Math Forum &
Virtual Math Teams Project
present:

Explore Dynamic Geometry Together
Introduction

Dynamic geometry is a new form of mathematics—and you can be a pioneer in it, exploring, discovering and creating new insights and tools. Dynamic geometry realizes some of the potential that has been hidden in geometry for thousands of years: by constructing dynamic-geometry figures that incorporate carefully designed mathematical relationships and dependencies, you can drag geometric objects around to investigate their general properties.

Geometry has always been about constructing dependencies into geometric figures and discovering relationships that are therefore necessarily true and provable. Dynamic geometry makes the construction of dependencies clear. The topics here will teach you to think about geometry this way and to design constructions with the necessary dependencies. The sequence of topics is designed to give you the basic knowledge to think about dynamic-geometric dependencies and to construct figures with them.

The activities in this workbook raise questions about geometric relationships in figures. You can answer them in terms of how a figure was designed and constructed with dependencies among its parts:

• Does that figure have to have that relationship?
• Does it remain true when dragged?
• Is the relationship necessarily true?
• How do you know if it is true?
• Can you demonstrate it or construct it?
• Can you understand, explain or prove why it has to be true?

In addition, the approach of these activities allows you to take advantage of the power of collaboration. Your “virtual math team” (VMT) can accomplish more than any one of you could on your own. You can chat about what you are doing, and why. You can discuss what you notice and wonder about the dynamic figures. Working in a team will prevent you from becoming stuck. If you do not understand a geometry term or a task description, someone else in the team may have a suggestion. If you cannot figure out the next step in a problem or a construction, discuss it with your team. Decide how to proceed as a team.

To harness the truly awesome power of collaborative dynamic geometry requires patience, playfulness and persistence. It will pay off by providing you with skills, tools and understanding that will be useful for a lifetime. You will need to learn how to construct complicated figures; this will be tricky and require practice. You will need to think about the hidden dependencies among dynamic points that make geometry work; this may keep you up at night. You will even create your own custom construction tools to extend the GeoGebra software for dynamic geometry; this will put you in control of mathematics.

The following activities present a special approach to dynamic geometry, which may be quite different from other approaches to learning geometry. They focus specifically on the core idea of dynamic geometry: how to design the construction of figures with dependencies. They can help you understand geometry better, whether you already studied geometry, are studying it now or will study it in school later. They will give you an understanding of necessary relationships in geometric figures. Dragging points around in pre-constructed GeoGebra apps can help you to discover and visualize relationships within a figure. However, such apps may also hide the dependencies that maintain the relationships. You should know how to construct those dependencies yourself. Chatting about the topics in a small group will help you to think about geometry and problem solving on your own in the future.

Each of the topics here is designed for an online team to work on together for about one hour. The sequence of topics introduces you and your team to GeoGebra and guides you in the exploration of dynamic geometry. Several tours are included at the end of this document, which provide tutorials in important features of the VMT and GeoGebra software. The pages of this document can be used as worksheets to keep notes on and if the instructions in the chat rooms become erased.
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Topic 0: Individual Warm-up Activity

You can start on this Warm-up activity by yourself at any time. Do this topic on the computer you will be using with your team—before your first collaborative group session. This will check that your computer is properly set up and that you are able to enter VMT chat rooms. It also introduces you to GeoGebra, our dynamic-geometry software system.

Here is a general procedure you might want to follow for each of the following topics: Before the time assigned for your group session, read the topic description in this document. It might suggest watching a brief video or taking a tour at the end of this document as important preparation. Think about the topic on your own. Then, in the group session, chat about the topic and work together on the various tasks. Discuss what you are doing in the chat and respond to the questions posed in the topic. You can mark down in your copy of this document what you noticed that surprised you and what you wondered about that you want to think more about later. Do not just rush through the topic; discuss what is important in it. The point is to learn about collaborative dynamic geometry, not just to get through the topic steps. When the session is over, try to work on your own on any parts that your team did not get to. Report to your team what you discover.

Each topic is designed for a team to spend about one hour discussing together. Some teams may want to spend more or less time on certain topics. Decide in your team if you want to go back to the chat room for a previous topic before continuing to a new topic. The rooms will remain available for you to go to either alone or with your team.

First, you can get a good introduction to using the GeoGebra software from this 5 minute YouTube clip: http://www.youtube.com/watch?v=2NqblD1P138. It gives a clear view of how to use GeoGebra. It also provides tips on GeoGebra tools that are equivalent to traditional geometry construction with a physical straightedge (for making line segments) and compass (for making circles).

Then, read “Tour 1: Joining a Virtual Math Team.” This will introduce you to using the VMT software to register, login, find a chat room, etc. The Tours are all found toward the end of this booklet, after all the Topics.

Section 0.01 Welcome!

Start in the Welcome tab (shown below) of the Warm-Up “Topic 0” chat room by reading the instructions and then dragging the objects shown. Press the “Take Control” button at the bottom, then select the “Move Tool” arrow in the Tool Bar near the top and click on a point in the geometry figure to drag it.

Next, create your own similar geometry objects using the buttons in the Tool Bar. Create points, lines, circles and triangles. Drag them around and notice how they change. To create a triangle, use the polygon tool and click on three points to define the triangle and then click again on the first point to complete the triangle.

You can change to one of the other tabs, like “#1 Team Member” and press the “Take Control” button there to create your own points and lines in your own empty GeoGebra workspace. Try out each of the tools in the Tool Bar.
If there is not an unused tab available to create your points and lines in, you can create a new tab with the “Add a tab +” button in the upper right corner of the VMT window. You can use the “ABC” text tool to construct a text box with your name and a title for your figure: Select the text tool and then click in the tab about where you want to have the text box. A form will open for you to enter your text.

Section 0.02 Helpful Hints

In the “Helpful Hints” tab, there are some hints for taking advantage of the limited space on your computer screen. Read these hints and adjust the size of your VMT window and its GeoGebra tab.

There are a number of Zoom Tools, which can be pulled down from the Move Graphic Tool (the crossed arrows on the right end of the Tool Bar).

If you are using a touchpad on your computer, you can zoom with a two-finger touchpad gesture. **Caution:** It is easy to move things around without wanting to as your fingers move on the touchpad. Things can quickly zoom out of sight. This can happen even when you do not have control of construction. Then you will not see the same instructions and figures that other people on your team see.

**Note:** Most Zoom Tools will only effect what **you** see on your computer screen, not what your teammates see on theirs. It is possible for you to create points on an area of your screen that your teammates do not see. If this happens, ask everyone if they want you to adjust everyone’s screen to the same zoom level and then select the menu item “GeoGebra” | “Share Your View.”

When you work in a team, it is important to have everyone agree before changes are made that affect everyone. If you want to delete a point or other object – especially one that you did not create – be sure that everyone agrees it should be deleted.

**Warning:** Be very careful when deleting points or lines. If any other objects are dependent on them, those objects will also be deleted. It is easy to unintentionally delete a lot of your group’s work.

Instead of deleting objects, you should usually hide them. Then the objects that are dependent on them will still be there and the dependencies will still be in effect. Use control-click (on a Mac) or right-click (in Windows) with the cursor on a point or line to bring up the context-menu. Select “Show Object” to
unclick that option and hide the object. You can also use this context-menu to hide the object’s label, rename it, etc.

You may want to save the current state of your GeoGebra tab to a .ggb file on your desktop sometimes so you can load it back if things are deleted. Use the menu “File” | “Save” to save your work periodically. Use the menu “File” | “Open” to load the latest saved version back into the current tab. Check with everyone in your team because this will change the content of the tab for everyone.

If the instructions in a tab are somehow erased, you can look them up in this document. You can also scroll back in the history of the tab to see what it looked like in the past. Finally, you can have your whole team go to a different room if one is available in the VMT Lobby that is not assigned to another team.

If you have technical problems with the chat or the figures in the tabs not showing properly, you should probably close your VMT window and go back to the VMT Lobby to open the room again.

You can use this booklet to keep notes on what you learn or wonder, either on paper or on your computer. Here is a space for notes on Topic 0:

Notes:
Topic 1: Creating Dynamic Points, Lines, Circles

Dynamic geometry is an innovative form of mathematics that is only possible using computers. It is based on traditional Euclidean geometry, but has interesting objects, tools, techniques, characteristics and behaviors of its own. Understanding dynamic geometry will help you think about other forms of geometry and mathematics.

In this topic, you will practice some basic skills in dynamic geometry. There are several tabs to work through in most topics; try to do them all with your team. Pace yourselves. Make sure that everyone in your team understands the important ideas in a tab and then have everyone move to the next tab.

First, if you did not already do this, watch this YouTube clip:

It gives a clear view of how to use GeoGebra. It provides important tips on GeoGebra tools that are equivalent to traditional geometry construction with a physical straightedge (for making line segments) and compass (for making circles).

Also look through “Tour 2: GeoGebra for Dynamic Math.” This will provide more details about the GeoGebra system for dynamic geometry. For instance, it will describe many of the buttons shown in the Tool Bar. You can always go back to these tours if you become stuck using VMT or GeoGebra.

As your teammates and you work on the Topics, discuss in the chat explicitly what you notice, what it means to you and what you wonder about. Chat about what geometric relationships among the objects you notice. Afterward, list what you wonder about these relations. Right answers are not the main goal of these activities, so use only the ideas that your teammates and you jointly developed, not ideas from the Internet or elsewhere.

Working Collaboratively. Since the goal of this course is for you to engage your teammates in thinking together about mathematical objects and relations and to collaborate productively, you need to communicate effectively. As you interact in VMT, keep in mind the following general guidelines for collaborative work:

Read other people’s chat postings to:
- Be prepared to refer to and connect to someone else’s ideas.
- Get thoughts on open questions.
- Get new perspectives.

Write your own chat postings to:
- Make your thinking available for the group.
- Develop your thinking.
- Get feedback on your ideas.
- Give feedback to others.

In general, try to include in your chat: (a) what you think should be done, (b) what you are doing and (c) the significance of what you did. Take turns doing steps. Work together as a group, rather than just trying to do the whole thing by yourself.

Section 1.01 Dynamic Points, Lines & Circles

Geometry begins with a simple point. A point is just the designation of a particular location. In dynamic geometry, a point can be dragged from its current location to any other location.

Everything in geometry is built up from simple points. For instance, a line segment is made up of all the points (the “locus”) along the shortest (direct, straight) path between two points (the endpoints of the segment).
A circle is all the points ("circumference,” “locus”) that are a certain distance (“radius”) from one point ("center"). Therefore, any line segment from the center point of a circle to its circumference is a radius of the circle and is necessarily the same length as every other radius of that circle. Even if you drag the circle and change its size and the length of its radius, every radius will again be the same length as every other radius of that circle.

In this tab, create some basic dynamic-geometry objects and drag them to observe their behavior. Take turns taking control and creating objects like the ones you see.

Don’t forget that you have to press the “Take Control” button to do actions in GeoGebra. Chat about who should take control for each step. Be sure you “Release Control” when you are done so someone else can take control.

When you drag point J between the two points, the locus of a line segment will be colored in. (This locus will only appear on your computer screen, so everyone in the team has to try it themselves).

The same for dragging point G around the locus of the circle.

With these simple constructions, you are starting to build up, explore and understand the system of dynamic geometry. You started with simple points and now you have line segments and circles too.

Note: You can change the “properties” of a dynamic-geometry object by first Taking Control and then control-clicking (on a Mac computer: hold down the “control” key and click) or right-clicking (on a Windows computer) on the object. You will get a pop-up menu. You can turn the Trace (locus) on/off, show/hide the object (but its constraints still remain), show/hide its label information, change its name or alter its other properties (like color and line style). Try these different options.
Section 1.02 Dynamic Dragging

When you construct a point to be on a line (or on a segment, or ray, or circle) in dynamic geometry, it is constrained to stay on that line; its location is dependent upon the location of that line, which can be dragged to a new location. Use the “drag test” to check if a point really is constrained to the line: select the Move tool (the first tool in the Tool Bar, with the arrow icon), click on your point and try to drag it; see if it stays on the line. Drag an endpoint of the line. Drag the whole line. What happens to the point?

Take control and construct some lines and segments with some points on them, like in the example shown in the tab. Notice how some points can be dragged freely, some can only be dragged in certain ways (we say they are partially “constrained”) and others cannot be dragged directly at all (we say they are fully “dependent”).

In GeoGebra, there are different ways to do things:

1. Construct a segment like AB by first constructing the two points with the point tool and then connecting them with the segment tool.
2. Then construct a segment by clicking at two locations with the segment tool.
3. Create a new point on the segment.
4. Create a new point off the segment and try to drag it onto the segment.
5. Create two segments that intersect and use the intersection tool to construct a point at the intersection.

6. Create two segments that intersect and use the point tool to construct a point at the intersection.

7. Create a new point off the segments and try to drag it onto the intersection.

8. Always use the drag test by dragging objects around to make sure they behave like you want them to.

Enter in your (paper or digital) copy of this document a summary of what you and your team noticed and wondered during your work on this Topic. You can also chat about these thoughts with your team. This will be a valuable record of your work. You may want to come back and think more about these entries later.

**Section 1.03 Extra: Dynamic Polygons**

If you have time, try this. It shows different ways to construct figures with several sides (polygons). Use the "drag test" to check how the different methods make a difference in the dependencies of the lengths of the sides. Then use the three methods to make your own figure with six sides.
If you did not have enough time to finish exploring this section, you can always come back later to this chat room—either with your team or by yourself.

What we noticed:

What meaning we give to what we noticed:

What we wondered:
**Topic 2: Copying Line Segments**

In this topic, you will learn the important technique of copying a length in dynamic geometry using the compass tool. This technique is tricky, but it is used a lot in dynamic geometry. Make sure that everyone in your team can do it and that everyone understands what it does.

First, get a good introduction to using the GeoGebra compass tool from this YouTube clip: [http://www.youtube.com/watch?v=AdBNfE0EVco](http://www.youtube.com/watch?v=AdBNfE0EVco).

Also look through “Tour 3: VMT to Learn Together Online.” This will provide more details about using VMT to collaborate in your team.

**Section 2.01 Copying Compass Circles versus Copy-and-Paste**

**Constructing** one segment to be the same length as another segment is different from just copying the segment. The compass tool can be used to construct a segment whose length is dependent on the length of another segment.

Compare copying a segment length with Copy-and-Paste to copying the same segment using the compass tool. First construct a segment like AB. Then select the segment and use the Edit menu to copy it and paste it to create a copy like A1B1. Next use the compass tool, first define its radius (by clicking on points A and B) and then locate a center C for it. Drag point A to change the length of segment AB. Does the copy A1B1 made with copy-and-paste change its length automatically? Does the radius CD change its length automatically when AB is changed? Why do you think this happens?
Section 2.02  Adding Segment Lengths

In dynamic geometry, you can construct figures that have complicated dependencies of some objects on other objects. Here you will construct one segment whose length is dependent on the length of two other segments.

Use the compass tool to copy the lengths of the line segments. Using the compass tool requires practice. Creating line segment length $DG = AB + BC$ provides a good visual image when you drag point B.

Discuss with your team how to do each step, especially step 2. Do you see how you can use the compass tool to lay out segments with given lengths (like $AB$ and $BC$) along a given line (like $DE$)? Discuss with your teammates the difference between the circle tool and the compass tool and let them add their ideas.

Enter below a summary of what you and your team noticed and wondered during your work on this Topic. Chat about these thoughts with your team.

What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 3: Constructing an Equilateral Triangle

The construction of an equilateral triangle illustrates some of the most important ideas in dynamic geometry. With this topic, you will explore that construction.

Before working on this topic with your team, it could be helpful to watch a brief YouTube clip that shows clearly how to construct an equilateral triangle:

http://www.youtube.com/watch?v=ORIaWNQSM_E

Also look through “Tour 4: GeoGebra Videos and Resources.”

Here are some suggestions that may help your team collaborate:

- Discuss things and ask questions.
- Include everyone’s ideas.
- Ask what your team members think and what their reasons are.
- Cooperate to work together.
- Listen to each other.
- Agree before deciding.
- Make sure all of the ideas are on the table.
- Try out the ideas put forth, no matter how promising or relevant.
- Voice all doubts, questions and critiques.
- Ensure everyone’s contributions are valued.
- Decide what to focus on, have ways of keeping track of and returning to other ideas or questions and use multiple approaches.
- Be sure that each team member knows how to construct figures in each task.

Section 3.01 Constructing an Equilateral Triangle

When Euclid organized ideas and techniques of geometry 2,300 years ago, he started with this construction of an equilateral triangle, whose three sides are constrained to always be the same lengths as each other and whose three angles are always equal to each other. This construction can be considered the starting point of Euclidean and dynamic geometry.

Have everyone in your team work on this. It shows—in one simple but beautiful example—the most important features of dynamic geometry. Using just a few points, segments and circles (strategically related), it constructs a triangle whose sides are always equal no matter how the points, segments or circles are dragged. Using just the basic definitions of geometry—like that the points on a circle are all the same distance from the center—it proves that the triangle must be equilateral (without even measuring the sides or the angles).

Your team should construct an equilateral triangle like the one already in the tab. Drag the one that is there first to see how it works. Take turns controlling the GeoGebra tools.
Euclid argued that both of the circles around centers A and B have the same radius, namely AB. The three sides of triangle ABC are all radii of these two circles. Therefore, they all have the same length. Do you agree with this argument (proof)? Are you convinced that the three sides of ABC have equal lengths – without having to measure them? If you drag A, B, or C and change the lengths of the sides are they always still equal? Do they have to be equal? Why or why not?

Euclid designed the construction so that the triangle ABC would necessarily be equilateral. He designed it so point C would be dependent on points A and B in a way that the distances from C to A and from C to B would both always be the same as the distance from A to B, making the side lengths all equal. He did that by locating point C at the intersection of circles centered on A and B with radii equal to the length of AB. Were you able to construct a triangle with that constraint? Do you see any other points that are equal distances from A and B?

Section 3.02 Where Are Perpendicular and Right Angles?

Let us look more closely at the relationships that are created in the construction of the equilateral triangle. In the next tab, more lines are drawn in. Explore some of the relationships that are created among line segments in this more complicated figure. What line segments do you think are equal length – without having to measure them? What angles do you think are equal without having to measure them? Try dragging different points; do these equalities and relationships stay dynamically? Can you see how the construction of the figure made these segments or angles equal?

Can you find different kinds of triangles in this construction? If a triangle always has a certain number of sides or angles equal, then it is a special kind of triangle. We know the construction of this figure defined an equilateral triangle, ABC. What other kinds of triangles did it define?

What kinds of angles can you find? Are there right angles? Are there lines perpendicular to other lines? Are they always that way? Do they have to be? Can you explain why they are?
What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 4: Programming Custom Tools

For constructing geometric figures and for solving typical problems in geometry, it is useful to have tools that do things for you, like construct midpoints of segments, perpendiculars to lines, and parallel sets of lines. GeoGebra offers about 100 tools that you can use from the tool bar, input bar or menu. However, you can also create your own custom tools to do additional things—like copy an angle, construct an isosceles triangle or locate a center of a triangle. Then you can build your own mini-geometry using a set of your own custom tools—like defining a complicated figure using custom tools for several of its parts.

Furthermore, programming your own tools gives you a good idea about how GeoGebra’s standard tools were created and why they work the way they do. By building up tools from the basic point, line and circle tools, you will understand better how geometry and its procedures work.

Section 4.01 Creating an Equilateral-Triangle Tool

Create a specialized custom tool for quickly generating equilateral triangles with the menu “Tools” | “Create New Tool …”

GeoGebra makes programming a tool easy. However, it takes some practice to get used to the procedure.

To program a tool, you have to define the Outputs you want (the points, lines, etc. that will be created by the custom tool) and then the Inputs that will be needed (the points, lines, etc. that a person will have to create to use the tool). For instance, to create a triangle using this custom tool, you will first select the custom tool as the active tool. Next, you will construct or select two points to define the base of the
triangle, AB, as inputs to the custom tool. Then the triangle line will appear automatically (as the output of the custom tool).

**Hint:** You can identify the output objects by selecting them with your cursor before or after you go to the menu “Tools” | “Create New Tool …”. Hold down the Command key (on a Mac) or the Control key (in Windows) to select more than one object. You can also identify the output objects from the pull-down list in the Output Objects tab. Similarly, you can identify input objects in the Input Objects tab by selecting them with the cursor or from the pull-down list. GeoGebra might identify most of the necessary input objects automatically. Give the tool a name that will help you to find it later and check the “Show in Toolbar” box so your tool will be included on the toolbar.

**Important:** To be able to use your custom tool in another tab or another chat room later, you have to save it now. Save your custom tool as a .ggt file on your desktop. Take control and use the GeoGebra menu “Tools” | “Manage Tools…” | “Save As.” Save your custom tool to your computer desktop, to a memory stick, or somewhere that you can find it later and give it a name like “Pat’s_Equilateral.ggt” so you will know what it is. When you want to use your custom tool in another tab or topic, take control, use the GeoGebra menu “File” | “Open…” then find and open the .ggt file that you previously saved. You should then be able to select your custom tool from the toolbar or from the menu “Tools” | “Manage Tools.” When your custom tool is available to you, it will also be available to your teammates when they are in that tab.

**Hint:** If a custom tool does not appear on your tool bar when you think it should be available, use the menu “Tools” | “Customize Toolbar”, find the custom tool in the list of Tools, and insert it on the toolbar list where you want it (highlight the group or the tool you want it to be listed after).

**Note:** The custom tools in GeoGebra have some limitations, unfortunately. Not all outputs of custom tools are completely dynamic. For instance, if you define a right-triangle tool, you will not be able to freely drag the new vertex of the right triangles that are created with this tool.


### Section 4.02 Creating a Bisector Tool

The procedure used to construct an equilateral triangle can be used to locate the midpoint of a segment and to construct a perpendicular to that segment, passing through the midpoint.

As you already saw, the construction process for an equilateral triangle creates a number of interesting relationships among different points and segments. In this tab, points A, B and C form an equilateral triangle. Segment AB crosses segment CD at the exact midpoint of CD and the angles between these two segments are all right angles (90 Degrees). We say that AB is the “perpendicular bisector” of CD—meaning that AB cuts CD at its midpoint, evenly in two sectors, and that AB is perpendicular (meaning, at a right angle) to CD.

Can your team create a custom tool to find a midpoint of a segment?

For many geometry constructions, it is necessary to construct a new line perpendicular to an existing line (like line FG). In particular, you may need to have the perpendicular go through the line at a certain point (like H). Can your team figure out how to do that?
Section 4.03  Creating a Perpendicular Tool

Now that you know how to construct a perpendicular, you can automate this process to save you work next time you need a perpendicular line. Create a custom tool to automatically construct a perpendicular to a given line through a given point by following the directions in the tab.

Members of the team should each create their own custom perpendicular tool. One person could create a tool to construct a perpendicular bisector through the midpoint of a given line (no third point would be needed as an input for this one). Another person could create a perpendicular through a given point on the line. A third person could create a perpendicular through a given point that is not on the line. Everyone should be able to use everyone else’s custom tool in this tab. Do the three tools have to be different? Does GeoGebra have three different tools for this? Do your custom tools work just like the GeoGebra standard perpendicular tools? Are there other cases for constructing perpendiculars?

GeoGebra has a perpendicular tool that works like your custom tool. If you just used the standard tool, you would not be aware of the hidden circles that determine the dependencies to keep the lines perpendicular during dragging. Now that you understand these dependencies, you can use either the standard tool or your custom tool. You will not see the hidden circles maintaining the dependencies of lines that are dynamically perpendicular, but you will know they are there, working in the background.

The tools of GeoGebra extend the power of dynamic geometry while maintaining the underlying dependencies. By defining your own custom tools, you learn how dynamic geometry works “under the hood.” In addition, you can extend its power yourself in new ways that you and your team think of.
Can you explain why points D and E were constructed? They are designed to form the base of equilateral triangles DEF and DEG, such that line FG goes through point C. How are points D and E dependent on C in a way that makes point C the midpoint of segment DE? Is this all necessary to construct a perpendicular to AB through C? What would have to be done differently if point C was not on line AB? What could be done differently if you wanted the perpendicular to go through the midpoint of segment AB?

### Section 4.04  Creating a Parallel Line Tool

One member of the group should create a custom perpendicular tool (like you did in the previous tab) in this tab or load a custom tool from the previous tab’s work. Now use this custom perpendicular tool (or the standard perpendicular tool) to create a custom parallel tool. See how tools can build on each other to create a whole system of new possible activities.

Note that if line CD is perpendicular to line AB, then any line perpendicular to line CD will be parallel to line AB. Imagine a rectangle (with all right angles). Its opposite sides are parallel to each other and perpendicular to the other sides, so the opposite sides are perpendicular to their perpendiculars.
What we noticed:

What meaning we give to what we noticed:

What we wondered:
A triangle is a relatively simple geometric construction: simply join three segments at their endpoints. Yet, there are many surprising and complex relationships possible in triangles with different dependencies designed into them.

**Section 5.01  Triangles with dependencies**

Here are some triangles constructed with different dynamic constraints. See if your team can figure out which ones were constructed to always have a certain number of equal sides, a certain number of equal angles and/or a certain number of right angles. In dynamic geometry, sometimes one triangle can be dragged to look like another type of triangle, but it does not have the same necessary relationships. For instance, a scalene triangle (with no special constraints) can be dragged to look like an equilateral triangle, but it does not have the lengths of its sides dependent on each other.

**Section 5.02  Constructing an Isosceles Triangle**

An isosceles triangle has two sides that are always the same length. Can you simplify the method of constructing an equilateral triangle so that the length of one side is dependent on the length of another side, but the third side can be any length? Can your team make a custom tool for constructing isosceles triangles? Can you make every possible isosceles triangle with this tool?
Section 5.03  Constructing a Right Triangle and a Right-Isosceles

Use your custom perpendicular tool to construct a right triangle.

Then combine the construction methods for an isosceles triangle and for a right triangle to design a triangle that has a right angle and two equal sides.

Be sure to take turns controlling the GeoGebra tools and chat about what you are doing. Work together—do not try to solve something yourself and then explain it to your team mates. Talk it out in the chat:

- Discuss what to do for each step in the chat.

- Do it in the GeoGebra tab.

- Discuss what you did and how it worked.

- Make sure everyone in the team understands what was done and can do it themselves.

- Discuss what you noticed and what you wondered about the construction.

- Discuss why it worked: why is the triangle you constructed always isosceles or always right-angled no matter how you drag it?
What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 6: Constructing Tools for Triangle Centers

There are a number of interesting ways to define the “center” of a triangle, each with its own interesting properties.

Section 6.01 The Centroid of a Triangle

The “centroid” of a triangle is the meeting point of the three lines from the midpoints of the triangle’s sides to the opposite vertex. Create a custom centroid tool.

Take control and use the GeoGebra menu “Tools” | “Manage Tools…” | “Save As.” Save your custom tool to your computer desktop or somewhere that you can find it later and give it a name like “Tasja’s_Centroid.ggt” so you will know what it is. When you want to use your custom tool in another tab or topic, take control, use the GeoGebra menu “File” | “Open…” then find and open the .ggt file that you saved. You should then be able to select your custom tool from the tool bar or from the menu “Tools” | “Manage Tools.” When your custom tool is available to you, it will also be available to your teammates when they are in that tab.

Section 6.02 The Circumcenter of a Triangle

The “circumcenter” of a triangle is the meeting point of the three perpendicular bisectors of the sides of the triangle. Create a custom circumcenter tool and save it.
Section 6.03  The Orthocenter of a Triangle

The “orthocenter” of a triangle is the meeting point of the three altitudes of the triangle. An “altitude” of a triangle is the segment that is perpendicular to a side and goes to the opposite vertex. Create a custom orthocenter tool and save it.

Section 6.04  The Incenter of a Triangle

The “incenter” of a triangle is the meeting point of the three angle bisectors of the angles at the triangle’s vertices. Create a custom incenter tool and save it.
Note: The incenter of a triangle is the center of a circle inscribed in the triangle. A radius of the inscribed circle is tangent to each side of the triangle, so you can construct a perpendicular from the incenter to a side to find the inscribed circle’s point of tangency – and then use this point to construct the inscribed circle.

What we noticed:

What meaning we give to what we noticed:

What we wondered:
**Topic 7: Exploring the Euler Segment and Circle**

The relationships presented in this topic seem quite surprising. However, they are results of the dependencies imposed in the triangle by the constructions of the different centers. Sometimes combinations of complex dependencies have surprising results.

**Section 7.01 The Euler Segment of a Triangle**

A Swiss mathematician named Euler (his last name is pronounced “oiler”) discovered a relationship among three of the centers that you created custom tools for. Can you discover what he did? He did this in the 1700s—without dynamic-geometry tools. Euler’s work renewed interest in geometry and led to many discoveries beyond Euclid’s.

**Note:** Take turns to re-create a custom tool for each of the triangle’s special points: centroid, circumcenter, orthocenter and incenter, as done in the previous tabs. Or else, load the custom tools you created before using the GeoGebra menu “File” | “Open…” then find and open the .ggt files that you saved. You should then be able to select your custom tools from the tool bar or from the menu “Tools” | “Manage Tools.” When your custom tools are available to you, they will also be available to your teammates in that tab.

![Diagram of a triangle with labels for centroid, circumcenter, orthocenter, and incenter.]

**Section 7.02 The Nine-Point Circle of a Triangle**

You can construct a circle that passes through a number of special points in a triangle. First construct custom tools for the four kinds of centers or open them from your .ggt files that you saved. Connect the orthocenter to the circumcenter: this is “Euler’s Segment.” The Centroid lies on this segment. A number of centers and related points of a triangle are all closely related by Euler’s Segment and its Nine-Point Circle.
Circle for any triangle. Create an Euler Segment and its related Nine-Point Circle, whose center is the midpoint of the Euler Segment.

You can watch a six-minute video of this segment and circle at: 
www.khanacademy.org/math/geometry/triangle-properties/triangle_property_review/v/euler-line.

The video shows a hand-drawn figure, but you can drag your dynamic figure to explore the relationships more accurately and dynamically.

Are you amazed at the complex relationships that this figure has? How can a simple generic triangle have all these special points with such complex relationships? Could this result from the dependencies that are constructed when you define the different centers in your custom tools?

What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 8: Visualizing Congruent Triangles

Two triangles are called “congruent” if all their corresponding angles and sides are equal size. However, you can constrain two triangles to be congruent by just constraining 3 of their corresponding parts to be equal – for certain combinations of 3 parts. Dynamic geometry helps you to visualize, to understand and to remember these different combinations.

In this topic, you will have to copy angles. You can copy an angle in dynamic geometry using the compass tool, but it is a bit more complicated than copying a segment.

Watch this YouTube clip: [http://www.youtube.com/watch?v=qngWUFgyyHc](http://www.youtube.com/watch?v=qngWUFgyyHc). It gives a clear view of how to use GeoGebra to copy an angle using tools that are equivalent to traditional geometry construction with a physical straightedge (for making line segments) and compass (for making circles).

Section 8.01 Corresponding Sides and Angles of Congruent Triangles

What constraints of sides and angles are necessary and sufficient to constrain the size and shape of a triangle? Two congruent triangles have 6 corresponding parts equal (3 sides and 3 angles), but you do not have to constrain all 6 parts to be equal in order to make sure the triangles are congruent.

For instance, two triangles with their corresponding 3 angles equal are called “similar” but they are not “congruent.” You can drag one of them to be larger than the other one. They will still have the same shape, but the corresponding side lengths of one will all be larger than the side lengths of the other triangle.

If two triangles have their corresponding angles constrained to be equal and then you constrain two corresponding sides to be equal length, will the triangles necessarily be congruent? Suppose you only constrained one of the corresponding sides to be equal? Explore different combinations of 4 or 5 of the 6 corresponding triangle parts being constrained to be equal. Which combinations guarantee that the triangles are congruent?
Section 8.02 Side-Side-Side (SSS)

If all three sides of one triangle are equal to the corresponding sides of another triangle, then the two triangles are congruent. This is called the “Side-Side-Side” (or “SSS”) rule.

![Image of constructing a triangle given three sides](image1)

1. Use the compass tool to copy the length of $AB$ to a segment $DE$.
2. Use the compass tool to copy the lengths of $AC$ and $BC$ to points $D$ and $E$.
3. Use the intersection tool to construct point $E$ at the intersection of the two circles.
4. Use the polygon tool to construct triangle $DEF$.
5. Is $DEF$ necessarily congruent to $ABC$?
6. Chat about how this is constrained.

Section 8.03 Side-Angle-Side (SAS)

If two sides and the angle between them of one triangle are equal to the corresponding sides and angle between them of another triangle, then the two triangles are congruent.

![Image of constructing a triangle given two sides and the angle between them](image2)

In this figure, two sides and the angle between them have been copied from triangle $ABC$ to construct triangle $FJK$.

To copy angle $BAC$, points $D$ and $E$ were constructed at equal distances along $AC$ and $AB$. The compass tool was used to transfer the distance $AD$ to $FH$ along $FG$. The compass was then used to transfer the distance $DE$ to $HI$, where $I$ is at the intersection of the two compass circles.

To copy side $AC$, the compass transferred the length of $AC$ to $FJ$ along $FG$. To copy side $AB$, the compass transferred the length of $AB$ to $FK$ along $FI$.

Use the drag test to make sure that triangles $ABC$ and $FJK$ are congruent. You can drag $FJK$ right on top of $ABC$ to see that all corresponding sides and angles are equal.
What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 9: Constraining Congruent Triangles

Use dynamic geometry to explore and visualize the different cases of constraints on triangles, in addition to SSS and SAS.

Section 9.01 Combinations of Sides and Angles

You can constrain two dynamic triangles to be congruent using a number of different combinations of equal corresponding sides and/or angles.

What combinations of constraints of sides and angles are necessary and sufficient to constrain the size and shape of a triangle?

Section 9.02 Angle-Side-Angle (ASA)

If two angles and the side included between them of one triangle are equal to the corresponding two angles and side between them of another triangle, then the two triangles are congruent. This is called the Angle-Side-Angle or ASA rule.
Section 9.03 Side-Side-Angle (SSA)

What if two corresponding sides and an angle are equal, but it is not the angle included between the two sides?

This is a tricky case.
Given triangle ABC, construct another triangle with an angle equal to ABC, a side along the angle equal to side AB, and a side opposite the angle equal to side AC.

1. Use the compass tool to copy angle ABC to angle HGI.
2. Use the compass tool to copy side AB to side GJ and 3. to copy side AC to side JK.
4. Now drag point K to meet the side extending GI.
5. Notice that for some shapes of triangle ABC, there are two points that satisfy the constraint SSA, but that only one of them constructs a triangle congruent to ABC.
6. Discuss this in the chat.

What we noticed:

What meaning we give to what we noticed:

What we wondered:
This topic presents a challenging problem for your team to construct.

It also includes a chance to prove that you succeeded. The proof uses what you learned about congruent triangles.

**Section 10.01 The Inscribed Triangles Problem**

Construct a pair of inscribed triangles. First, explore the given figure. Note the dependencies in the figure. Then construct your own pair of inscribed triangles that behaves the same way.

If you solved this, did you construct the lengths of the sides of the outer triangle to be directly dependent upon each other? Did you construct the lengths of the sides of the inner triangle to be directly dependent upon each other? Do you think that the triangles are equilateral? How do you know?

**Section 10.02 Proofs about Triangles**

Constructing figures in dynamic geometry—like the inscribed triangles—requires thinking about dependencies among points, segments and circles. You can talk about these dependencies in the form of proofs, which explain why the relationships among the points, segments and circles are always, necessarily true, even when any of the points are dragged around.

Many proofs in geometry involve showing that some triangles are congruent to others. You can prove that the inscribed triangles are equilateral by proving that certain triangles are congruent to each other.
Chat about what you can prove and how you know that certain relationships are necessarily true in your figure. Explain your proof to your team. Does everyone agree?

What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 11: Building a Hierarchy of Triangles

Now you can use GeoGebra tools (or your own custom tools) to construct triangles with different constraints. How are the different kinds of triangles related to each other? Which ones are special cases of other kinds?

Section 11.01 Constructing Constrained Triangles

How many different kinds of dynamic triangles can your team construct? You might want to make a list or table of the different possible constraints on the sides and angles of triangles. How many angles can be equal? How many sides can be equal? How many angles can be right angles?

Can your team construct each kind of triangle by designing the necessary dependencies into the construction process?

Section 11.02 The Hierarchy of Triangles

How are the different kinds of triangles related to each other?

There are different ways of thinking about how triangles are related in dynamic geometry.

In dynamic geometry, a generic or “scalene” triangle with no special constraints on its sides or angles may be dragged into special cases, like a right triangle or an equilateral triangle. However, it does not have the constraints of a right angle vertex or equal sides built into it by its construction, so it will not necessarily retain the special-case characteristics when it is dragged again.

You can think of a hierarchy of kinds of triangles: an equilateral triangle can be viewed as a special case of an isosceles acute triangle, which can be viewed as a special case of an acute triangle, which can be viewed as a special case of a scalene triangle.

Can your team list all the distinct kinds of triangles?
Can your team connect them in a hierarchy diagram? Create a hierarchy diagram like the one shown in the tab. Add more kinds of triangles to it if you found some. You may want to reorganize the structure of the diagram.

A triangle always has 3 sides and 3 angles, but it can have additional constraints.
* The largest angle of an 'acute triangle' is less than 90°.
* The largest angle of a 'right triangle' is equal to 90°.
* The largest angle of an 'obtuse triangle' is greater than 90°.
* A 'scalene' triangle may not have any equal angles (or sides).
* An 'isosceles triangle' has at least 2 equal angles (or sides).
* An 'equilateral triangle' has 3 equal angles (or sides).
* An 'isosceles-right' triangle has the constraints of an isosceles triangle and those of a right triangle.

1. Discuss in the chat any other kinds of triangles possible.
2. Add them to the hierarchy.
3. Discuss in chat what the hierarchy shown here represents.

When you study quadrilaterals (figures with four sides) in geometry, you should create a hierarchy of the different types of quadrilaterals with different constraints: How many angles can be equal? How many sides can be equal? How many angles can be right angles? How many pairs of sides can be parallel? There are many types of constrained quadrilaterals, many without any common names.

What we noticed:

What meaning we give to what we noticed:

What we wondered:
Topic 12: Solving Geometry Problems

Here is a set of challenge problems for your team. Have fun!
If the team does not solve them during its session, try to solve them on your own and report your findings in the next team session.

Section 12.01 Treasure Hunt

Can you discover the pot of gold in this tale told by Thales de Lelis Martins Pereira, a high school teacher in Brazil? You might want to construct some extra lines in the tab.

Section 12.02 Square and Circle

Can you determine the radius of the circle? You should not have to measure. What if the side of the square is “s” rather than “8”?

Hint: To solve this kind of problem, it is usually useful to construct some extra lines and explore triangles and relationships that are created. If you know basic algebra, you might set up some equations based on the relationships in the figure.
Section 12.03 Crossing an Angle

Can you construct the segment crossing the angle with the given midpoint?

*Hint:* This is a challenging problem. Try to add some strategic lines and drag the figure in the tab to see what would help to construct the segment EF at the right place.
Congratulations on completing “Explore Dynamic Geometry Together”!

If your virtual math team has successfully completed the previous topics, then you are ready to go off on your own—either as a team or individually—to explore geometry using GeoGebra. Actually, GeoGebra also supports algebra, 3-d geometry, trigonometry, pre-calculus, calculus and other forms of mathematics. So you will be able to use it for the rest of your life as a tool for better understanding mathematical concepts and relationships.

You are welcome to continue to use VMT for collaborative events. You can create your own rooms and invite people to them to explore topics that you define. You can also download GeoGebra from www.GeoGebra.org to your computer or iPad to use on your own.

Enjoy!

What we noticed:

What meaning we give to what we noticed:

What we wondered:
TOURS: Tutorials on VMT and GeoGebra
Tour 1: Joining a Virtual Math Team

In this tour, you will explore the VMT-with-GeoGebra environment and learn how to use it. You will learn about many special features of the VMT system, which you will need to use in work on the topics.

The Virtual Math Teams (VMT) Environment

The VMT system has been developed to support small groups of people to discuss mathematics online. It has tabs and tools to help individuals, small groups (about 2-6 people) and larger groups (like classes) to explore math collaboratively.

Register and Login to VMT

Go to the VMT Lobby at http://vmt.mathforum.org

Log in. If you do not have a VMT login, then first register. If you are using VMT in a class, your instructor may have already registered you and assigned your username and password. If not, then choose a username that you want to be known by online in VMT. To protect your privacy, you should select a username that is different from your real name. (Work in VMT is publicly available and anyone can see your username and what you say in VMT chats.)

Be sure to choose the project that is defined for your class or group.

Look Around the VMT Lobby

Interface of the VMT Lobby.
In the center of the VMT Lobby is a list of math subjects. For each subject, if you click on the little triangle in front of the subject name, you can view activity topics related to that subject. For each topic, if you click on the little triangle in front of the activity name, you can view the links to chat rooms for discussing that topic. Find the room where you are supposed to meet with your group. Click on the link (the name is a live link) for that room to open a window with the chat room.

On the left of the Lobby is a list of links to other functions. The link, “List of All Rooms,” displays the list of math subjects, like you see in the figure above. The link, “My Profile,” allows you to change your login name, password, or information about you. The link, “My Rooms,” lets you see links to chat rooms that you have been invited to by your teacher or a friend, as well as rooms that you have been in before.

You can use the “VMT Sandbox” link to open a practice chat room. However, it is better to meet with the members of your group in a chat room that has been created for your group to do an activity. You should be able to find it in the “List of all Rooms” under your project, the subject “Dynamic Geometry,” the topic (start with “Topic 01”), and the name of your group. It may also be listed under “My Rooms” or you might have been given a direct link to the room.

Enter a VMT Chat Room

When you click on a chat room link to open it, your computer will download VMT files. This may take a couple minutes, especially the first time it is done on your computer. You will see a dialog box window asking if you want to open the file with Java Web Start. Just select “Open with Java Web Start” and press the OK button. (See “Tour 10: Technical Problems” at the end of this document if you have problems at this point.)

Dialog box for Java Web Start.

It is important to try to log into VMT using the computer you will be using with your team before you are going to meet with your team online the first time. Teachers having their students use a school computer lab for VMT should meet with their school technical support personnel well in advance, and test each of the computers to make sure they can access VMT on the Internet and have all the necessary software and permissions.

You will learn more about how to use the VMT tools in future tours. For now, just click on the tab for GeoGebra and proceed with the topic.
Tour 2: GeoGebra for Dynamic Math

Go to the VMT Chat Room and Open the GeoGebra Tab

Open the GeoGebra tab in your VMT chat room and identify the parts listed in the figure below. You will be using this GeoGebra tab most of the time in the topic activities.

The GeoGebra tab interface in VMT.

Take Turns

This is a multi-user version of GeoGebra. What you see in the team’s GeoGebra tab is the same as what everyone in the VMT chat room with you also sees in their GeoGebra tab (except that they may have their view options set differently, like having the tab opened wider or smaller than you do).

Two people cannot be creating and manipulating objects at the same time in GeoGebra, so you have to take turns. While someone else is constructing or dragging, you can be watching and chatting.

Use the chat to let people know when you want to “take control” of the GeoGebra construction. Use the chat to tell people what you notice and what you are wondering about the construction.

Decide in the chat who will go first. That person should press the “Take Control” button and do some dragging and constructing. Then “Release Control” and let the others take a turn.
Before you start to drag or construct, say in the chat what you plan to do. After you release control, say in the chat what you discovered if anything surprised you. You can also ask other people in your group questions about what they constructed and how they did it.

There is a history slider on the left side of the GeoGebra tab. You can only use the history slider in the GeoGebra tab when you are not “in control.” Sliding the history slider shows you previous versions of constructions in the GeoGebra tab, so you can review how your group did its work.

**Create a Practice Tab**

To create a new GeoGebra tab for yourself in the VMT chat room, use the “+” button in the upper-right corner above the other tabs.

This way, you can create your own GeoGebra tab, where you can practice doing things in GeoGebra before you get together with your team in the team’s GeoGebra tab. You can use your own tab to try out the construction tools described below. At the beginning of each activity, there may be tasks for you to try yourself in your own tab; then you will discuss them in chat and share your figures in the team GeoGebra tab. Anyone can view any tab, so you can post a chat invitation to other people to go to your GeoGebra tab and see what you have done. You can even let someone else “take control” in your tab to help you construct something or to explore your construction. After your group constructs something in the group GeoGebra tab, you should make sure that you can do it yourself by doing the construction in your own tab.

**Some Drawing Tools in GeoGebra**

When you open a GeoGebra tab, the Tool Bar may look something like this:

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For some topics, the Tool Bar has been simplified, so only the tools needed for that topic are available. Notice that you may be able to “pull down” many different tools by clicking on the small arrow at the bottom of each icon in the tool bar. For instance, from the third icon, you can select the Line Tool, the Segment Tool, and the Ray Tool. If your tool bar does not look like this, then change the perspective to the “Geometry” perspective from the “View” menu or the pop-up menu on the right edge of the tab. If there are grid lines, you can remove them with the Grid button below the tool bar. If there are coordinate axes, you can remove them with the coordinates button below the tool bar. You can change the color or thickness of a selected line with the other buttons there.

Make sure that the menu “Options” | “Labeling” | “New Points Only” is checked so that new points you create will have their names showing.

Here are some of the first tools you will be using in GeoGebra:

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These tools correspond to the traditional Euclidean geometry construction tools of straightedge and compass. The first several tools let you construct dynamic points and lines (including lines, segments,
rays and circles), much as you would with a pencil and paper using a straightedge for the lines, segments and rays or a compass for the circles.

Check out this video for an overview and some tips on the use of these tools: http://www.youtube.com/watch?v=2NqblDIP138

Here is how to use these tool buttons. Try each one out in the construction area of your own GeoGebra tab. First click on the button for the tool in the tool bar, and then click in the construction area to use the tool. The tool will remain selected in the tool bar until you select another one:

Use the Move Tool to select a point that already exists (or segment or circle) and drag it to a new position. Everyone will see the object being dragged.

Use the Point Tool to create some points. Each place you click with the Point Tool will leave a point. These points will appear in the GeoGebra tab of everyone in your chat room. By convention, points are named with capital letters.

Use the Intersection Tool to mark the intersection of two objects—like a line and a circle—with a new point. When you click on the intersection of two objects, both objects should get thicker to show they have been selected. You can also select the two objects separately, one after another and the new point will be on their intersection. If you click at a location where three objects meet, you will get a pull-down menu to select the two objects that you want.

Note on intersections: a circle and a line or two circles can intersect in two points. If the two intersecting objects move apart, a point defined at their intersection will disappear, but it will come back if they move together and intersect again. Sometimes, they move to a configuration in which the two intersecting objects are just tangent to each other and then they will continue to intersect in two points. After they pass through the point of tangency, the two points of intersection (which are sometimes labeled with a 1 and a 2) may switch. This is called the “continuity problem” (see Kortenkamp, 1999 in the “Further Readings” Tour). If you use the Intersection Tool and click on one intersecting object at a time, this may place points at both intersections. You can turn on “Continuity” in the menu Options | Advanced (Preferences) | Gears tab (Advanced) | checkbox for Continuity On. This may help a point move continuously through a tangent configuration. By playing with the Intersection Tool and Continuity, you can often get complicated situations to work the way you want—but not always. This is a foundational mathematical limitation of all dynamic geometry implementations.

Use the Line Tool to create a line with no endpoints. A line has to pass through two points. You can either select two existing points or click with the Line tool to create the points while you are constructing the line. By convention, lines (as well as segments, rays, circles and polygons) are named with lowercase letters.

Use the Segment Tool to connect two points with a line segment. You can also create points as you click for the ends of the segment. See what happens when two segments use the same point for one of their endpoints.

Use the Ray Tool to connect two points with a ray. First click for the starting point of the ray and then click for a point along the ray. You can also select existing points for the endpoint and the other point.

Use the Circle Tool to draw a circle. You must click to place a point where you want the center to be and then click again for a point on the circumference of the circle. You can also use existing points for the center and the other point.
Use the Compass to draw a circle whose radius is equal to the distance between two points and whose center is at a third point. First, click on two points to define the length of the radius. Then without releasing the cursor, drag the circle to the point where you want its center to be. This tool is like a mechanical compass, where you first set the size of the opening and then fix one end at a center and draw a circle around it. The Compass tool is very handy for copying a length from one part of a construction to another in a way that will be preserved through any dragging; if you change the original length, the copied length will change automatically to still be equal to the original one.

*The following tools can be used for modifying the display of a construction to make it easier to see what is going on with the dependencies of the construction.*

The Polygon Tool is used to display a two-dimensional polygon. For instance, if three segments connecting three points form a triangle, then you can use the Polygon tool to display a filled-in triangle. Click on the vertex points in order around the polygon and then complete the figure by clicking on the first point again.

Show/Hide Label. Select this tool. Then click on an object to hide its label (or display it if it was hidden).

Show/Hide Object. Select this tool. Then click on an object to hide it (or to display it if it was hidden).

Use the Angle tool to display an angle with its measurement. Click on the three points that form an angle *in clockwise order*—if you do it in counterclockwise order it will display the exterior angle, which you probably do not want. You can also click on the two lines that form the angle in clockwise order.

Use the Move Graphic tool to drag the whole construction area.

To delete a point, either use the Delete tool or select the object and press the “delete” key on your keyboard. *Hint:* Before you delete something that someone else created, be sure to ask in the chat if everyone agrees that it should be deleted. *Warning:* When you delete an object, anything dependent on that object will also be deleted; it may be better to just hide the object.

Insert Text. This tool can be used to place text on the drawing surface. You can add a title, a comment, etc. First, an input box will open for you to type your text; when you select OK, it appears in a text box.

You can use the Zoom in and Zoom out tools to change the scale of your view of the construction area. On a Mac computer, you can also use two-finger gestures for zooming; on a Windows computer, you can use a mouse scroll wheel or right button. Changing your view with the zoom tools will not affect what others see in their views.

**The Algebra View**

A good way to view the locations, lengths, areas or other values of all the GeoGebra objects is to open the Algebra View from the GeoGebra “View” | “Algebra” menu. This opens a window listing all the free and
dependent objects that you have constructed. You can un-attach this window with the little window icon that is above the Algebra View:

![Image of GeoGebra interface]

Top of the Algebra View and the Graphics View in GeoGebra.

**The “Drag Test”**

This is where dynamic geometry gets especially interesting. Select an object in the construction area with the Move tool. Drag the object by holding down the Move Tool on the object and moving it. Observe how other parts move with the selected object. That is because the other parts are “dependent” on the part you are dragging. For instance, a segment depends on its end-points; when the points move, the segment must also move. If two segments both depend on the same point, then they will always move together; if you drag one of the two segments, it will drag the common end-point, which will drag the other segment. Dragging is an important way to check that parts have the correct connections or “dependencies” on other parts. GeoGebra lets you construct objects that have dependencies that are important in geometry and in other branches of mathematics.

A thorough explanation of a simple construction with a dependency is given in a YouTube video using GeoGebra tools that are equivalent to straightedge and compass:

http://www.youtube.com/watch?v=AdBNfEOEVco

**Explore!**

Construct some lines that share the same points. Think about how the figures are connected. State what you think will happen if certain objects are dragged. Then try it out. Take control and drag part of a figure. Discuss the dependencies in chat.

**Hint**

If two elements share a point – for instance, if a line segment starts at a point on a circle, then we say there is a “dependency” between the segment and the circle. That is, the position of the segment depends on the position of the circle, and when you move one, the other also moves. *Geometry is all about such dependencies.* A dynamic-math environment lets you see how the dependencies work and lets you explore them. Check out these videos of complicated dependencies:

http://www.youtube.com/watch?v=Oyj64QnZle4&NR=1
http://www.youtube.com/watch?v=-GgOn66knqA&NR=1
Tour 3: VMT to Learn Together Online

In this tour, you will explore the VMT-with-GeoGebra environment and learn how to use it to collaborate. You will learn about many special features of the VMT system, which you will need to use in the topic activities.

Enter a VMT Whiteboard Tab

When the VMT chat room is open, it may look something like this:

Interface of a VMT chat room with a whiteboard tab.

Note that this screen image shows a VMT shared whiteboard tab, not a GeoGebra tab. The whiteboard is useful for free-hand sketches and textboxes. You may want to use a whiteboard tab to collaboratively write a summary of your team’s work in GeoGebra tabs. You can always add a shared whiteboard tab if there is none in your VMT chat room by using the “Add a tab +” button above the tab names.

See What is Going On

See the list of users present in the upper right. It shows all the people who are currently logged into this chat room.

Awareness messages near the bottom of the window state who is currently typing a chat message or drawing in the shared whiteboard. You should see all the messages that anyone posted in the chat room and all the drawings that anyone did in the whiteboard as soon as they finish typing (after they post the
message by pressing the Return key on their keyboard) or drawing (and after they click on the whiteboard background).

**Post a Greeting Message**

Type in the chat input box. Press the Return or Enter key on your keyboard to post your message for others to read. Your message should appear above in the chat messages area with your login name and the current time. Other people in the same chat room will also see your message.

**Look Back in Chat and Whiteboard History**

Load old messages if you are joining a room where people have already been chatting. Use the reload icon (two curved arrows). You can scroll back in the chat if there are too many messages to be displayed at once.

The whiteboard also has a history slider so you can see how the images in the whiteboard tab changed over time.

**Reference a Previous Chat Message**

Point to a previous chat message by double clicking on the previous message when you type a new chat message. This will create an arrow from your new chat message to the previous chat message. Everyone will see this arrow when your message is posted or if they click on your message later.

If a reference arrow exists and you want to delete it, then press the ESC key on your keyboard before you post the message. (You cannot delete a reference arrow after you post the message.)

**Leave a Message on a Shared Whiteboard**

Click on the different tabs to see the different work areas. A Summary Tab is just a Whiteboard Tab. Your group can use a Summary Tab to summarize your work on an activity.

Go back to the Whiteboard Tab. Open a textbox (the icon for this is in the middle of the Whiteboard tool menu; it has an “A” in it; if you roll your cursor over it, it says “Add a textbox”.) Type a message in the textbox. Double-click on someone else’s textbox to edit and add to what they wrote.

You can draw a square or circle and change its color, outline, etc.

**Reference an Object in the Whiteboard**

You can also create an arrow from your new chat message to an object or an area on the whiteboard, just like you did from a new chat message to a previous one.

Point to the square or circle with the Reference Tool. First, click on the Referencing Tool (the pointing hand in the whiteboard tool bar — see screenshot of “VMT chat room with Whiteboard tab” above). Then select the square or circle — or else drag the cursor to select a rectangular area around the square or circle. Finally, type a chat message and post it. You should see a line connecting your chat posting to the object in the whiteboard. This is handy to use when you want to make a comment or ask a question about an object in the Whiteboard or in GeoGebra.
You can also use the Referencing Tool to point to an object in a GeoGebra tab.

**Draw Two Triangles**

Draw an equilateral triangle (where all three sides are of equal length) on the shared whiteboard. Or draw a right triangle (where one angle is a 90-degree right angle) on the shared whiteboard.

Try to move these triangles around.

What do you notice about them? Is it hard to rotate or move the triangle around? What would you like to be able to do?

If you drag one end of a line to change the lengths of the sides, are the triangles still equilateral or right triangles?

GeoGebra has other ways of constructing triangles. You will be doing a lot of that in the topics.

**Open Extra Tabs**

Use the “+” button (above the upper right corner of the tab) to create a new tab. This is handy if a whiteboard tab or a GeoGebra tab becomes filled up and you want to open a new one without erasing everything.

**Get Some Help on Math Notation**

Go to the Help tab to learn more about VMT. For instance, look up how to enter Mathematical Equations/Expressions in the chat, in Whiteboard textboxes and in the VMT wiki. They use the $ to indicate math notation. You can cut and paste these expressions between the chat, Whiteboard textboxes, and the VMT wiki.

**Review Your Team’s Work**

Use the history sliders on the left side of the whiteboard and on the left side of the chat to get an overview of what your group has done and discussed if you come in late or return on another day. Discuss a summary of your work with your group. You can put a textbox with this summary in your Summary Tab. You can even start with an outline or a first draft of a summary in the Summary Tab and then have everyone discuss it in the chat and edit it in the Summary Tab.

Try to create a reference from a chat posting about an idea in the summary to the sentence in the Summary tab.

What have you learned in this activity? What do you wonder about it? What did you not understand or what do you want to know about? Ask the other people in your group—they may have some answers for you or be able to help you find the answers.
GeoGebra was created to harness the power of personal computers to help people learn about how exciting geometry can be as an interactive and creative world of exploration and expression. The original developer of GeoGebra discusses his vision and the worldwide response to it in this YouTube video:

http://www.youtube.com/watch?v=w7lgMx8-1e0

Another video shows students engrossed in artistic, evolving and three-dimensional images of mathematical phenomena constructed in the GeoGebra environment:

http://www.youtube.com/watch?v=9lrZAYHpGfk

A third video provides a sampling of advanced GeoGebra constructions, showing the boundless possibilities of the system for representing mathematical objects:

www.youtube.com/watch?v=rZnKMwicWM

Check out this video for an overview and some tips on the use of the GeoGebra tools that are equivalent to traditional straightedge and compass:

http://www.youtube.com/watch?v=2NqblDIPi38

A thorough explanation of a simple construction with a dependency is given in a YouTube video using GeoGebra tools that are equivalent to straightedge and compass:

http://www.youtube.com/watch?v=AdBNfEOEVco

Here is a video showing how to construct an equilateral triangle with those tools in GeoGebra:

http://www.youtube.com/watch?v=ORlauWNQSM_E

Check out these videos of complicated dependencies:

http://www.youtube.com/watch?v=Oyj64QnZle4&NR=1
http://www.youtube.com/watch?v=-GgOn66knqA&NR=1

There are a large number of YouTube tutorials for GeoGebra. Some of them are collected on the GeoGebra channel:

http://www.youtube.com/geogebrachannel

A good place to begin these videos is:

www.youtube.com/watch?v=2NqblDIPi38

There is a GeoGebra wiki site with resources for students and teachers:

www.geogebra.org
http://www.geogebratube.org

GeoGebra becomes even more powerful in its multi-user version, as part of the VMT (Virtual Math Teams) software environment. Here are some YouTube demos of important aspects of the VMT-with-GeoGebra system:
The multi-user version of GeoGebra—each person sees the actions of the others as they happen:

http://youtube.googleapis.com/v/4oBBynYVrY0

GeoGebra's history slider—you can go back and forth to see how a diagram evolved step by step in a GeoGebra or Whiteboard tab of VMT:

http://youtube.googleapis.com/v/DRlDnadcFRE

The VMT Replayer—you can replay an entire session, including all the tabs. The chat is coordinated with the drawings as you scroll or replay. You can speed up the replaying at multiple speeds. You can stop and step through, action-by-action, forward or backward to analyze the group interaction in detail:

http://youtube.googleapis.com/v/3IzkcVSyYjM

The three videos on VMTwG are integrated in a PowerPoint slide show introduction to VMTwG, available at: http://GerryStahl.net/pub/vmtdemo.pptx
Tour 5: Further Reading and Browsing

GeoGebra
www.geogebra.org -- main website for downloading stand-alone GeoGebra
www.geogebratube.org -- website for sharing and downloading GeoGebra constructions and applets
wiki.geogebra.org -- website with GeoGebra help pages, tutorials and user forum for questions

Geometer’s Sketchpad

Geometry

Euclid

Virtual Math Teams Research Project

The Math Forum
Creative Commons Share Alike Copyright
The first topics in this document are based largely on: Introduction to GeoGebra by Judith and Markus Hohenwarter, modified: November 9, 2011, for GeoGebra 4.0

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In addition, this document has drawn many ideas from the other sources listed in this section.
Some Common Problems while Starting Up VMT

• **VMT Cannot Find Java**
  Look in your applications library. If you do not have the latest version of Java, download it from the Internet.

• **VMT Cannot Find Java Web Start**
  Look in your applications library. If you do not have the latest version of Java WebStart, download it from the Internet.

Some Common Problems While Using VMT

• **If Your View of VMT or the Shared GeoGebra Construction Becomes Dysfunctional**
  If your view of the shared GeoGebra construction becomes dysfunctional or you do not think you are receiving and displaying chat messages, then close the VMT chat room window. Log in to the VMT Lobby again and enter the chat room again. Hopefully, everything will be perfect now. If not, press the Reload button if there is one. If all else fails, read the Help manual, which is available from the links on the left side of the VMT Lobby.

• **After Adding/Removing the Algebra View or Changing the Perspective, Part of the GeoGebra Tab is Blank**
  Press the REFRESH button at the bottom left of the VMT window.

• **Unable to Take Control Even Though Nobody Else Has Control**
  Make sure the history slider (on the left) is at the current event (all the way down). You cannot take control while scrolling through the history. On rare occasions, the control mechanism breaks and that tab can no longer be used.

• **When Trying to Open a VMT Chat Room, the Password Field is Blank and the Logon Fails**
  This can happen when your VMT-Lobby session has expired. Go to the VMT Lobby and logout. Then log back in and try again to enter your room. If that does not work, try closing your browser, then logging back into the lobby. As a last resort, reboot your computer.

• **Your Username is Refused When You Try to Enter a Chat Room That You Recently Left**
  Sometimes when a chat room crashes, your username is still logged in and you cannot use that username to enter the room again. After a few minutes, that username will be automatically logged out and you will be able to enter the room again with that username. Alternatively, you can register a new username and enter the chat room with the new username.

Technical Requirements to Start a VMT Chat Room

• **VMT is a Java WebStart application, so Java WebStart must be enabled. Note, on Macs you may need to go to the Java Control Panel (or Preferences depending on the version) and explicitly enable Java WebStart.**

• **VMT downloads a .jnlp Java WebStart file, so .jnlp must be an allowed file type to download.**

• **When VMT starts it will download the needed Java jars from the VMT server. So downloading jars must be allowed.**
• If VMT does not start when a .jnlp file is downloaded, then the .jnlp file extension needs to be associated with the Java WebStart program (javaws).
• It should also be possible to start VMT by finding the .jnlp file in the browser downloads folder and double clicking it.
• The firewall (for instance at a school) must allow vmt.mathforum.org
• The firewall (for instance at a school) may need to open port 8080
• To use the VMT Lobby, JavaScript must be enabled
• It might be helpful to list vmt.mathforum.org as a trusted site for java downloads

Contact Us
Problems or questions? Email us at: vmt.help@mathforum.org.
Notes & Sketches

This space is for your notes. Paste in views of your constructions. List files of constructions or custom tools that you have saved. Jot down interesting things you have noticed, questions you have wondered about or conjectures you might want to explore in the future. Collect more dynamic-math activities here.