Chapter 7

The Organization of Graphical, Narrative and Symbolic Interactions

Murat Perit Çakir

MPC48@Drexel.edu

Abstract: In order to collaborate effectively in group discourse on a topic like mathematical patterns, group participants must organize their activities so that they have a shared understanding of the significance of their utterances, inscriptions and behaviors—adequate for sustaining productive interaction. Some methodologies applied in CSCL research—such as the widespread coding-and-counting quantitative analysis genre—systematically ignore the sequentiality of actions and thereby miss the implicit referencing, which is essential to shared understanding. The VMT Project attempts to capture and analyze the sequential organization of references and inter-relationships among whiteboard inscriptions, chat postings, mathematical expressions and other elements of virtual math team activities in order to understand the mechanisms of group cognition.

Here, we report the results of a micro-ethnographic case study of collaborative math problem-solving activities mediated by the VMT multimodal online environment. We employ ethnomethodological conversation analysis techniques to investigate moment-to-moment details of the interaction practices through which participants organize their chat utterances and whiteboard actions as a coherent whole. In particular, we observe that the sequential construction of shared drawings and the deictic references that link chat messages to features of those drawings and to prior chat content are instrumental in the achievement of shared understanding (intersubjective reciprocity) among the team members. We characterize this foundational precondition of collaboration as the co-construction of an indexical field that functions as a common ground for group cognition. The integration of graphical, narrative and symbolic semiotic modalities in this manner also facilitates joint problem solving by allowing group members to invoke and operate with multiple realizations of their mathematical artifacts, a characteristic of deep learning of math.

Keywords: Dual-interaction space, multimodal interaction, unit of analysis, persistence, animation, reference

Computer-supported collaborative learning is centrally concerned with the joint organization of interaction by small groups of students in online environments. The term "collaborative learning" is a gloss for *interaction that is organized for the joint achievement of knowledge-building tasks* such as problem solving in domains like school mathematics. Rather than using the term "collaborative learning," which carries vague and contradictory connotations, we prefer the term "group cognition" to refer to activities where several students organize their joint interaction to achieve such collective cognitive accomplishments as planning, deducing, designing, describing, problem solving, explaining, defining, generalizing, representing, remembering and reflecting as a group.

In this chapter, we present a case study of an 18-minute-long excerpt from the VMT Spring Fest 2006. We look at some ways in which the students organized their joint efforts. Our observations here are consistent with our impressions from more than a hundred student-hours of interaction in the VMT data corpus.

The issue that we address in the following pages is: *How do the students in our case study organize their activity so they can define and accomplish their tasks as a group within their online environment?* This is necessarily a pivotal question for a science of CSCL (see Chapter 28). It involves issues of meaning making, shared understanding and common ground that have long been controversial in CSCL.

The problem of coordination is particularly salient in the VMT software environment, which is an instance of a dual-interaction space (Dillenbourg, 2005)(see also Chapter 15), requiring organization across multiple media, each with their own affordances. We have found that the key to joint coordination of knowledge building is sequential organization of a network of indexical and semantic references within the group discourse (see Chapter 26). We therefore analyze sequential interaction at the group level of description, using ethnomethodologically inspired chat interaction analysis rather than quantitative coding, in order to maintain and study this sequential organization. Thereby, we arrive at a view of mathematical knowledge building as the coordination of visual, narrative and symbolic inscriptions as multiple realizations of co-constructed mathematical objects.

While we have elsewhere presented theoretical motivations for focusing on *group discourse organization* as fundamental for CSCL, in this chapter we foreground our *analysis of empirical data* from a VMT session. We derive a number of characteristics of the joint organization of interaction from the details of the case study. The characteristics we describe are to some extent specific to the technological affordances of the VMT environment, to the pedagogical framing of the chat session and even to the unique trajectory of this particular group interaction. Nevertheless, the characteristics are indicative of what takes place—with variations—in similar settings. After the analytic centerpiece of the chapter, we discuss *methodological implications* for CSCL analysis, including what it means to take the *group* as the unit of analysis. We then contrast our approach to leading *alternative approaches* in CSCL. This discussion focuses particularly on multi-modal interaction in a *dual-interaction space* and on related conceptions of *common ground*, concluding with summary remarks on *sequential analysis*. The chapter proceeds through the following topics:

- The problem of group organization in CSCL
- A case study of a virtual math team
- Implications for CSCL chat interaction analysis
- The group as the unit of analysis
- Other approaches in CSCL to analyzing multimodal interaction
- Grounding through interactional organization
- Sequential analysis of the joint organization of interaction

The Problem of Group Organization in CSCL

A central issue in the theory of collaborative learning is how students can solve problems, build knowledge, accomplish educational tasks and achieve other cognitive accomplishments *together*. How do they share ideas and talk about the same things? How do they know that they are talking about, thinking about, understanding and working on things in the same way? Within CSCL, this has been referred to as the problem of the "attempt to construct and maintain a shared conception of a problem" (Roschelle & Teasley, 1995), "building common ground" (Baker et al., 1999; Clark & Brennan, 1991) or "the practices of meaning making" (Koschmann, 2002). We have been interested in this issue for some time. *Group Cognition* (Stahl, 2006) documents a decade of background to the VMT research reported here: its Chapter 10 (written in 2001) argued the need for a new approach. It pointed out that some CSCL methods of analysis—which reduce subtle networks of linguistic interaction to counts of codes—reify the flow of discourse and miss the temporal structure that is important for understanding the meaning making.

Knowledge building in CSCL has traditionally been supported primarily with asynchronous technologies (Scardamalia & Bereiter, 1996). Within appropriate educational cultures, this can be effective for long-term refinement of ideas by learning communities. However, in small groups and in many classrooms, asynchronous media encourage mere exchange of individual opinions more than co-construction of progressive trains of joint thought. We have found informally that synchronous interaction can more effectively promote group cognition—the accomplishment of "higher order" cognitive tasks through the coordination of contributions by individuals within the discourse of a small group. We believe that the case study in this chapter demonstrates the power of group interaction in a largely synchronous environment; the coordination of interaction in an asynchronous interaction would be quite different in nature as a result of very different interactional constraints.

In CSCL settings, interaction is mediated by a computer environment. Students working in such a setting must enact, adapt or invent ways of coordinating their understandings by means of the technological affordances that they find at hand (Dohn, 2009). The development and deployment of these methods is not usually an explicit, rational process that is easily articulated by either the participants or analysts. It occurs tacitly, unnoticed, taken-for-granted. In order to make it more visible to us as analysts, we have developed an environment that makes the coordination of interaction more salient and captures a complete record of the group interaction for detailed analysis. In trying to support online math problem solving by small groups, we have found it important to provide media for both linguistic and graphical expression. This resulted in what is known within CSCL as a *dual-interaction space*. In our environment, students must coordinate their text chat postings with their whiteboard drawings. A careful analysis of how they do this reveals as well their more general methods of group organization.

The analysis of our case study focuses on episodes of interaction through which an online group of students co-constructs mathematical artifacts across dual-interaction spaces. It looks closely at how group members put the multiple modalities into use, how they make their chat postings and drawing actions intelligible to each other, and how they achieve a sense of coherence among actions taking place across the modalities to which they have access. We base our discussion, analysis and design of the affordances of the online environment on the methodical ways the features of the software are put into use by the students.

In Chapter 6 above, we saw how the problem-solving work of a virtual math team is accomplished through the co-construction and maintenance of a *joint problem space* (Teasley & Roschelle, 1993). This figurative space—that supports group interaction and the shared understanding of that interaction by the participants—not only grounds the *content* of the team's discourse and work, but also ties together the *social* fabric of the relations among the team members as actors. In addition, we saw that the joint problem space has a third essential dimension: *time* or sequence. The construction of the joint problem space constitutes a shared temporality through bridging moves that span and thereby order discontinuous events as past, present and future (Sarmiento-Klapper, 2009). This can be seen, for instance in the use of tenses in group-remembering discourses. More generally, the joint problem space provides a framework of sequential orderings, within which temporal deictic references, for example, can be resolved.

In this chapter, we further investigate how a virtual math team achieves a group organization of its activities such that the group can proceed with a sense of everyone understanding each other and of working collaboratively as a group. We do this through a fine-grained analysis of the group's interaction in a VMT session in which they formulate, explore and solve a geometry problem. Their work takes place in graphical, narrative and symbolic media—supported technologically by the shared whiteboard, text chat and wiki pages of the VMT environment. We pay particular attention to how graphical inscriptions, textual postings and symbolic expressions in the different media are closely coordinated by the group members, despite the differences of the media.

We pursue a micro-ethnographic approach to analyzing the activities of the group members in their own terms. They set themselves a task, propose how to proceed step by step and explain to each other how to understand their actions. We try to follow the explanations, which are available in the inscriptions, postings and expressions—particularly when the sequentiality of these allows the complex references among them to be followed.

The establishment of group order in small-group interaction is always strongly dependent upon the media, which mediate interaction. In the case of VMT chats, there is an intricate set of technological media, including text chat, a shared whiteboard, a community wiki and graphical references from chat to whiteboard. The central part of this chapter explores the different characteristics of the VMT media by observing how the students use them. Of particular interest are the ways in which a group coordinates activities in the different graphical and textual media. From a math-education perspective, it is also insightful to see how the visual and narrative understandings feed into the development and understanding of symbolic expressions.

By the end of the chapter, we will see how the group organization of graphical, narrative and symbolic interactions continuously produce the joint problem space of the group's effort. This coordination is revealed through sequential analysis, in which the consequence of one action in one medium following another in another medium is seen as mutually constitutive of the meaning of those actions. The sequential web of activity across the VMT media—woven by semantic and indexical references among them—forms the joint problem space within which problem content, participant relationships and temporal progress are all defined in a way that is shared by the group. We can see the "indexical field" formed by the group activities as the source of grounding that supports the intersubjectivity of the group effort. In contrast to psychological or psycholinguistic models of common ground, the fact that team members believe they have understandings in common about what each other is saying and doing is not a result of exchanging individual mental opinions, but is a function of the indexical organization of the group interaction.

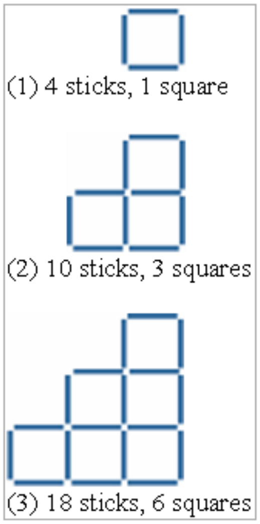
The joint problem space—as the foundation of group cognition—is not a mental construct of a set of individuals who achieve cognitive convergence or common (identical) ground through comparing mental models anymore than it is a figment of some form of group mind. Rather, it is a system of interconnected meanings formed by a weaving of references in the group discourse itself (Chapter 26). In this chapter, we analyze the methods the students used to co-construct this indexical field.

In our case study, the organization of group meaning making takes place across media—in accordance with the specific affordances of the different media. Furthermore, the grounding of the students' symbolic mathematical understanding can be seen as related to their visual and narrative understandings—or, rather, the various understandings are intricately interwoven and support each other. We trace this interweaving through our approach to the interactional analysis of sequential coordination at the group unit of analysis.

A Case Study of a Virtual Math Team

The excerpts we present in this chapter are obtained from a problem-solving session of a team of three students who participated in the VMT Spring Fest 2006. This event brought together several teams from the US, Scotland and Singapore to collaborate on an open-ended math task on geometric patterns. Students were recruited anonymously through their teachers. Members of the teams generally did not know each other before the first session. Neither they nor we knew anything about each other (e.g., age or gender) except chat handle and information that may have been communicated during the sessions. Each group participated in four sessions during a two-week period, and each session lasted over an hour. An adult from the research project moderated each session; the facilitators' task was to help the teams when they experienced technical difficulties, not to participate in the problem-solving work.

During their first session, all the teams were asked to work online on a particular pattern of squares made up of sticks (see Figure 7-1). For the remaining three sessions the teams were asked to come up with their own shapes, describe the patterns they observed as mathematical formulas, and share their observations with other teams through a wiki page. This task was chosen because of the possibilities it afforded for many different solution approaches ranging from simple counting procedures to more advanced methods involving the use of recursive functions and exploring the properties of various number sequences. Moreover, the task had both algebraic and geometric aspects, to allow us to observe how participants put many features of the VMT software system into use. The open-ended nature of the activity stemmed from the need to agree upon a new shape made by sticks. This required groups to engage in an open-ended problem-solving activity, as compared to traditional situations where questions are given in advance and there is a single "correct" answer—presumably already known by a teacher. We used a traditional pattern problem (Moss & Beatty, 2006; Watson & Mason, 2005) to seed the activity and then left it up to each group to decide the kinds of shapes they found interesting and worth exploring further.



N	Sticks	Squares
1	4	1
2	10	3
3	18	6
4	?	?
5	?	?
6	?	?
...	...	...
N	?	?

Session I

1. Draw the pattern for N=4, N=5, and N=6 in the whiteboard. Discuss as a group: How does the graphic pattern grow?

2. Fill in the cells of the table for sticks and squares in rows N=4, N=5, and N=6. Once you agree on these results, post them on the VMT Wiki

3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

Sessions II and III

1. Discuss the feedback that you received about your previous session.

2. WHAT IF? Mathematicians do not just solve other people's problems — they also explore little worlds of patterns that they define and find interesting. Think about other mathematical problems related to the problem with the sticks. For instance, consider other arrangements of squares in addition to the triangle arrangement (diamond, cross, etc.). What if instead of squares you use other polygons like triangles, hexagons, etc.? Which polygons work well for building patterns like this? How about 3-D figures, like cubes with edges, sides and cubes? What are the different methods (induction, series, recursion, graphing, tables, etc.) you can use to analyze these different patterns?

3. Go to the VMT Wiki and share the most interesting math problems that your group chose to work on.

Figure 7-1. Task description.

All the problem-solving sessions were conducted in the VMT environment. The VMT online system has two main interactive components that conform to the typical layout of systems with dual-interaction spaces: a shared drawing board that provides basic drawing features on the left, and a chat window on the right (Figure 7-2). The online environment has features specifically designed to help users relate the actions happening across dual-interaction spaces (Chapter 15). One of the unique features of this chat system is the referencing support mechanism that allows users to visually connect their chat postings to previous postings or objects on the whiteboard via arrows (see the last posting in Figure 7-2 for an example of a message-to-whiteboard reference). The referential links attached to a message are displayed until a new message is posted. Messages with referential links are indicated by an arrow icon in the chat window, and a user can see where such a message is pointing by clicking on it at any time.

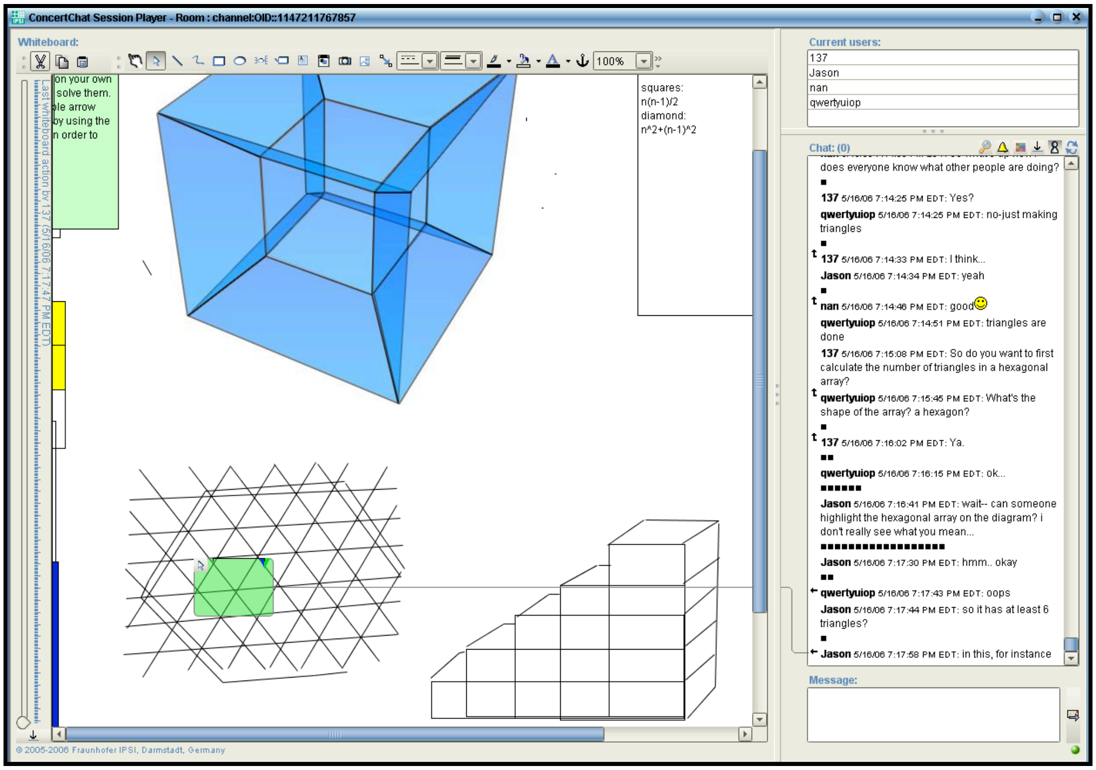


Figure 7-2. A screen-shot of the VMT environment.

In addition to the explicit referencing feature, the system displays small boxes in the chat window to indicate actions performed on the whiteboard. This awareness mechanism allows users to observe how actions performed in both interaction spaces are sequenced with respect to each other. Moreover, users can click on these boxes to move the whiteboard back and forth from its current state to the specific point in its history when that action was performed. Chat messages and activity markers are color coded to help users to keep track of who is doing what in the online environment. In addition to standard awareness markers that display who is present in the room and who is currently typing, the system also displays textual descriptions of whiteboard actions in tool-tip messages that can be observed by holding the mouse either on the object in the whiteboard or on the corresponding square in the chat window.

Studying the meaning-making practices enacted by the users of CSCL systems inevitably requires a close analysis of the process of collaboration itself (Dillenbourg et al., 1996; Stahl, Koschmann & Suthers, 2006). In an effort to investigate the organization of interactions across the dual-interaction spaces of the VMT environment, we consider the small group as the unit of analysis (Stahl, 2006), and we appropriate methods of ethnomethodology and conversation analysis to conduct sequential analysis of group interactions at a micro-level (Psathas, 1995; Sacks, 1962/1995; ten Have, 1999). Our work is informed by studies of interaction mediated by online text-chat with similar methods (Garcia & Jacobs, 1998; 1999; O'Neill & Martin, 2003), although the availability of a shared drawing area and explicit support for deictic references in our online environment substantially differentiate our study from theirs.

The goal of this line of analytic work is to discover the commonsense understandings and procedures group members use to organize their conduct in particular interactional settings (Coulon, 1995). Commonsense understandings and procedures are subjected to analytical scrutiny because they are what "enable actors to recognize and act on their real world circumstances, grasp the intentions and motivations of others, and achieve mutual understandings" (Goodwin & Heritage, 1990, p. 285). Group members' shared competencies in organizing their conduct not only allow them to produce their own actions, but also to interpret the actions of others (Garfinkel & Sacks, 1970). Since group members enact these understandings visibly in their situated actions, researchers can discover them through detailed analysis of the members' sequentially organized conduct (Schegloff & Sacks, 1973).

We conducted numerous VMT Project data sessions, where we subjected our analysis of the excerpts below to intersubjective agreement (Psathas, 1995). This chapter presents the outcome of this group effort together with the actual transcripts so that the analysis can be subjected to external scrutiny. During the data sessions we used the VMT Replayer tool, which allows us to replay a VMT chat session as it unfolded in real time based on the timestamps of actions recorded in the log file. The order of actions—chat postings, whiteboard actions, awareness messages—we observe with the Replayer as researchers exactly matches the order of actions originally observed by the users. This property of the Replayer allowed us to study the sequential unfolding of events during the entire chat session, which is crucial in making sense of the complex interactions mediated by a CSCL environment (Koschmann, Stahl & Zemel, 2007).

In this case study, we focus on a sequence of excerpts obtained from a single problem-solving session of a virtual math team. We are concerned with how the actors contribute to the group meaning making as they proceed. This example involves the use and coordination of actions involving both the whiteboard and chat environment. It therefore served as a useful site for seeing how actors, in this local setting, were able to engage in meaningful coordinated interaction.

The team has three members: Jason, 137 and Qwertyuiop, who are upper-middle-school students (roughly 14 years old) in the US. In the following subsections we will present how this team co-constructed a mathematical artifact they referred to as the "**hexagonal array**" through a coordinated sequence of actions distributed between the chat and whiteboard spaces, and how they subsequently explored its properties by referring to and annotating shared drawings on the whiteboard. In particular, we will highlight how whiteboard objects and previous chat postings were used as semiotic resources during the collaborative problem-solving activity. This will show how chat and whiteboard differ in terms of their affordances for supporting group interaction. We will see how these differences are enacted and used in complementary ways by team members to achieve mutual intelligibility of their actions across multiple interaction spaces.

Availability of the Production Processes

Log 7-1 is taken from the beginning of the team's third session. The team has already explored similar patterns of sticks and become familiar with the features of the VMT online environment during their prior sessions. The drawing actions at the beginning of this excerpt were the first moves of the session related to math problem solving.

Log 7-1

Line	Time	Chat handle	Chat message or <*whiteboard action*>
	7:07:52 - 7:11:00	137	<*137 draws a hexagon shape and then splits it up into regions by adding lines. Figure 7-3 shows some of the key steps in 137's drawing performance*>
1	7:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?
	7:11:16 - 7:11:49	137	<*137 deletes the set of lines he has just drawn*>
2	7:11:51	Qwertyuiop	just a grid?....
	7:11:54 - 7:12:01	137	<*137 moves some of the older drawings away*>
3	7:12:07	137	Yeah...
4	7:12:17	Qwertyuiop	ok…
	7:12:23 - 7:14:07	Qwertyuiop	<Qwertyuiop draws a grid of triangles in the space opened up by 137. Figure 7-4 shows some of the steps in Qwertyuiop's drawing actions>

At the beginning of this excerpt, 137 performs a series of drawing actions. 137's actions on the whiteboard include the drawing of a hexagon first, then three diagonal lines and finally lines parallel to the diagonals and to the sides of the hexagon whose intersections eventually introduce some triangular and diamond-shaped regions. Moreover, 137 also performs some adjustment moves—for instance between the 4^{th} and 5^{th} snapshots in Figure 7-3—to ensure that three non-parallel lines intersect at a single point, and the edges of the hexagon are parallel to the lines introduced later as much as possible. Hence, this sequence of drawing actions suggests a particular organization of lines for constructing a hexagonal shape. (Figure 7-3 shows six snapshots corresponding to intermediary stages of 137's drawing actions: 137 initiates his drawing actions with six lines that form the hexagon in stage 1. Then he adds three diagonal lines in step 2. The 3^{rd} snapshot shows the additional two lines drawn parallel to one of the diagonals. The 4^{th} snapshot shows a similar set of two parallel lines added with respect to another diagonal. The 5^{th} snapshot shows slight modifications performed on the new set of parallel lines to ensure intersections at certain places. The 6^{th} snapshot shows the final stage of 137's drawing.)

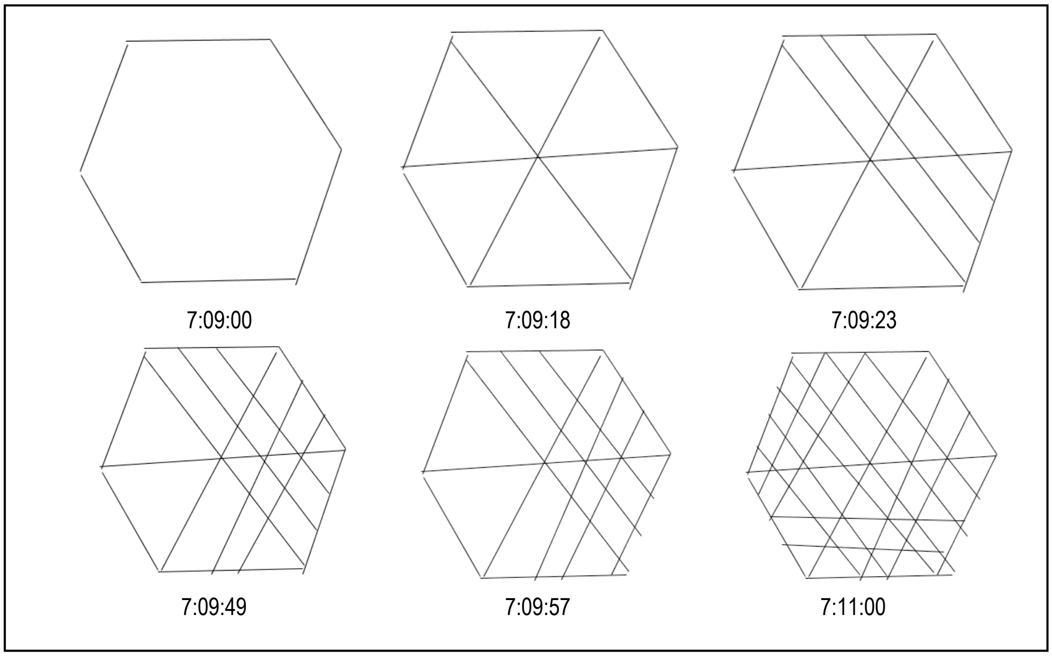


Figure 7-3. Six stages of 137's drawing actions obtained from the Replayer tool.

137's chat posting in line 1 that follows his drawing effort (which can be read as a self-critical, sarcastic "**great**") suggests that he considers his illustration inadequate in some way. He makes this explicit by soliciting help from other members to produce "**a diagram of a bunch of triangles**" on the whiteboard, and then removing the diagram he has just produced (the boxes following this posting in Figure 7-5 correspond to deletion actions on the whiteboard). By removing his diagram, 137 makes that space available to other members for the projected drawing activity. Qwertyuiop responds to 137's query with a request for clarification regarding the projected organization of the drawing ("**just a grid?**"). After 137's acknowledgement, Qwertyuiop performs a series of drawing actions that resemble the latter stages of 137's drawing actions, namely starting with the parallel lines tipped to the right first, then drawing a few parallel lines tipped to the left, and finally adding horizontal lines at the intersection points of earlier lines that are parallel to each other (see Figures 7-4 and 7-5). Having witnessed 137's earlier actions, the similarity in the organizations of both drawing actions suggest that Qwertyuiop has appropriated some key aspects of 137's drawing strategy, but modified/re-ordered the steps (e.g., he didn't start with the hexagon at the beginning) in a way that allowed him to produce a grid of triangles as a response to 137's request.

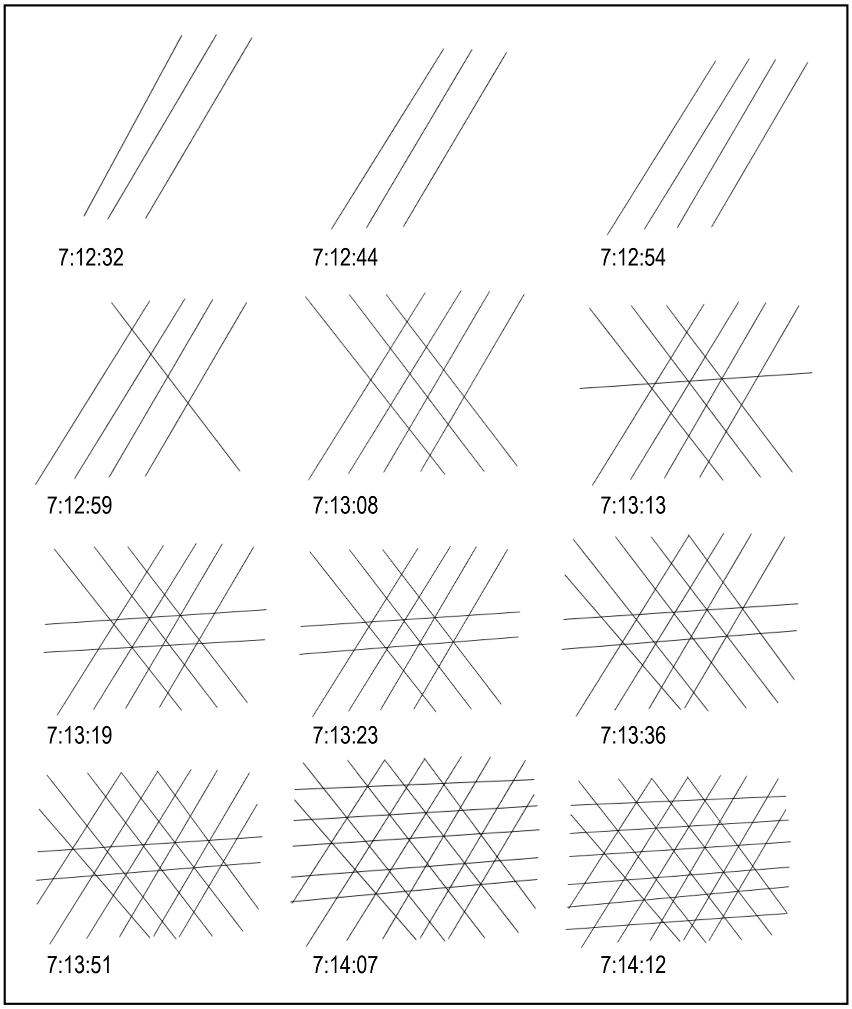


Figure 7-4. The evolution of Qwertyuiop's drawing in response to 137's request.

The key point we would like to highlight in this episode is that *the availability of the sequencing of the drawing actions that produces a diagram on the shared whiteboard can serve as a vital resource for collaborative sense-making*. As seen in Log 7-1, 137 did not provide any explanation in chat about his drawing actions or about the shape he was trying to draw. Yet, as we have observed in the similarity of Figures 7-3 and 7-4, the orderliness of 137's actions has informed Qwertyuiop's subsequent performance. The methodical use of intersecting parallel lines to produce triangular objects is common to both drawing performances. Moreover, Qwertyuiop does not repeat the same set of drawing actions, but selectively uses 137's steps to produce the relevant object (i.e., a grid of triangles) on the whiteboard. Qwertyuiop does not initially constrain his representational development by constructing a hexagon first, but allows a hexagon (or other shapes made with triangles) to emerge from the collection of shapes implied by the intersecting lines. Thus, Qwertyuiop's performance shows us that he is able to *notice a particular organization* in 137's drawing actions, and he has *selectively appropriated and built upon* some key aspects of 137's drawing practice. As we will see in the following logs,[1] the group's subsequent use of this drawing will provide us additional evidence that Qwertyuiop's diagram serves as an adequate response to 137's request.

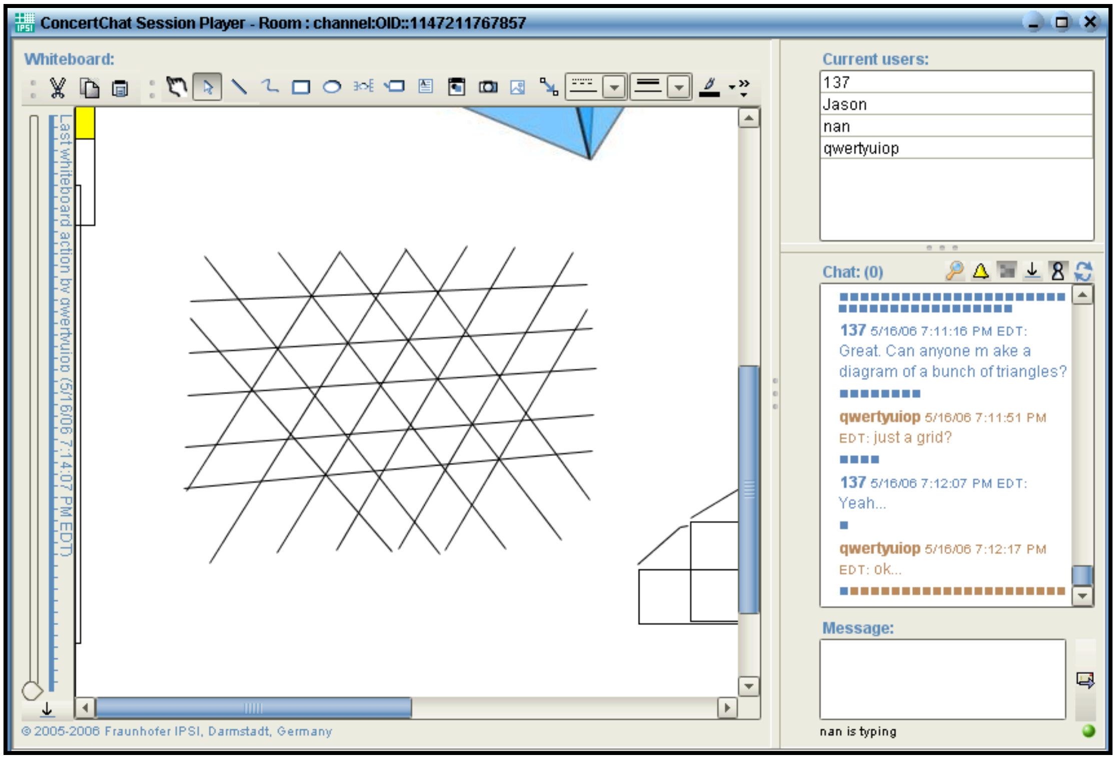


Figure 7-5. The interface at the 12th stage of Figure 7-4.

This excerpt highlights a fundamental difference between the two interaction spaces: whiteboard and chat contributions differ in terms of the availability of their production process. As far as chat messages are concerned, participants can only see who is currently typing,[2] but not what is being typed until the author decides to send the message. A similar situation applies to *atomic* whiteboard actions such as drawing an individual line or a rectangle. Such actions make a single object appear in the shared drawing area when the user releases the left mouse button; in the case of editable objects such as textboxes, the object appears on the screens of the computers of all chat participants when the editor clicks outside the textbox. However, the

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[1] For instance, after Qwertyuiop declares the completion of the grid in line 11, 137 anchors Qwertyuiop's drawing to the background at 7:15:47 (see Log 7-3). Since such a move preserves the positions of the selected objects and the objects affected by the move includes only the lines recently added by Qwertyuiop, 137's anchoring move seems to give a particular significance to Qwertyuiop's recent drawing. Hence, 137's anchoring move can be treated as an (implicit) endorsement of Qwertyuiop's drawing effort in response to his previous request.

[2] While a participant is typing, a social awareness message appears under the chat entry box on everyone else's screen stating that the person "is typing" (see Figure 5). When the typist posts the message, the entire message appears suddenly as an atomic action in everyone's chat window.

construction of most shared diagrams includes the production of multiple atomic shapes (e.g., many lines), and hence the sequencing of actions that produce these diagrams is available to other members. As we have observed in this excerpt, the availability of the drawing process can have interactionally significant consequences for math-problem-solving chats due to its instructionally informative nature. For instance, in Figure 7-4 transitions from stages 1 to 2 and 7 to 8 show modifications performed to achieve a peculiar geometric organization on the shared workspace. In short, the whiteboard affords an *animated evolution* of the shared space, which makes the *visual reasoning process* manifest in drawing actions *publicly available* for other members' inspection.

Mutability of Chat and Whiteboard Contents

Another interactionally significant difference between the chat and the whiteboard interaction spaces, which is evidenced in the excerpt above, is the difference in terms of the mutability of their contents. Once a chat posting is contributed, it cannot be changed or edited. Moreover, the sequential position of a chat posting cannot be altered later on. If the content or the sequential placement of a chat posting turns out to be interactionally problematic, then a new posting needs to be composed to repair that. On the other hand, the object-oriented design of the whiteboard allows users to re-organize its content by adding new objects and by moving, annotating, deleting, reproducing existing ones. For instance, the way 137 and Qwertyuiop repaired their drawings in the excerpt above by re-positioning some of the lines they drew earlier to make sure that they intersect at certain points and/or that they are parallel to the edges of the hexagon illustrates this difference. Such demonstrable tweaks make the mathematical details of the construction work visible and relevant to observers, and hence serve as a vital resource for joint mathematical sense making. By seeing that Qwertyuiop successively and intentionally adjusts lines in his whiteboard drawing to appear more parallel or to intersect more precisely, the other group members take note of the significance of the arrangement of lines as parallel and intersecting in specific patterns.

While both chat and whiteboard in VMT support persistence, visibility and mutability, they do so in different ways. A chat posting scrolls away gradually, whereas a drawing may be rearranged or even erased by anyone at any time. Chat conventions allow one to replace (i.e., follow) a mistyped posting with a new one, much as conversational conventions allow spoken utterances to be retracted, repaired or refined. The mechanisms of the two mediational technologies are different and the characteristics of their persistence, visibility and mutability differ accordingly. Collaborative interaction in the dual-space environment is sensitively attuned to these intricate and subtle differences.

Monitoring Joint Attention

The excerpt in Log 7-2 immediately follows the one in Log 7-1, where the team is oriented to the construction of a triangular grid after a failed attempt to embed a grid of triangles inside a hexagon. As Qwertyuiop is adding more lines to the grid, the facilitator (Nan) posts two questions addressed to the whole team in line 5. The question not only queries about what is happening now and whether everybody knows what others are currently doing, but the placement of the question at this point in interaction also problematizes the relevance of what has been happening so far. 137's response in lines 6 and 8 treat the facilitator's question as a problematic intervention. Qwertyuiop's response indicates he is busy with making triangles and hence may not know what others are doing. Jason acknowledges that he is following what has been going on in line 9. These responses indicate that the team members have been following (perhaps better than the facilitator) what has been happening on the whiteboard so far as something relevant to their task at hand.

Log 7-2

5	**7:14:09**	**nan**	**so what's up now? does everyone know what other people are doing?**
	7:14:12	**Qwertyuiop**	**< *Qwertyuiop adds a line to the grid of triangles*>**
6	**7:14:25**	**137**	**Yes?**
7	**7:14:25**	**Qwertyuiop**	**no-just making triangles**
	7:14:32	**Qwertyuiop**	**< *Qwertyuiop adds a line to the grid of triangles*>**
8	**7:14:33**	**137**	**I think... [*REF to line 6*]**
9	**7:14:34**	**Jason**	**Yeah**
	7:14:36	**Qwertyuiop**	**< *Qwertyuiop adds a line to the grid of triangles*>**
10	**7:14:46**	**nan**	**good :-)**
11	**7:14:51**	**Qwertyuiop**	**Triangles are done**
12	**7:15:08**	**137**	**So do you want to first calculate the number of triangles in a hexagonal array?**

In this excerpt, the facilitator calls on each participant to report on his/her understanding of the activities of other participants. There was an extended duration in which no chat postings were published while whiteboard actions were being performed by Qwertyuiop. Because it is not possible for any participant to observe other participants directly, it is not possible to monitor a class of actions others may perform that (1) are important for how we understand ongoing action but (2) do not involve explicit manipulation of the VMT environment, actions like watching the screen, reading text, inspecting whiteboard constructs, etc. The only way to determine if those kinds of actions are occurring is to explicitly inquire about them using a chat posting.

Past and Future Relevancies Implied by Shared Drawings

Following Qwertyuiop's announcement in line 11 of Log 7-2 that the drawing work is complete, 137 proposes that the team calculate "**the number of triangles**" in a "**hexagonal array**" as a possible question to be pursued next. Although a hexagon was previously produced as part of the failed drawing, this is the first time someone explicitly mentions the term "hexagonal array" in this session. What makes 137's proposal potentially intelligible to others is the availability of referable resources such as whiteboard objects, and the immediate history of the production of those objects such that the proposal can be seen to be embedded in a sequence of displayed actions. 137's use of "**So**" to introduce his proposal presents it as a consequence of, or a making explicit of, what preceded. His suggestion of it as a "**first**" (next) move implies that the drawings opened up multiple mathematical tasks that the group could pursue, and that the proposed suggestion would be a candidate for a next move. In other words, the objects on the whiteboard and their visually shared production index a horizon of past and future activities. The indexical terms in 137's proposal (like "**hexagonal array**") not only rely on the availability of the whiteboard objects to propose a relevant activity to pursue next, but also modify their sense by using linguistic and semantic resources in the production to label or gloss the whiteboard object and its production. This allows actors to orient in particular ways to the whiteboard object and the procedures of its co-construction—providing a basis for coordinated joint activity. The joint activity acquires a temporal structure that is defined by the details of chat wording, the animation of graphical construction and the sequentiality of proposing.

Methods for Referencing Relevant Objects in the Shared Visual Field

Bringing relevant mathematical objects to other members' attention often requires a coordinated sequence of actions performed in both the chat and whiteboard interaction spaces. The episode following 137's proposal (Log 7-3) provides us with an appropriate setting to illustrate how participants achieve this in interaction. Following 137's proposal in line 12, both Qwertyuiop and Jason post queries for clarification in lines 13 and 16, respectively, which indicate that the available referential resources were insufficient for them to locate what 137 is referring to with the term "**hexagonal array**." Jason's query in the chat is particularly important here since it explicitly calls for a response to be performed on the shared diagram, i.e., in a particular field of relevance in the other interaction space. Following Jason's query, 137 begins to perform a sequence of drawing actions on the shared diagram. He adds a few lines that gradually begin to enclose a region on the triangular grid[3] (see Figure 7-6).

Log 7-3

11	**7:14:51**	**Qwertyuiop**	**Triangles are done**
12	**7:15:08**	**137**	**So do you want to first calculate the number of triangles in a hexagonal array?**
13	**7:15:45**	**Qwertyuiop**	**What's the shape of the array? a hexagon? *<REF to 12>***
	7:15:47	**137**	***<137 locks the triangular grid that Qwertyuiop has just drawn>***
14	**7:16:02**	**137**	**Ya *<REF to line 13>***
15	**7:16:15**	**Qwertyuiop**	**ok….**
	7:16:18 - 7:16:35	**137**	***<137 performs a few drawing actions and then erases them>***
16	**7:16:41**	**Jason**	**wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...**

[3] In the meantime, Qwertyuiop also performs a few drawing actions near the shared drawing, but his actions do not introduce anything noticeably different since he quickly erases what he draws each time.

	7:16:45 - 7:17:28	137	*<137 adds new lines to the grid on the whiteboard which gradually forms a contour on top of the grid. Figure 7-6 shows some of the performed by 137>*
17	7:17:30	Jason	Hmm.. okay
18	7:17:43	Qwertyuiop	Oops <*REF to Whiteboard*>
19	7:17:44	Jason	so it has at least 6 triangles?
20	7:17:58	Jason	in this, for instance <*REF to Whiteboard*>
	7:18:03 - 7:18:17	137	*<137 completes the contour by adding more lines, which forms a hexagon>*
21	7:18:53	137	How do you color lines?
22	7:19:06	Jason	There's a little paintbrush icon up at the top
23	7:19:12	Jason	it's the fifth one from the right
	7:19:13 - 7:19:20	137	*<137 begins to change the color of the lines that form the contour to blue>*
24	7:19:20	137	Thanks.
25	7:19:21	Jason	There ya go :-)
	7:19:25 - 7:19:48	137	*<137 finishes the coloring. Now the contour is highlighted in blue>*
26	7:19:48	137	Er... That hexagon.
27	7:20:02	Jason	so... should we try to find a formula i guess

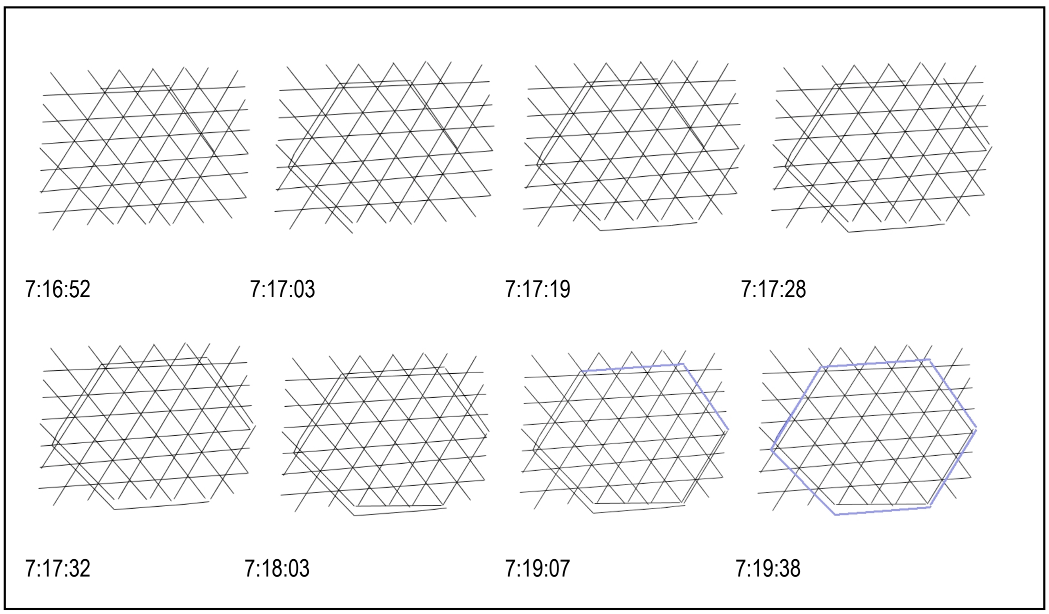

Figure 7-6. Snapshots from the sequence of drawing actions performed by 137.

When the shared diagram reaches the stage illustrated by the 4^{th} frame in Figure 7-6, Jason posts the message "**hmmm... okay**" in line 17, which can be read as an acknowledgement of 137's performance on the whiteboard as a response to his recent chat query. Since no chat message was posted after Jason's request in line 16, and the only shared actions were 137's work on the whiteboard, Jason's chat posting can be read as a response to the ongoing drawing activity on the whiteboard. As it is made evident in his posting, Jason is treating the evolving drawing on the shared diagram as a response to his earlier query for highlighting the hexagonal array on the whiteboard: the question/answer adjacency pair is spread across the two interaction spaces in an unproblematic way.

Following provisional acknowledgement of 137's drawing actions on the whiteboard, Jason posts a claim in line 19. This posting is built as a declarative: "**so it has at least 6 triangles**," with a question mark appended to the end. The use of "**so**" in this posting invites readers to treat what follows in the posting as a consequence of the prior actions of 137. In this way, Jason is (a) proposing a defeasible extension of his understanding of the sense of 137's actions and (b) inviting others to endorse or correct this provisional claim about the hexagonal array by presenting this as a query using the question mark.

In line 20 Jason provides further specificity to what he is indexing with the term "**it**" in line 19 by highlighting a region on the grid with the referencing tool of the VMT system. The textual part of the posting makes it evident that the highlighted region is an instance of the object mentioned in line 19. Moreover, the 6 triangles highlighted by the explicit reference recognizably make up a hexagon shape altogether. Hence, Jason's explicit reference seems to be pointing to a particular stage (indexed by "**at least**") of the hexagonal array that the team is oriented to (see Figure 7-7).

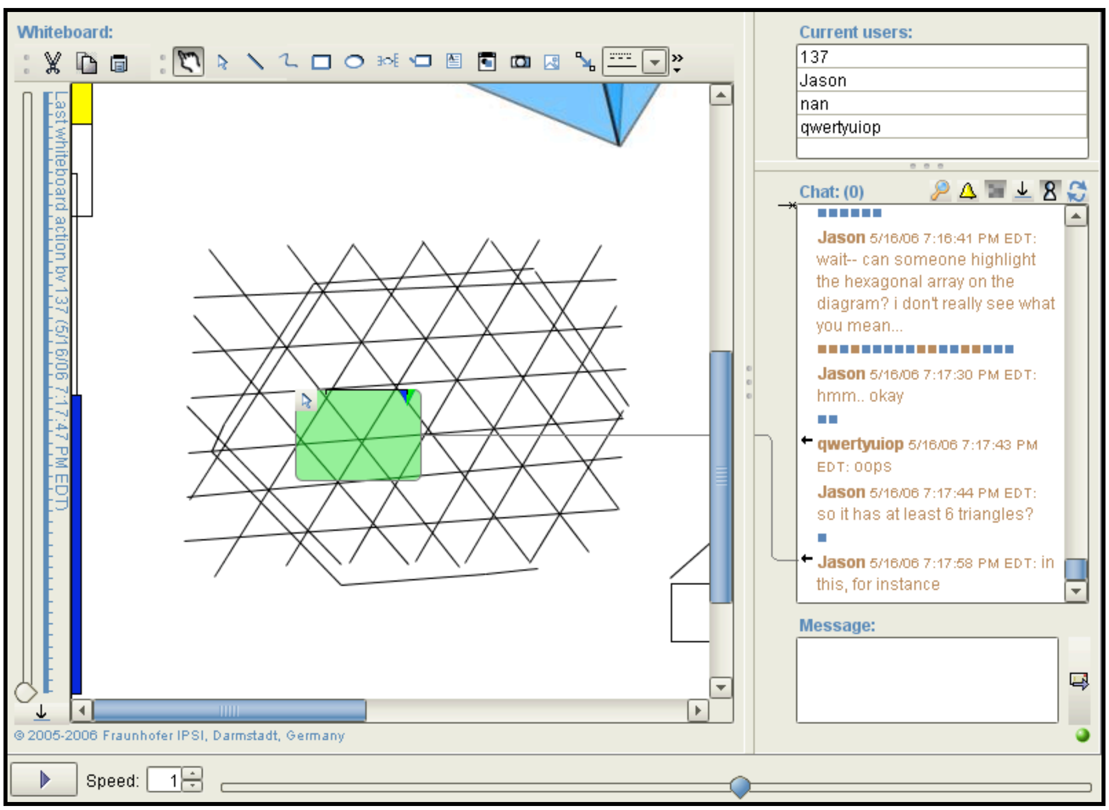

Figure 7-7. Use of the referencing tool to point to a stage of the hexagonal array.

In other words, having witnessed the production of the hexagonal shape on the whiteboard as a response to his earlier query, Jason displays his competence by demonstrating his recognition of the hexagonal pattern implicated in 137's graphical illustration. 137's drawing actions highlight a particular stage of a growing pattern made of triangles—stage N=3, as we will see in Figure 7-9. However, recognizing the stick-pattern implicated in 137's highlighting actions requires other members to project how the displayed example can be grown and/or shrunk to produce other stages of the hexagonal array. Thus, Jason's description of the shape of the "hexagonal array" at a different stage—N=1—is a public display of his newly achieved comprehension of the significance of the math object in the whiteboard and the achievement of "indexical symmetry" among the parties involved with respect to this math object (see Chapter 14).

Although Jason explicitly endorsed 137's drawing as an adequate illustration, the small boxes in the chat stream that appear after Jason's acknowledgement in line 17 show that 137 is still oriented to and operating on the whiteboard. In line 21, 137 solicits other members' help regarding how he can change the color of an object on the board, which opens a side sequence about a specific feature of the whiteboard system. Based on the description he got, 137 finishes marking the hexagon by coloring all its edges with blue, and he posts "**that hexagon**" in line 25. This can be read as a chat reference to the whiteboard shape enclosed by the blue contour, and as a response to other members' earlier requests for clarification.

In this excerpt, we have observed two referential methods enacted by participants to bring relevant graphical objects on the whiteboard to other group members' attention. In the first case, 137 *marked the drawing* with a different color to identify the contour of a hexagonal shape. As evidenced in other members' responses, this was designed to make the hexagonal array embedded in a grid of triangles visible to others. Jason demonstrated another method by using the explicit referencing tool to support his *textual description* of the first stage of the pattern. Both mechanisms play a key role in directing other members' attention to features of the shared *visual field* in particular ways. This kind of deictic usage isolates components of the shared drawing and constitutes them as relevant objects to be attended to for the purposes at hand. As we shall see, these guided shifts in visual focus of the group have strategic importance for the group's mathematical work. Hence, such referential work establishes a fundamental *relationship between the narrative and mathematical terminology used in text chat and the animated graphical constructions produced on the whiteboard*. The shared sense of the textual terms and the inscriptions co-evolve through the referential linkages established as the interaction sequentially unfolds in both interaction spaces.

In Log 7-3, the group tentatively proposes a major mathematical insight—that a hexagon can be viewed as six symmetric triangular areas. It is a visual achievement. It emerges from a visual inspection by Jason of 137's graphical diagram, based on Qwertyuiop's method of visually representing hexagons as patterns of triangularly intersecting lines. By literally focusing his eyes on a smallest hexagon in the larger array and counting the number of triangles visible within a hexagonal border, Jason discovers that there are at least six triangles at the initial stage of a hexagon with one unit on each side. We will see how the group visualizes the generalization of this picture to other stages. However, it is already interesting to note that Jason not only observes the composition of a small hexagon out of six triangles, but he conveys this to the rest of the group in both media. He posts chat line 19 and then references from chat line 20 to a visually highlighted view in the whiteboard, so that his *visual* understanding can be shared by the group as well as his *narrative* description in his claim. The next step for the group will be to formulate a *symbolic* mathematical expression of this claim.

Whiteboard Visualizations, Chat Narratives and Wiki Symbolisms

The excerpt in Log 7-4 immediately follows Log 7-3. The way 137 uses both interaction spaces in this episode highlights another important aspect of collaborative problem-solving work in an environment like VMT. Since participants can contribute to only one of the interaction spaces at a time, they cannot narrate their whiteboard actions simultaneously with chat postings, as can be done with talking about a whiteboard in a face-to-face setting. However, as we will observe in 137's use of the whiteboard in the following excerpt, participants can achieve a similar interactional organization by coordinating their actions in such a way that whiteboard actions can be seen as part of an exposition performed in chat.

Log 7-4

27	**7:20:02**	**Jason**	**so... should we try to find a formula i guess**
28	**7:20:22**	**Jason**	**input: side length; output: # triangles**
29	**7:20:39**	**Qwertyuiop**	**It might be easier to see it as the 6 smaller triangles.**
30	**7:20:48**	**137**	**Like this? <*REF to line 29*>**
	7:20:53	**137**	***<137 draws a red line>***
	7:20:57	**137**	***<137 draws a red line>***
	7:21:00	**137**	***<137 draws a red line>***
31	**7:21:02**	**Qwertyuiop**	**Yes**
32	**7:21:03**	**Jason**	**Yup**
	7:21:03	**137**	***<137 moves the second red line>***
	7:21:05	**137**	***<137 moves the second red line again. It is positioned on the grid now>***
33	**7:21:29**	**Qwertyuiop**	**Side length is the same...**
34	**7:22:06**	**Jason**	**Yeah**

Jason brings the prior activity of locating the hexagonal array on the shared drawing to a close with his so-prefaced posting in line 27, where he invokes the task of finding a formula that was mentioned by 137 earlier. Jason provides further specificity to the formula he is referring to in the next line (i.e., given the side length as input the formula should return the number of triangles as output). In line 29 Qwertyuiop takes up Jason's proposal by suggesting the team consider the hexagonal array as six smaller triangles to potentially simplify the task at hand. In the next line, 137 posts a question phrased as "**like this?**" which is addressed to Qwertyuiop's prior posting, as indicated by the use of the referential arrow. Next, we observe the appearance of three red lines on the shared diagram, which are all added by 137. Here, 137 demonstrates a particular way of splitting the hexagon into six parts: the image on the left of Figure 7-8 corresponds to the sequence of three whiteboard actions represented as three boxes in the chat excerpt. After 137 adds the third line whose intersection with the previously drawn red lines recognizably produces six triangular regions on the shared representation, Qwertyuiop and Jason both endorse 137's demonstration of a particular way of splitting up the hexagonal shape.

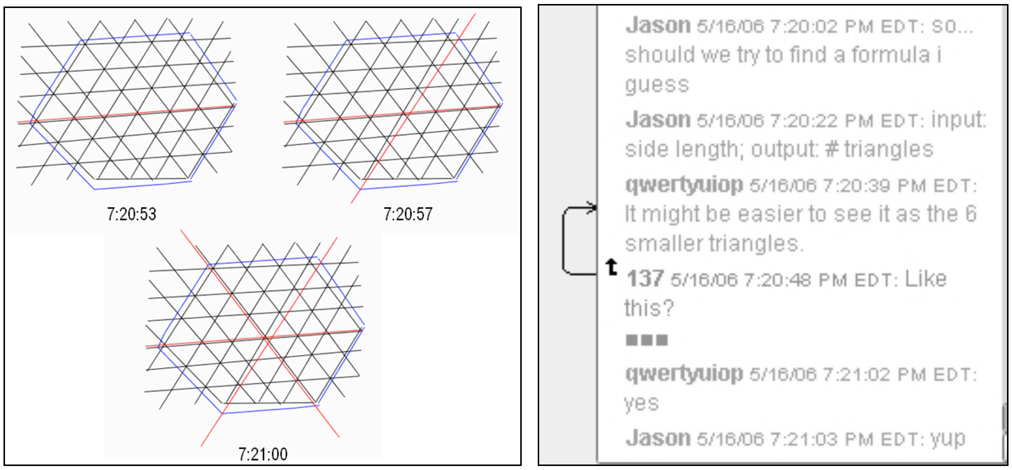

Figure 7-8. 137 splits the hexagon into 6 parts.

One important aspect of this organization is directing other members' attention to the projected whiteboard activity as a relevant step in the sequentially unfolding exposition in chat. For instance, the deictic term "**this**" in 137's chat line 30 refers to something yet to be produced, and thereby projects that there is more to follow the current posting, possibly in the other interaction space. Moreover, the use of the referential link and the term "**like**" together inform others that what is about to be done should be read in relation to the message to which 137 is responding. Finally 137's use of a different color marks the newly added lines as recognizably distinct from what is already there as the background, and hence noticeable as a demonstration of what is implicated in recent chat postings.

Again, the progress in understanding the mathematics of the problem is propelled through visual means. In response to Jason's proposal of finding a formula, Qwertyuiop suggests that "**it might be easier to see it**" in a certain way. Jason's proposed approach might be difficult to pursue because no one has suggested a concrete approach to constructing a formula that would meet the general criteria of producing an output result for any input variable value. By contrast, the group has been working successfully in the visual medium of the whiteboard drawing and has been literally able to "see" important characteristics of the math object that they have co-constructed out of intersecting lines. Jason has pointed out that at least six triangles are involved (in the smallest hexagon). So, Qwertyuiop proposes building on this insight. 137 asks if the way to see the general case in terms of the six small triangles as proposed by Qwertyuiop can be visualized by intersecting the hexagon array with 3 intersecting lines to distinguish the six regions of the array. He does this through a visual construction, simply referenced from the chat with his "**Like this?**" post.

By staring at the final version of the array (the 3rd stage in Figure 7-8), all members of the group can see the hexagon divided into six equal parts at each stage of the hexagonal pattern. Near the intersection of the red lines, they can see a single small triangle nestled in each of the six regions. As will be evidenced in Log 7-5, within the larger hexagon delimited by the blue lines, they can see a set of 1+3+5=9 small triangles in each of the six larger triangular regions. Similarly, midway between stage N=1 and stage N=3, one can visually observe 1+3=4 small triangles in each region. The new view, scaffolded by 137's red lines, entails *visual reasoning* that leads to mathematical deductions. As soon as Qwertyuiop and Jason see 137's construction, they both concur with it as the easier way to see the mathematical pattern of triangles in the hexagonal array. The visual reasoning supported by whiteboard and narrated textually in the chat will lead in the next episode to symbolic reasoning for posting in the wiki.

A first glance at the chat logs might suggest that the group is narrating their problem-solving process in the chat and illustrating what they mean by "napkin" drawings in the whiteboard, to use Dillenbourg & Traum's (2006) metaphor. However, a second look reveals that the most significant insight and sharing is occurring in the whiteboard, more along the lines of a visual "model" metaphor. Perhaps the best way to describe what is going on is to say that the group is very carefully coordinating their work in the dual space as a whole to achieve a shared progression of understanding of the pattern problem. This is accomplished with an efficiency and effectiveness that could not be achieved in either a purely textual chat system or a purely graphical whiteboard. Although in this view the chat and whiteboard both function as symmetric parts of a coordinated whole—in which chat references drawing, and drawing illustrates chat—it is important to differentiate their roles as well.

Using Representations of Specific Instances as a Resource for Generalization

Immediately following the previous excerpt the team moves on to figuring out a general formula to compute the number of triangles in a hexagonal pattern. In line 34 of Log 7-5, Jason relates the particular partitioning of the hexagon illustrated on the whiteboard to the problem at hand by stating that the number ("#") of triangles in the hexagon will equal 6 times ("x6") the number of triangles enclosed in each partition. In the next posting, 137 seems to be indexing one of the six partitions with the phrase "each one." Hence, this posting can be read as a proposal about the number of triangles included in a partition. The sequence of numbers in the expression "1+3+5" calls others to look at a partition in a particular way. While 137 could have simply said here that there are 9 triangles in each partition, he instead organizes the numbers in summation form and offers more than an aggregated result. His expression also demonstrates a systematic method for counting the triangles. In other words, his construction is designed to highlight a particular orderliness in the organization of triangles that form a partition. Moreover, the sequence includes increasing consecutive odd numbers, which implicitly informs a certain progression for the growth of the shape under consideration.

Log 7-5

34	7:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
35	7:22:19	137	Each one has 1+3+5 triangles.
36	7:22:23	Jason	but then we're assuming just regular hexagons
37	7:22:29	Qwertyuiop	the “each polygon corrisponds to 2 sides” thing we did last time doesn't work for triangles
38	7:23:17	137	It equals 1+3+...+(n+n-1) because of the “rows”?
39	7:24:00	Qwertyuiop	yes- 1st row is 1, 2nd row is 3...
40	7:24:49	137	And there are n terms so... n(2n/2)
41	7:25:07	137	or n^2 <*REF to line 40*>
42	7:25:17	Jason	Yeah
43	7:25:21	Jason	then multiply by 6
44	7:25:31	137	To get 6n^2 <*REF to line 43*>

About a minute after his most recent posting, 137 offers an extended version of his sequence as a query in line 38. The relationship between the sequence for the special case and this one is made explicit through the repetition of the first two terms. In the new version the “…” notation is used to substitute a series of numbers following the second term up to a generic value represented by “**n+n-1**,” which can be recognized as a standard expression for the n^{th} odd number. Hence, this representation is designed to stand for something more general than the one derived from the specific instance illustrated on the whiteboard. 137 attributes this generalization to the concept of “**rows**,” and solicits other members’ assessment regarding the validity of his version (by ending with a question mark). 137’s use of the term **rows** seems to serve as a pedagogic device that attempts to locate the numbers in the sequence on the n^{th} stage of the hexagonal pattern (see Figure 7-9 for an analyst’s illustration of the generalized hexagonal pattern). For stages 1, 2 and 3, the hexagonal shape has 6*(1) = 6, 6*(1+3) = 24, 6*(1+3+5) = 54 triangles, respectively.

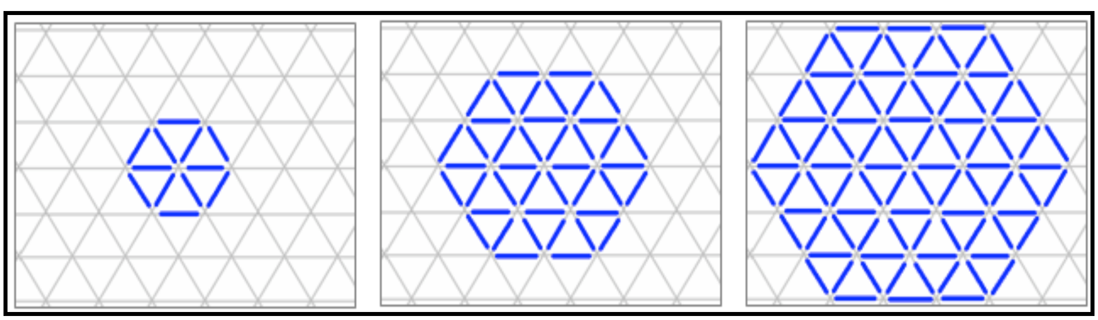

Figure 7-9. A reconstruction of the first three iterations of the geometric pattern.

Qwertyuiop’s endorsement of 137’s proposal comes in line 39. He also demonstrates a row-by-row iteration on a hexagon, where each number in the sequence corresponds to a row of triangles in a partition. In other words, Qwertyuiop elaborates on 137’s statement in line 38 of the chat by displaying his understanding of the relationship between the rows and the sequence of odd numbers. Although he does not explicitly reference it here, Qwertyuiop may be viewing the figure in the whiteboard to see the successive rows. The figure is, of course, also available to 137 and Jason to help them follow Qwertyuiop's chat posting and check it.

Then 137 proposes an expression for the sum of the first n odd numbers in line 40.[4] Jason agrees with the proposed expression and suggests that it should be multiplied by 6 next. In the following line, 137 grammatically completes Jason's posting with the resulting expression. In short, by virtue of the agreements and the co-construction work of Jason and 137, the team demonstrates its endorsement of the conclusion that the number of triangles would equal $6n^2$ for a hexagonal array made of triangles. As the group collaboratively discovered, when n equals the stage number (as "input" to the formula), the number of triangles is given by the expression $6n^2$.

The way team members orient themselves to the shared drawing in this episode illustrates that the drawings on the whiteboard have a figurative role in addition to their concrete appearance as illustrations of specific cases. The particular cases captured by concrete, tangible marks on the whiteboard are often used as a resource to investigate and talk about general properties of the mathematical objects indexed by them.

Another important aspect of the team's achievement of a general expression in this episode is the way they transformed a particular way of *counting* the triangles in one of the partitions (i.e., a geometric observation) into an algebraic mode of investigation. This shift from a visual method led the team members to recognize that a particular sequence of numbers can be associated with the way the partition grows in subsequent iterations. The shift to this symbolic mode of engagement, which heavily uses the shared drawing as a resource, allowed the team to go further in the task of generalizing the pattern of growth by invoking algebraic resources. In other words, *the team made use of multiple realizations (graphical and linguistic) of the math object (the hexagonal array) distributed across the dual interaction space to co-construct a general formula* for the task at hand.

Chat Versus Whiteboard Contributions as Persistent Referential Resources

In all of the excerpts we have considered so far, the shared drawing has been used as a resource within a sequence of related but recognizably distinct activities. For instance, the group has oriented itself to the following activities: (1) drawing a grid of triangles, (2) formulating a problem that relates a hexagonal array to a grid of triangles, (3) highlighting a particular hexagon on the grid, (4) illustrating a particular way to split the shape into six smaller pieces, and (5) devising a systematic method to count the number of triangles within one of the six pieces. As the group oriented to different aspects of their joint task, the shared diagram was modified on the whiteboard and annotated in chat accordingly. Yet, although it had been modified and annotated along the way, the availability of this shared drawing on the screen

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[4] 137 makes use of Gauss's method for summing this kind of series, adding the first and last term and multiplying by half of the number of terms: $(1 + n + n - 1)*n/2=2n*n/2=n^2$. This method was used by the group and shared in previous sessions involving the stair pattern that is still visible in the whiteboard.

and the way participants organize their discussion around it highlights its persistent characteristic as an ongoing referential resource. In contrast, none of the chat postings in prior excerpts were attributed a similar referential status by the participants. As we have seen, in each episode the postings responded or referred either to recently posted chat messages or to the visual objects in the shared space.

The textual chat postings and the graphical objects produced on the whiteboard differ in terms of the way they are used as referential resources by the participants. The content of the whiteboard is persistently available for reference and manipulation, whereas the chat content is visually available for reference for a relatively shorter period. This is due to the linear growth of chat content, which replaces previous messages with the most recent contributions inserted at the bottom of the chat window. Although one can make explicit references to older postings by using the scroll-bar feature, the limited size of the chat window affords a referential locality between postings that are visually (and hence temporally) close to each other.

By contrast, objects drawn in the whiteboard tend to remain there for a long time. They are often only erased or moved out of view when space is needed for drawings related to a new topic. While they may be modified, elaborated or moved around, whiteboard objects may remain visible for an entire hour-long session or even across sessions. Like the chat, the whiteboard has a history scrollbar, so that any past state of the drawing can be made visible again—although in practice students rarely use this feature. Although both media technically offer a persistent record of their contents, the visual locality of the whiteboard—the fact that graphical objects tend to stay available for reference from the more fleeting chat—qualifies it as the more persistent medium as an interactional resource. This notion of persistence does not imply that the shared sense of whiteboard objects is fixed once they are registered to the shared visual field. As they continue to serve as referential resources during the course of the problem-solving effort, the sense of whiteboard objects may become increasingly evident and shared, or their role may be modified as participants make use of them for varying purposes.

Implications for CSCL Chat Interaction Analysis

In this case study we investigated how a group of three upper-middle-school students put the features of an online environment with dual-interaction spaces into use as they collaboratively worked on a math problem they themselves came up with. Our analysis has revealed important insights regarding the affordances of systems with dual-interaction spaces. First, we observed that the whiteboard can make visible to everyone the animated evolution of a geometric construction, displaying the *visual reasoning* process manifested in drawing actions. Second, whiteboard and chat contents differ in terms of *mutability* of their contents, due to the object-oriented design of the whiteboard, which allows modification and annotation of past contributions. Third, the media differ in terms of the *persistence* of their contents: whiteboard objects remain in the shared visual field until they are removed, whereas chat content gradually scrolls off as new postings are produced. Although contents of both spaces are persistently available for reference, due to linear progression of the chat window, chat postings are likely to refer to visually (and hence temporally) close chat messages and to graphical whiteboard objects. Finally, the whiteboard objects *index* a horizon of past and future activities as they serve as an interactional resource through the course of recognizably distinct but related episodes of chat discussion.

Our analysis of this team's joint work has also revealed methods for the organization of collaborative work, through which group members co-construct mathematical meaning sedimented in semiotic objects distributed across the dual interaction spaces of the VMT environment. We observed that bringing relevant math artifacts referenced by indexical terms such as "**hexagonal array**" to other members' attention often requires a coordinated sequence of actions across the two interaction spaces. Participants use explicit and verbal references to guide each other about how a new contribution should be read in relation to prior contents. Indexical terms stated in chat referring to the visible production of shared objects are instrumental in the reification of those terms as meaningful mathematical objects for the participants. Verbal references to co-constructed objects are often used as a resource to index complicated and abstract mathematical concepts in the process of co-constructing new ones. Finally, different representational affordances of the dual interaction spaces allow groups to develop multiple realizations of the math artifacts to which they are oriented. Shared graphical inscriptions and chat postings are used together as semiotic resources in mutually elaborating ways. Methods of coordinating group interaction across the media spaces also interrelate the mathematical significances of the multiple realizations.

Overall, we observed that actions performed in both interaction spaces constitute an evolving historical context for the joint work of the group. What gets done now informs the relevant actions to be performed next, and the significance of what was done previously can be modified depending on the circumstances of the ongoing activity. As the interaction unfolds sequentially, the sense of previously posted whiteboard objects and chat statements may become evident and/or refined. In this way, the group's joint problem space is maintained.

Through the sequential coordination of chat postings and whiteboard inscriptions, the group successfully solved their mathematical challenge, to find a formula for the number of small triangles in a hexagonal array of any given side-length. Their interaction was guided by a sequence of proposals and responses carried out textually in the chat medium. However, the sense of the terms and relationships narrated in the chat were largely instantiated, shared and investigated through observation of visible features of graphical inscriptions in the whiteboard medium. The mathematical object that was visually co-constructed in the whiteboard was named and described in words within the chat. Finally, a symbolic expression was developed by the group, grounded in the graphic that evolved in the whiteboard and discussed in the terminology that emerged in the chat. The symbolic mathematical result was then posted to the wiki, a third medium within the VMT environment. The wiki is intended for sharing group findings with other groups as part of a permanent archive of work by virtual math teams.

Our case study in this chapter demonstrates that it is possible to analyze how math problem solving—and presumably other cognitive achievements—can be carried out by small groups of students. The students can define and refine their own problems to pursue; they can invent their own methods of working; they can use unrestricted vocabulary; they can coordinate work in multiple media, taking advantage of different affordances. Careful attention to the sequentiality of references and responses is necessary to reveal *how* the group coordinated its work and how that work was driven by the reactions of the group members' actions to each other. Only by focusing on the sequentiality of the actions can one see how the visual, narrative and symbolic build on each other as well as how the actions of the individual students respond to each other. Through these actions, the students co-construct math objects, personal understanding, group agreement and mathematical results that cannot be attributed to any one individual, but that emerge from the interaction as complexly sequenced.

This analysis illustrates a promising approach for CSCL research to investigate aspects of group cognition that are beyond the reach of alternative methods that systematically ignore the full sequentiality of their data.

The Group as the Unit of Analysis

For methodological reasons, quantitative approaches—such as those reviewed in the next section—generally (a) constrain (scaffold) subject behaviors, (b) filter (code) the data in terms of operationalized variables and (c) aggregate (count) the coded data. These acts of standardization and reduction of the data eliminate the possibility of observing the details and enacted processes of unique, situated, indexical, sequential, group interaction (Stahl, 2006, ch. 10). An alternative form of interaction analysis is needed to explore the organization of interaction that can take place in CSCL settings.

In this chapter, we focused on small-group interactions mediated by a multimodal interaction space. Our study differs from similar work in CSCL by our focus on groups larger than dyads whose members are situated outside a controlled lab environment, and by our use of open-ended math tasks where students are encouraged to come up with their own problems. Moreover, we do not impose any deliberate restrictions on the ways students access the features of our online environment or on what they can say. Our main goal is to investigate how small groups of students construe and make use of the "available features" of the VMT online environment to discuss mathematics with peers from different schools outside their classroom setting. In other words, we are interested in studying interactional achievements of small groups in complex computer mediations "in the wild" (Hutchins, 1996).

Our interest in studying the use of an online environment with multiple interaction spaces in a more naturalistic use scenario raises serious methodological challenges.

In an early VMT study where we conducted a content analysis of collaborative problem-solving activities mediated by a standard text-chat tool in a similar scenario of use, we observed that groups larger than dyads exhibit complex interactional patterns that are difficult to categorize based on a theory-informed coding scheme with a fixed/predetermined unit of analysis (Chapter 20). In particular, we observed numerous cases where participants post their messages in multiple chat turns, deal with contributions seemingly out of sequence and sustain conversations across multiple threads that made it problematic to segment the data into fixed analytic units for categorization. Moreover, coming to agreement on a code assignment for a unit that is defined *a priori* (e.g., a chat line) turned out to be heavily dependent upon how the unit can be read in relation to resources available to participants (e.g., the problem description) and to prior units (Chapter 22). In other words, the sense of a unit not only depends on the semantic import of its constituent elements, but also on the occasion in which it is situated (Heritage, 1984). This often makes it possible to apply multiple categories to a given unit and threatens the comparability of cases that are labeled with the same category. More importantly, once the data is reduced to codes and the assignments are aggregated, the complex sequential relationships among the units are largely lost. Hence, the coding approach's attempt to enforce a category to each fixed unit without any consideration to how users sequentially organize their actions in the environment proved to be too restrictive to adequately capture the interactional complexity of chat (Chapter 23). Moreover, the inclusion of a shared drawing area in our online environment made the use of a standard coding schema even harder due to increased possibilities for interaction. The open-ended nature of the tasks we use in our study makes it especially challenging to model certain types of actions and to compare them against ideal solutions.

The issue of unit of analysis has theoretical implications. In text chat it is tempting to take a single posting as the unit to be analyzed and coded, because a participant defined this as a unit by posting it as a message and because the chat software displays it as a visual unit. However, this tends to lead the analyst to treat the posting as a message from the posting individual—i.e., as an expression of a thought in the poster's mind, which must then be interpreted in the minds of the post readers. Conversation analysis has argued for the importance of *interactions* among participants as forming more meaningful units for analysis. These consist of sequences of multiple utterances by different speakers; the individual utterances take each other into account. For instance, in a question/answer "adjacency pair," the question elicits an answer and the answer responds to the question. To take a pair of postings such as a question/answer pair as the analytic unit is to treat the interaction within the group as primary. It focuses the analysis at the level of the group rather than the individual. As mentioned, in online text chat, responses are often separated from their referents, so the analysis is more complicated. In general, we find that the important thing is to trace as many references as possible between chat postings or whiteboard actions in order to analyze the interaction of the group as it unfolds (Chapter 26). As seen in our case study, it is through the co-construction of a rich nexus of such references that the group weaves its joint problem space.

Analysis at the group unit of analysis focuses on the co-construction, maintenance and progressive refinement of the joint problem space. This is a distinctive analytic task that takes as its data only what is shared by the group. Whatever may go on in the physical, mental or cultural backgrounds of the individual participants is irrelevant unless it is brought into the group discourse. Because the students know nothing about the gender, age, ethnicity, accent, appearance, location, personality, opinions, grades or skills of the other participants other than what is mentioned or displayed in the chat interaction, these "factors" from the individual and societal levels can be bracketed out of the group analysis. Survey and interview data is unnecessary; individual learning trajectories are not plotted. The VMT Project has been designed to make available to the analyst precisely what was shared by the student group, and nothing else.

Relatedly, the notion of common ground (see section on grounding below) as an abstract placeholder for registered cumulative facts or pre-established meanings has been critiqued in the CSCL literature for treating meaning as a fixed/denotative entity transcendental to the meaning-making activities of inquirers (Koschmann, 2002). The common ground that supports mutual understanding in group cognition or group problem solving is a matter of semantic references that unfold sequentially in the momentary situation of dialog, not a matter of comparing mental contents (Stahl, 2006, pp. 353-356). Committing to a reference-repair model (Clark & Marshall, 1981) for meaning making falls short of taking into account the dynamic, constitutive nature of meaning-making interactions that foster the process of inquiry (Koschmann et al., 2001).

As we saw in the preceding case study, the understanding of the mathematical structure of the hexagon area did not occur as a mental model of one of the students that was subsequently externalized in the chat and whiteboard and communicated to the other students. It emerged in the discourse media in a way that we could witness as analysts. It consisted of the layering of inscriptions (textual and graphical) that referenced one another. The referential network of group meaning can be observed in the way that deictic and indexical expressions are resolved. The three students each contribute to the progressive development of the shared meaning by responding appropriately to the on-going state of the discourse. This is a matter of linguistic skill—including ability in discussing mathematical matters—not of articulating mental representations. It is surprising from a rationalist perspective how poor students are at explaining (Chapter 26), reproducing (Koschmann & LeBaron, 2003) or even recalling (Chapter 6) what they did in the group when they are no longer situated in the moment.

Given these analytical and theoretical issues, we opted for an alternative to the approaches reviewed below that involve modeling of actions and correct solution paths or treating shared understanding as alignment of pre-existing individual representations and opinions. In this chapter we built on our previous work on referencing math objects in a system with chat and a whiteboard (Chapter 17); we presented a "micro-ethnographic" (Streeck & Mehus, 2005) case study using interaction analysis (Jordan & Henderson, 1995). We focused on the *sequence of actions* in which the group co-constructs and makes use of *semiotic resources*

(Goodwin, 2000) distributed across dual interaction spaces to *do* collaborative problem-solving work. In particular, we focused on the joint organization of activities that produce graphical drawings on the shared whiteboard and the ways those drawings are used as resources by actors as they collaboratively work on an open-ended math task. Through detailed analysis at the group unit of analysis, we investigated how actions performed in one workspace inform the actions performed in the other and how the group coordinates its actions across both interaction spaces.

Other Approaches in CSCL to Analyzing Multimodal Interaction

In this section we review previous investigations by other CSCL researchers. Their studies focus on the interactions mediated by systems with multimodal interaction spaces to support collaborative work online. Our review is not meant to be exhaustive, but representative of the more advanced analytical approaches employed. We have selected sophisticated analyses, which go well beyond the standard coding-and-counting genre of CSCL quantitative reports, in which utterances are sorted according to a fixed coding scheme and then statistics are derived from the count of utterances in each category. Unlike the simple coding-and-counting studies, the approaches we review attempt to analyze some of the structure of the semantic and temporal relationships among chat utterances and workspace inscriptions in an effort to get at the fabric of common ground in dual-interaction online environments.

Multimodal interaction spaces—which typically bring together two or more synchronous online communication technologies such as text chat and a shared graphical workspace—have been widely used to support collaborative learning activities of small groups (Dillenbourg & Traum, 2006; Jermann, 2002; Mühlpfordt & Wessner, 2005; Soller & Lesgold, 2003; Suthers et al., 2001). The way such systems are designed as a juxtaposition of several technologically independent online communication tools carries important interactional consequences for the users. Engaging in forms of joint activity in such online environments requires group members to use the technological features available to them in methodical ways to make their actions across multiple spaces intelligible to each other and to sustain their joint problem-solving work.

The communicative processes mediated by multimodal interaction spaces have attracted increasing analytical interest in the CSCL community. A workshop held at CSCL 2005 specifically highlighted the need for more systematic ways to investigate the unique affordances of such online environments (Dillenbourg, 2005). Previous CSCL studies that focus on the interactions mediated by systems with two or more interaction spaces can be broadly categorized under: (1) prescriptive approaches based on models of interaction and (2) descriptive approaches based on content analysis of user actions.

(1) The *prescriptive modeling approach* builds on a content-coding approach by devising models of categorized user actions performed across multimodal interaction spaces, for example:

(a) Soller & Lesgold's (2003) use of hidden Markov models (HMM) and

(b) Avouris et al's (2003) object-oriented collaboration analysis framework (OCAF).

In these studies, the online environment is tailored to a specific problem-solving situation so that researchers can partially automate the coding process by narrowing the possibilities for user actions to a well-defined set of categories. The specificity of the problem-solving situation also allows researchers to produce models of idealized solution cases. Such ideal cases are then used as a baseline to make automated assessments of group work and learning outcomes.

(2) The *descriptive approach* informed by content analysis also involves categorization of user actions mediated by multimodal interaction spaces, applying a theoretically informed coding scheme. Categorized interaction logs are then subjected to statistical analysis to investigate various aspects of collaborative work such as:

(c) The correlation between planning moves performed in chat and the success of subsequent manipulations performed in a shared workspace (Jermann, 2002; Jermann & Dillenbourg, 2005),

(d) The relationship between grounding and problem-solving processes across multiple interaction spaces (Dillenbourg & Traum, 2006),

(e) A similar approach based on cultural-historical activity theory (Baker et al., 1999), and

(f) The referential uses of graphical representations in a shared workspace in the absence of explicit gestural deixis (Suthers, Girardeau & Hundhausen, 2003).

We will now review each of these studies:

(a) Soller and Lesgold's modeling approach involves the use of Hidden Markov Models (HMM) to automatically detect episodes of effective knowledge sharing (Soller & Lesgold, 2003) and knowledge breakdowns (Soller, 2004). The authors consider a programming task where triads are asked to use object-oriented modeling tools to represent relationships among well-defined entities. The task follows a jigsaw design where each group member receives training about a different aspect of the shared task before meeting with other members. The group sessions are hosted in the Epsilon online environment, which includes a text-chat area and a shared workspace. The workspace provides basic shapes that allow users to diagrammatically represent entities and relationships. Participants are required to select a sentence opener to categorize their contributions before posting them in the chat window. The authors manually extract segments from their corpus where each member gets the opportunity to share the unique knowledge element he/she was trained in with other group members. Some of these episodes are qualitatively identified as ideal cases that exemplify either an instance of effective knowledge sharing or a knowledge breakdown, completely based on the results of post-tests. For instance, a segment is considered an effective knowledge-sharing episode provided a chance for demonstrating the unique knowledge element comes during the session, the presenter correctly answers the corresponding questions in both pre- and post-tests, and the explanation leads at least one other member to correctly answer the corresponding question(s) in the post-test. The sequence of categorized actions (including chat postings and workspace actions) that correspond to these ideal cases is used to train two separate HMMs for the breakdown and effective knowledge sharing cases, respectively. An HMM computes the probability of a certain kind of action immediately following another; it thus captures certain aspects of sequentiality. These models are then used to automatically classify the remaining episodes and to assess team performance. However, the method is seriously limited to recognizing connections among actions to those based on immediate sequences of codes. While this can capture adjacency pairs that are important to conversation, it misses more distant responses, interrupted adjacency pairs, temporal markings and semantic indexes. The authors apparently make no specific distinction between workspace and chat actions as they build their HMMs over a sequence of interface actions. Moreover, the relationship between object diagrams constructed in the workspace and the explanations given in chat do not seem to be considered as part of the analysis. Hence, it is not clear from the study how a successful knowledge-sharing episode is achieved in interaction and whether the way participants put the affordances of both interaction spaces into use as they explain the materials to each other have had any specific influence on that outcome. Although they were reported to be successful in classifying manually segmented episodes, HMMs computed over a sequence of categorized actions seem to obscure these interactional aspects of the coordination of chat and workspace.

(b) The modeling approach outlined in Avouris et al. (2003) and Komis et al. (2002) proposes a methodology called the object-oriented collaboration analysis framework (OCAF) that focuses on capturing the patterns in the sequence of categorized actions through which dyads co-produced objects in a shared task space. The collaborative tasks the authors used in their online study included the construction of database diagrams with well-defined ontological elements such as entities, relationships and attributes. In this problem-solving context the final representation co-constructed in the shared workspace counted as the group's final solution. The OCAF model aims to capture the historical evolution of the group's solution by keeping track of who contributed and/or modified its constituent elements during the course of an entire chat session. The authors not only consider direct manipulation acts on specific elements but also chat statements through which actors propose additions/modifications to the shared diagram or agree/disagree with a prior action. The chat and drawing actions are categorized in terms of their functional roles (e.g., agree, propose, insert, modify, etc.). The mathematical model includes the sequence of categorized actions and the associations among them. The model is then used to gather structural properties of interactions (e.g., how contributions are distributed among dyads, what functional role each contribution plays) and to trace how each action performed in the interface is related to other actions. This modeling approach differs from similar approaches in terms of its specific focus on the objects co-constructed in the shared workspace. The model captures the sequential development of the shared object by keeping track of the temporal order of contributions made by each user. However, it is not clear from the study how the model could deal with the flexibility of referential work. For instance a chat posting may refer to multiple prior postings or to a sub-component of a more complicated entity-relationship diagram by treating several elemental objects as a single object. In other words, a model trying to capture all possible associations between individual actions in a bottom-up fashion may miss the flexibility of referential work and obscure the interactional organization.

(c) Jermann (2002) employs a coding scheme to study the correlation between planning moves in the chat area and the success of subsequent manipulations performed on the shared simulation in the Traffic Simulator environment. The shared task involved students tuning red-green periods of four traffic lights in the simulation to figure out an optimal configuration to minimize the waiting time of cars at intersections. The workspace could be manipulated in specific ways by users. The workspace also includes a dynamic graph that shows the mean waiting time for the cars. The goal of the task is to keep the mean value below a certain level for two minutes. The study included additional experimental cases where dynamically updated bar charts are displayed to provide feedback to users about their level of participation. The logs of recorded sessions are coded in terms of their planning and regulatory content. The nature of the task allowed authors to numerically characterize different types of work organizations in terms of the distribution of manipulations performed on four possible traffic lights. The authors complement this characterization with number of messages posted, number of manipulations done and the types of messages as captured in the coding scheme. The study reported that dyads who coordinated their actions across both interaction spaces by planning what to do next (i.e., task regulation) and discussing who should do what (i.e., interaction regulation) in chat before manipulating the simulation performed better (i.e., achieved the objective more quickly). The interaction meters were not reported to have significant effects on promoting task and interaction regulation. The work of high performance groups are characterized with phrases like “posted more messages,” “more frequent postings,” “talked relatively more than they executed problem solving actions,” “monitor results longer,” “produced elaborated plans more frequently” in reference to the tallied codes, frequency of messages and duration of activity. Although the main argument of the chapter highlights the authors’ interest in sequential unfolding of regulatory moves, the way the employed quantitative approach isolates and aggregates the actions obscures the temporal connections and sequential mechanisms constituting different forms of regulation moves.

(d) Dillenbourg & Traum (2006) employ a similar methodology to study the relationship between grounding and problem solving in an online environment including a shared whiteboard and a text-chat area. In this study the participants were grouped into dyads and asked to collaboratively work on a murder-mystery task. The authors framed their analysis along the lines of Clark & Brennan’s (1991) theory of grounding (at least applied at the micro level of individual utterances) and theories of socio-cognitive conflict. The study reports two kinds of uses of the dual spaces to facilitate grounding during problem solving: a “napkin” model and a “mockup” model. The authors hypothesized that the whiteboard would be mainly used to disambiguate dialogues in the chat window via basic illustrations (i.e., the napkin model). However, the authors report that the dyads used the whiteboard for organizing factual information as a collection of text boxes, and the chat component was mainly used to disambiguate the information developed on the whiteboard (i.e., the mockup model). The authors attributed this outcome to the nature of the task, which required users to keep track of numerous facts and findings about the murder case, and the difference between the two media in terms of the *persistency* of their contents. Since participants organized key factual information relevant to the problem at hand on the shared whiteboard during their experiments, the authors attributed a shared external memory status to this space and claimed that it facilitated grounding at a broader level by offering a more persistent medium for storing agreed upon facts. The study succeeds in highlighting the important role of medium persistence, even if it does not specify the methods by which students exploited such temporal persistence.

(e) Baker et al. (1999) provide a theoretical account of collaborative learning by bringing together the processes of grounding and appropriation from psycholinguistics and cultural-historical activity theory (CHAT), respectively. In their study they focus on the interactions mediated by the C-Chene software system where dyads are tasked to co-construct energy models that account for storage, transfer and transformation of energy (Baker & Lund, 1997). The models for energy-chains are constructed in a shared workspace that allows the addition of annotated nodes and directed edges. Participants also have access to a chat area that can be customized with sentence openers, which are claimed to promote reflective contributions, reduce typing effort and minimize off-task discussion. The interface is designed to allow only one user to produce a contribution in a given interaction interval. The users need to press a button to switch between dual interaction spaces. Hence the possibility of parallel or overlapping work (e.g., one user drawing on the board as the other is typing a message) is ruled out on the grounds that this would hinder collaboration. The dyads also could not overlap in typing since they need to take turns to use the dialog box where they type their messages. However, it is possible for a user to interrupt his/her partner through a special prompt, which asks whether it is okay to take the turn. If the partner agrees, then the turn is passed to the other user. The study reported that dyads who used the structured interface exhibited more reflective and focused discussion. The authors point to limitations involved with constraining user actions to fixed categories, but they argued that some of the sentence openers they used correspond to generic speech acts that were used for multiple purposes in the course of interaction.

(f) Suthers et al. (2003) investigate the *referential* uses of shared representations in dyadic online discourse mediated by the Belvedere system. This environment has a chat area as well as a shared workspace where dyads can co-construct evidence maps to represent their arguments as a set of categorized textboxes linked to each other (Suthers et al., 2001). The study compares face-to-face and online cases to investigate how dyads use the system as a conversational resource in each case as they work on a shared task that involves developing hypotheses about the spreading of a disease at a remote island. Categories for deictic uses such as finger pointing, cursor-based deixis, verbal deixis and direct manipulation of objects are identified and applied to the session logs. Based on the distributions of these categories for each case, the authors report that dyads in the online case made use of verbal deixis and direct manipulation of shared objects to compensate for the limitations of the online environment to achieve referential relationships across dual interaction spaces. Moreover, the study reports that such referential links are more likely to be observed between temporally proximal actions. For instance, a chat posting including a deictic term is likely to be read in relation to a node recently added to the shared representation.

Our review of relevant work in the CSCL literature highlights some common threads in terms of methodological approaches and theoretical orientations.[5] First, these studies all focus on the group processes of collaboration, rather than treating it as a mere experimental condition for comparing the individuals in the groups. Second, they employ a content-coding approach to categorize actions occurring in multiple interaction spaces. In most cases, representational features like sentence openers or nodes corresponding to specific ontological entities are implemented in the interface to guide/constrain the possibilities for interaction. Such features are also used to aid the categorization of user actions. The categorization schemes are applied to recorded logs and subjected to statistical analysis to elicit interaction patterns.

The analytic thrust of these studies is to arrive at quantitative results through statistical comparisons of aggregated data. To accomplish this, they generally have to restrict student actions in order to control variables in their studies and to facilitate the coding of student utterances within a fixed ontology. We fear that this unduly restricts the interaction, which must be flexible enough to allow students to invent unanticipated behaviors. The restrictions of laboratory settings make problematic experimental validity and generalization of results to real-world contexts. Even more seriously, the aggregation of data—grouping utterances by types or codes rather than maintaining their sequentiality—ignores the complexity of the relations among the utterances and actions. According to our analysis, the temporal and semiotic relations are essential to understanding, sharing and coordinating meaning, problem solving and cognition. While quantitative approaches can be effective in testing model-based hypotheses, they seem less appropriate both for exploring the problem of interactional organization and for investigating interactional methods, which we feel are central to CSCL theory.

Despite the accomplishments of these studies, we find that their approaches introduce systematic limitations. Interactional analysis is impossible because coherent excerpts from recorded interactions are excluded from the analysis itself. (Excerpts are only used anecdotally, outside of the analysis, to introduce the features

[5] We do not intend to minimize the contributions of the particular papers or authors reviewed. On the contrary, we have selected exemplary CSCL studies in order to make a methodological comparison. The quantitative studies may be effective in pursuing their research questions, but their approaches are inadequate for understanding common ground qualitatively. Some of these authors have also adopted case-study approaches more recently; to take only examples from one of the labs, see the studies of deixis, interactional up-take and narrative structure in (Dwyer & Suthers, 2006; Suthers, 2006; Yukawa, 2006).

of the system to the reader, to illustrate the categorization schemes employed or to motivate speculative discussion). Moreover, most studies like these involve dyads working on specific problem-solving contexts through highly structured interfaces in controlled lab studies in an effort to manage the complexity of collaboration. The meanings attributed by the researchers to such features of the interface need to be discovered/unpacked by the participants as they put them into use in interaction—and this critical process is necessarily ignored by the methodology. Finally, most of these papers are informed by the psycholinguistic theory of common ground, and are unable to critique it systematically. By contrast—as we shall see in the following section—our analysis of the joint organization of interaction in the case study positions us to understand how the group grounds its shared understanding in interactional terms at the group level.

Grounding through Interactional Organization

The coordination of visual and linguistic methods (across the whiteboard and chat workspaces) plays an important role in the establishment of common ground through the co-construction of references between items in the different media within the VMT environment. Particularly in mathematics—with its geometric/algebraic dual nature—symbolic terms are often grounded in visual presence and associated visual practices, such as counting or collecting multiple units into a single referent (Goodwin, 1994; Healy & Hoyles, 1999; Livingston, 2006; Sfard, 2008; Wittgenstein, 1944/1956). The visually present can be replaced by linguistic references to objects that are no longer in the visual field, but that can be understood based on prior experience supported by some mediating object such as a name—see the discussion of mediated memory and of the power of names in thought by Vygotsky (1930/1978; 1934/1986). A more extended analysis of the co-construction of mathematical artifacts by virtual math teams, the complementarity of their visual, semantic and symbolic aspects, their reliance on pre-mathematical practices and processes of reification into concepts are beyond the scope of this chapter and require comparison of multiple case studies (see Çakir, 2009). However, for this chapter it is important to understand something of how the interactional organization that we have observed here functions to ground the group's understanding of their math object (the hexagonal array) as a shared group achievement.

As implied in the OCAF study (Avouris et al., 2003) mentioned in the previous section, investigating grounding and problem-solving processes in online dual-interaction environments like VMT requires close attention to the relationships among actions performed in multiple interaction spaces. Our case study illustrates some of the practical challenges involved with producing mathematical models that aim to exhaustively capture such relationships. For instance, the hexagonal array that was co-constructed by the team draws upon a triangular grid that is formed by three sets of parallel lines that intersect with each other in a particular way. In other words, these objects are layered on top of each other by the participants to produce a shape recognizable as a hexagon. Despite this combinatoric challenge, a modeling approach can still attempt to capture all possible geometric relationships among these graphical objects in a bottom-up fashion. However, when all chat messages referring to the whiteboard objects are added to the mix, the resulting model may obscure rather than reveal the details of the interactional organization through which group members discuss more complicated mathematical objects by treating a collection of atomic actions as a single entity. Terminology co-constructed in the chat-and-whiteboard environment—like "**hexagonal array**"—can refer to complexly defined math objects. What is interesting about the student knowledge building is how they aggregate elements and reify them into higher-order, more powerful units (Sfard, 2008). A model should mirror this rather than to simply represent the elements as isolated.

The challenges involved with the modeling approach are not limited to finding efficient ways to capture all relationships among actions and identifying meaningful clusters of objects. The figurative uses of the graphical objects present the most daunting challenge for such an undertaking. For instance, the team members in our case study used the term "**hexagonal array**" to refer to a mathematical object implicated in the witnessed production of prior drawing actions. As we have seen in the way the team used this term during their session, "**hexagonal array**" does not simply refer to a readily available whiteboard illustration. Instead it is used as a *gloss* (Garfinkel & Sacks, 1970) to talk about an imagined pattern that grows infinitely and takes the shape illustrated on the whiteboard only at a particular stage. In the absence of a fixed set of ontological elements and constraints on types of actions a user can perform, modeling approaches that aim to capture emergent relationships among semiotic objects distributed across multiple interaction spaces need to adequately deal with the retrospective and prospective uses of language in interaction. Rather than relying upon a generic approach to modeling imposed by the researchers, our ethnographic approach aims to discover the unique "model"—or, better, the specific meaning—that was constructed *by the group* in its particular situation.

In another study discussed earlier, Dillenbourg & Traum (2006) offer the napkin and mockup models in their effort to characterize the relationship between whiteboard and chat spaces. In short, these models seem to describe two use scenarios where one interaction space is subordinated to the other during an entire problem-solving session. The complex relationships between the actions performed across both interaction spaces in our case made it difficult for us to describe the interactions we have observed by committing to only one of these models, as Dillenbourg & Traum did in their study. Instead, we have observed that in the context of an open-ended math task, groups may invoke either type of organization, depending upon the contingencies of their ongoing problem-solving work. For instance, during long episodes of drawing actions where a model of some aspect of the shared task is being co-constructed on the whiteboard (as in our first excerpt), the chat area often serves as an auxiliary medium to coordinate the drawing actions, which seems to conform to the mockup model. In contrast, when a strategy to address the shared task is being discussed in chat (as in the excerpt where the group considered splitting the hexagon into six regions), the whiteboard may be mainly used to quickly illustrate the textual descriptions with annotations or rough sketches, in accordance with the napkin model. Depending on the circumstances of ongoing interaction, participants may switch from one type of organization to another from moment to moment. Therefore, instead of ascribing mockup and napkin models to entire problem-solving sessions, we argue that it would be more fruitful to use these terms as glosses or descriptive categories for types of interactional organizations group members may invoke during specific episodes of their interaction.

Another provocative observation made by Dillenbourg & Traum is that the whiteboard serves as a kind of shared external memory where group members keep a record of agreed-upon facts. In their study the dyads were reported to post text notes on the whiteboard to keep track of the information they had discovered about a murder-mystery task. This seems to have led the authors to characterize the whiteboard as a placeholder and/or a shared working memory for the group, where agreed-upon facts or "contributions" in Clark's sense are persistently stored and spatially organized. As Dillenbourg & Traum observed, the scale of what is shared in the course of collaborative problem solving becomes an important issue when a theory operating at the utterance level like contribution theory (Clark & Marshall, 1981) is used as an analytic resource to study grounding processes that span a longer period of time. Dillenbourg & Traum seem to have used the notion of persistence to extend common ground across time to address this limitation. In particular, they argued that the whiteboard grounds the solution to the problem itself rather than the contributions made by each utterance. In other words, the whiteboard is metaphorically treated as a physical manifestation of the common ground. We certainly agree with this broadening of the conceptualization of common ground, although we do not see the whiteboard as just a metaphor or externalization of a mental phenomenon. Rather, *common ground is established in the discourse spaces* of text chat and graphical whiteboard. Their differential forms of persistence provide a continuing resource for sharing, modifying and remembering the group meaning of joint artifacts and products of group cognition.

In our case study, we have observed that the whiteboard does not simply serve as a kind of shared external memory where the group keeps a record of agreed upon facts, opinions, hypotheses or conclusions. The shared visible communication media are places where the group does its work, where it cognizes. Ideas, concepts, meanings, etc. can *subsequently* be taken up by individuals into their personal memories as resources for future social or mental interactions. There is no need to reduce group meaning to identical individual mental contents or to hypothesize a mysterious "group mind" as the location of common ground—the location is the discourse medium, with all its particular affordances and modes of access.

In our sessions, the whiteboard was primarily used to draw and annotate graphical illustrations of geometric shapes, although users occasionally posted textboxes on the whiteboard to note formulas they had found (see Figure 7-2 above). While the whiteboard mainly supported visual reasoning—and textual discussion or symbolic manipulation occurred chiefly in the chat stream—actions were carefully, systematically coordinated across the media and integrated within an interactionally organized group-cognitive process. As we have illustrated in our analysis, the fact that there were inscriptions posted on the whiteboard did not necessarily mean that all members immediately shared the same sense of those graphical objects. The group members did considerable interactional work to achieve a shared sense of those objects that was adequate for the purposes at hand. For instance, the crosshatched lines that Qwertyuiop originally drew became increasingly meaningful for the group as it was visually outlined and segmented and as it was discussed in the chat and expressed symbolically.

Hence, the whiteboard objects have a different epistemic status in our case study than in Dillenbourg & Traum's experiment. Moreover, the participants did not deem all the contents of the whiteboard relevant to the ongoing discussion. For instance, Figure 7-2 above shows a snapshot of the entire whiteboard as the team was discussing the hexagonal pattern problem. The figure shows that there are additional objects in the shared scene like a blue hypercube and a 3-D staircase, which are remnants of the group's prior problem-solving work. Finally, the sense of previously posted whiteboard objects may be modified or become evident as a result of current actions (Suchman, 1990).

In other words, group members can not only reuse or reproduce drawings, but they can also make subsequent sense of those drawings or discard the ones that are not deemed relevant anymore. Therefore, the technologically extended notion of common ground as a placeholder for a worked-out solution suffers from the same issues stated in Koschmann & LeBaron's (2003) critique of Clark's theory. As an abstract construct transcendental to the meaning-making practices of participants, the notion of common ground obscures rather than explains the ways the whiteboard is used as a resource for collaborative problem solving.

Instead of using an extended version of common ground as an analytical resource we frame our analysis using the notion of "indexical ground of deictic reference," which is a notion we appropriated from linguistic anthropology (Hanks, 1992). In face-to-face interaction, human action is built through the sequential organization of not only talk but also coordinated use of the features of the local scene that are made relevant via bodily orientations, gesture, eye gaze, etc. In other words, "human action is built through simultaneous deployment of a range of quite different kinds of semiotic resources" (Goodwin, 2000, p. 1489). Indexical terms and referential deixis play a fundamental role in the way these semiotic resources are interwoven in interaction into a coherent whole.

Indexical terms are generally defined as expressions whose interpretation requires identification of some element of the context in which it was uttered, such as who made the utterance, to whom it was addressed, when and where the utterance was made (Levinson, 1983). Since the sense of indexical terms depends on the context in which they are uttered, indexicality is necessarily a relational phenomenon. Indexical references facilitate the mutually constitutive relationship between language and context (Hanks, 1996). The basic communicative function of indexical-referentials is "to individuate or single out objects of reference or address in terms of their relation to the current interactive context in which the utterance occurs" (Hanks, 1992, p. 47).

The specific sense of referential terms such as *this*, *that*, *now*, *here* is defined locally by interlocutors against a shared indexical ground. Conversely, the linguistic labels assigned to highlighted features of the local scene shapes the indexical ground.

Hence, the indexical ground is not an abstract placeholder for a fixed set of registered contributions. Rather, it signifies an emergently coherent field of action that encodes an interactionally achieved set of background understandings, orientations and perspectives that make references intelligible to interlocutors (Zemel et al., 2008).

Despite the limitations of online environments for supporting multimodality of embodied interaction, participants make substantial use of their everyday interactional competencies as they appropriate the features of such environments to engage with other users. For instance, Suthers et al.'s (2003) study reports that deictic uses of representational proxies play an important role in the interactional organization of online problem-solving sessions mediated by the Belvedere system. The authors report that participants in the online case devised mechanisms that compensate for the lack of gestural deixis with alternative means, such as using verbal deixis to refer to the most recently added text nodes and visual manipulation of nodes to direct their partner's attention to a particular node in the shared argument map.

In contrast to the Belvedere system, VMT offers participants additional resources such as an explicit referencing mechanism, a more generic workspace that allows producing and annotating drawings, and an awareness feature that produces a sense of sequentiality by embedding indicators for drawing actions in the sequence of chat postings. Our case study shows that despite the online situation's lack of the familiar resources of embodied interaction, team members can still achieve a sense of shared access to the meaningful objects displayed in the dual interaction spaces of the VMT environment. Our analysis indicates that coherence among multiple modalities of an online environment like VMT is achieved through group members' development and application of shared methods for using the features of the system to coordinate their actions in the interface.

Through coordinated use of indexical-referential terms and highlighting actions, team members help each other to literally "see" the objects implicated in the shared visual field (Goodwin, 1994) and to encode them with locally specified terminology for subsequent use. They demonstrate how to "read" graphical as well as textual objects through the way the objects are built up sequentially and are spatially arranged in relation to each other through sequences of actions. The deictic references that link chat messages to features of graphical inscriptions and to prior chat content are instrumental in the sequential achievement of indexical symmetry, intersubjectivity or common ground.

Sequential Analysis of the Joint Organization of Interaction

To sum up, the focus of our ethnomethodological inquiry is directed towards documenting how a virtual team achieved intersubjectivity and coherence among their actions in an online CSCL environment with multiple interaction spaces. We looked at the moment-to-moment details of the practices through which participants organize their chat utterances and whiteboard actions as a coherent whole in interaction—a process that is central to CSCL. We observed that referential practices enacted by the users are essential, particularly in the coordinated use of multimodalities afforded by environments like VMT. The referential uses of available features are instrumental not only in allocating other members' attention to specific parts of the interface where relevant actions are being performed, but also in the achievement of reciprocity (intersubjectivity, common ground, shared understanding, group cognition) among actions in the multiple interaction spaces, and hence a sense of sequential organization across the spaces.

In our case study, we have seen the establishment of an indexical ground of deictic references co-constructed by the group members as an underlying support for the creation and maintenance of their joint problem space. We have seen that nexus of references created interactionally as group members propose, question, repair, respond, illustrate, make visible, supply symbols, name, etc. In the VMT dual-media environment, the differential persistence, visibility and mutability of the media are consequential for the interaction. Group members develop methods of coordinating chat and drawing activities to combine visual and conceptual reasoning by the group and to co-construct and maintain an evolving shared indexical ground of their discourse.

In this chapter, we have *reconceptualized the problem of common ground from an issue of sharing mental representations to a practical matter of being able to jointly relate semiotic objects to their indexed referents*. The references do not reside in the minds of particular actors, but have been crafted into the presentation of the chat postings and drawing inscriptions through the details of wording and sequential presentation. The references are present in the data as affordances for *understanding* by group participants as well as by analysts (Stahl, 2006, ch. 17). The *meaning* is there in the visual presentation of the communication objects and in the network of interrelated references (Chapter 26), rather than in mental re-presentations of them. The understanding of the references is a matter of normally tacit social practice, rather than of rationalist explicit deduction. *The references can be explicated by analysis, but only if the structure of sequentiality and indexicality is preserved in the data analysis and only if the skill of situated human understanding is applied*.

In our case study of an 18-minute excerpt taken from a four-hour group chat, three students construct a diagram of lines, triangles and hexagons, propose a math pattern problem, analyze the structure of their diagram and derive an algebraic formula to solve their problem. They propose their own creative problem about mathematical properties; gradually construct a complex mathematical object; explore related patterns with visual, narrative and symbolic means; express wonder; gain mathematical insight and appreciate their achievement. They do this by coordinating their whiteboard and chat activities in a synchronous online environment. Their accomplishment is precisely the kind of educational math experience recommended by mathematicians (Livingston, 2006; Lockhart, 2008; Moss & Beatty, 2006). It was not a mental achievement of an individual, but a group accomplishment carried out in computer-supported discourse. *By analyzing the sequentiality and indexicality of their interactions, we explicated several mechanisms of the group cognition by which* *the students coordinated the group meaning of their discourse and maintained an effective joint problem space*.

The coordination of visual and textual realizations of the mathematical objects that the students co-construct provides a grounding of the algebraic formulas the students jointly derive using the line drawings that they inspect visually together. As the students individualize this experience of group cognition, they can develop the deep understanding of mathematical phenomena that comes from seeing the connections among multiple realizations (Sfard, 2008)(Chapter 3). Our case study does not by any means predict that all students can accomplish similar results under specific conditions, but merely demonstrates that such group cognition is possible within a synchronous CSCL setting and that a fine-grained sequential analysis of interaction can study how it is collaboratively accomplished.

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