 | 18:35:40 |
| 73 | Aznx | Yeah, I know. | 18:35:43 | 18:35:46 |
| 74 | bwang8 | and just slightly modify it to get this | 18:35:45 | 18:35:59 |
|  | bwang8 | ((Deletess some objects on whiteboard )) | 18:36:15 | 18:36:27 |
|  | bwang8 | ((Creates a line on whiteboard )) |  | 18:36:31 |
| 75 | Aznx | Aditya, you get this right? | 18:36:27 | 18:36:31 |
|  | bwang8 | ((Creates some lines on whiteboard )) | 18:36:32 | 18:36:37 |
|  | Quicksilver | ((Initiates a chat message but deletes without posting)) | 18:36:35 | 18:36:39 |
|  | bwang8 | ((Creates some lines on whiteboard )) | 18:36:39 | 18:36:43 |

Appendix B: Chat Log

|  | Quicksilver | ((Moves some objects on whiteboard )) | 18:36:44 | 18:37:05 |
| :---: | :---: | :---: | :---: | :---: |
|  | Gerry | START:TextEditing |  | 18:37:44 |
| 76 | Quicksilver | What does the n represent? | 18:37:39 | 18:37:45 |
|  | bwang8 | ((Initiates a chat message but deletes without posting)) | 18:37:35 | 18:37:52 |
| 77 | bwang8 | the given | 18:37:54 | 18:37:57 |
| 78 | bwang8 | N | 18:37:58 | 18:37:58 |
| 79 | Aznx | Yeah. | 18:38:00 | 18:38:02 |
|  | bwang8 | ((Initiates a chat message but deletes without posting)) | 18:38:02 | 18:38:04 |
| 80 | Aznx | In the problem. | 18:38:03 | 18:38:05 |
|  | Gerry | END:TextEditing |  | 18:38:13 |
|  | Gerry | ((Creates a textbox on whiteboard )) |  | 18:38:13 |
|  | Gerry | ((Resizes some objects on whiteboard )) |  | 18:38:17 |
|  | Quicksilver | ((Resizes some objects on whiteboard )) |  | 18:38:20 |
|  | Aznx | ((Initiates a chat message but deletes without posting)) | 18:38:10 | 18:38:21 |
| 81 | Aznx | Oh | 18:38:36 | 18:38:37 |
| 82 | bwang8 | The number of squares is just (1+N)*N/2 | 18:38:06 | 18:38:38 |
|  | Aznx | ((Creates some lines on whiteboard )) | 18:38:48 | 18:38:44 |
| 83 | Quicksilver | We need that as well. | 18:38:46 | 18:38:50 |
|  | Aznx | ((Creates a line on whiteboard )) |  | 18:38:51 |
| 84 | Gerry | I put BWang's formula on the whiteboard | 18:38:36 | 18:38:52 |
|  | Aznx | ((Creates a line on whiteboard )) |  | 18:38:55 |
|  | Aznx | START:TextEditing |  | 18:39:04 |
|  | Aznx | END:TextEditing |  | 18:39:19 |
|  | Aznx | ((Creates a textbox on whiteboard)) |  | 18:39:19 |
|  | Aznx | ((Moves some objects on whiteboard)) |  | 18:39:28 |
| 85 | Aznx | So how do we submit this? | 18:39:39 | 18:39:45 |


[^0]:    ${ }^{1}$ It might be worth noting that the three co-authors conducted all but one of the research meetings to plan this report in the same environment and using the same tools as the participants.

[^1]:    2 If the message is not posted, the interval is marked in Appendix B as "Initiates a chat message but deletes without posting."

[^2]:    ${ }^{3}$ There is probably more that could be said here. There is something about the selection of the $\mathrm{N}=3$ case which gives enough scope for development of the more generic understanding that Bwang is seeking to achieve. In part, one could attribute the selection of the N=3 case to the peculiar affordances of the whiteboard demonstration. For $\mathrm{N}<3$, the "four stage" presentation would not have been as effective or could have led to confusion on the part of others witnessing the construction. Because whiteboard actions cannot be narrated like traditional mathematical boardwork (cf., Greiffenhagen, 2008), running through $\mathrm{N}=1$ and $\mathrm{N}=2$ before getting to $\mathrm{N}=3$ would have been time consuming and pedantic.

