

Social practices of group cognition in virtual math teams

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The Virtual Math Teams (VMT) project is an effort to promote interest in mathematics by teenagers around the world in a way that is not necessarily linked with formal schooling. It invites people to meet online in small groups of about three to six participants to chat about mathematics. The Math Forum, which hosts VMT, is a leading online site for math information and services, which has grown during the past 15 years to now serve several million visitors a month. While it offers a variety of math-related resources, the Math Forum had not featured a strong, on-going collaborative learning service until VMT.

The VMT project (mathforum.org/vmt/) explores a model for informal networked collaborative learning (see Section 1). As researchers in the VMT project, my colleagues and I are particularly interested in analyzing the way that learning takes place in a context like VMT; we focus on the group cognition that can be observed in the logs of math chats (Section 2). To support this analysis, the VMT project has developed a methodology for recording and analyzing social practices that contribute to group cognitive accomplishments (Section 3). Our research using the VMT experimental platform and methodology has resulted in an increased understanding of social practices that are developed and used by participants in the online informal collaborative learning environment (Section 4). In particular, we will summarize in this chapter the results of four case studies of VMT interactions that illustrate the variety of social practices that occur in this context (Sections 5, 6, 7, 8).

The purpose of this chapter is to provide an overview of what has been learned to date from about a thousand student-hours of VMT sessions, as published in a number of more detailed analytic case studies. In particular, it aims to describe how online text chat is carried on with the use of innovative social practices and how this enables teams of students to accomplish group cognitive results.

1. An online math discourse community outside of school

We are building the foundations for a global online community of people who are interested in mathematics. Our focus is on students, rather than professionals or graduate students, so we feature math problems that can be solved with basic knowledge of algebra and geometry. The math-education research community stresses the importance of math students discussing their reasoning (NCTM, 1989; Sfard, 2002), but school classrooms continue to be dominated by problem solving by individuals. So we are creating a place where students can explore and discuss math with other students, either independent of or in parallel with classroom routines.

The VMT project is a five-year effort to design an online math discourse community. Starting very simply in 2003 from a successful online math problem-of-the-week service (mathforum.org/pow/) and taking advantage of popular off-the-shelf chat software to make it collaborative, we have since then gradually evolved a more sophisticated environment involving carefully scripted pedagogical interventions, open-ended math issues and custom software—guided by extensive analysis of student behaviors through cycles of trials.

While the ubiquity of networked computers connected through the Internet from homes and schools creates an exciting opportunity for students around the world to explore math together, the practical difficulties are enormous. We are interested in facilitating the development of high-level thinking skills and the deep understanding that comes from engaging in effective dialog (Wegerif, 2006, 2007) and merging personal perspectives (Stahl, 1993, 2006b), but we find that students are accustomed to using text chat and the Internet for superficial socializing. Furthermore, their habits of learning are overwhelmingly skewed toward passive acquisition of knowledge from authority sources like teachers and books, rather than from self-regulated or collaborative inquiry. Finally, attempts to invent technological solutions have failed for lack of regard for issues of social practice. Designers of online environments have too often been driven more by technological capabilities than by careful analysis of the needs of people trying to work or learn together. Moreover, they have assumed that online environments will be used as intended by the designers rather than as adapted by the users through creative social practices constrained by institutional contexts. Our experience to date suggests three stubborn challenges that need to be addressed:

- How to deepen the learning that takes place, given that most current examples of social networking and learning in online communities remain shallow.
- How to introduce inquiry learning by student-centered informal online communities into social contexts dominated by formal schooling.
- How to integrate (a) pedagogical scaffolding, (b) technological affordances and (c) motivational sociability into a coherent service that fosters a growing community.

In order to address these needs, we have been using our emergent online community as a laboratory for studying the social practices of group cognition “in the wild.” Virtual math teams are small groups of students who meet in a chat room to discuss mathematical topics. These are typically three to six teenage students who interact for about an hour at a time. The chat rooms are set up by staff of the Math Forum. New students are invited through Math Forum initiatives, although students can subsequently set up their own rooms and invite friends or the online public. These meetings may be encouraged by teachers, but they occur online while the students are at home, in a library or elsewhere. No teacher is present in the room, although a facilitator may be present to provide guidance in learning how to use the online environment. In the long run, these small, short-lived teams may evolve to become part of a global community of math discourse.

As designers of educational chat environments, we are particularly interested in how small groups of students construct their interactions in chat media with different possible configurations of technical features (Lonchamp, 2006). How do they learn about the meanings that designers embed in the environment and how do they negotiate the practices that they will adopt to turn technological possibilities into practical means for mediating their interactions? How can we design with students the technologies, pedagogies and communities that will result in desirable collaborative experiences for them?

The analysis of social practices summarized in this chapter points to the potential of text-based chat to provide an effective medium for computer-supported collaborative learning outside of school settings. We have found that in many contexts chat is more engaging than the asynchronous media often used in education. However, text messaging and chat as normally practiced by teenagers is customarily a medium of informal socializing, not of group knowledge building. Creating a virtual place, a technological infrastructure and a set of social practices to foster more serious group cognition requires coordinated design based on detailed analysis of usage in settings like virtual math teams. If the group experience is a positive one for the participants, they may want to return. Many chats end with people making plans to get together again. In some experiments, the same groups attended multiple

sessions. Eventually, we would like to see a community of users form, with teams re-forming repeatedly and with old-timers helping new groups to form and to learn how to collaborate effectively.

As the researchers who build the VMT service, we are studying how students do mathematics collaboratively in online chat environments. We are particularly interested in the social practices that they develop to conduct their interactions in such an environment. Taken together, these practices define a culture, a shared set of ways to make sense together. The practices are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to the participants. These practices define the ways in which chat groups interactively manage resources and conduct activities.

2. Group cognition in math chats

Through the use of the kinds of practices we have analyzed, small groups construct their collaborative experience. The chat takes on a flow of interrelated ideas for the group, analogous to an individual's stream of consciousness. The referential structure of this flow provides a basis for the group's experience of intersubjectivity, common ground and a shared world.

Our goal in the VMT project is to provide a service to students that will allow them to have a rewarding experience collaborating with their peers in online discussions of mathematics. We can never know exactly what kind of subjective experience they had, let alone predict how they will experience life under conditions that we design for them. Our primary access to information related to their group experiences comes from our chat logs. The logs capture most of what student members see of their group on their computer screens. We can even replay the logs so that we see how they unfolded sequentially in time. Of course, we are not engaged in the interaction the way the participants were and recorded experiences never quite live up to the live version because the engagement is missing. We do test out the environments ourselves and enjoy the experience, but we experience math and collaboration differently than do middle-school students.

We also interview students and their teachers, but teenagers rarely reveal much of their life to adults. So we try to understand how collaborative experiences are structured as interpersonal interactions. Our focus is not on the individuals as subjective minds, but on the social group as constituted by the interactions that take place within the group.

During VMT chats, students work on math problems and themes. In solving problems and exploring math worlds or phenomena, the groups construct sequences of mathematical reasoning that are analogous to proofs. Proofs in mathematics have an interesting and subtle structure. One must distinguish: (a) the problem situation; (b) the exploratory search for the solution; (c) the effort to reduce a haphazard solution path to an elegant, formalized proof; (d) the statement of the proof; and (e) the lived experience of following the proof (Livingston, 1986, 1987). Each of these has its own structures and social practices. Each necessarily references the others. To engage in mathematics is to become ensnarled in the intricate connections among them. To the extent that these aspects of doing math have been distinguished and theorized, it has been done as though there is simply an individual mathematician at work. There has been virtually no research into how these could be accomplished and experienced collaboratively—despite the fact that talking about math has for some time been seen as a priority in math education.

More generally, we investigate how these groups construct and make sense of their shared experience of collaborating online (Stahl, 2007). While answers to many questions in computer-mediated interaction have been formulated largely in terms of *individual* psychology, questions of collaborative experience require consideration of the *group* as the unit of analysis. Naturally, groups include individuals as contributors and interpreters of content, but the group interactions have structures and elements of their own that call for special analytic approaches (see Section 5). When groups work well, they can succeed in accomplishing high-order cognitive tasks—like inquiry, problem-solving, generalization and insight—as a group (see Section 6). We call this *group cognition* (Stahl, 2006b).

Problem solving by groups often takes place through interactions involving multiple people. Key ideas do not simply reflect mental representations of an individual, but arise through interactions in which people and groups respond to each other. Online, the sequentiality of chat messages can become confused without the turn-taking conventions of face-to-face communication. Both participants and analysts must learn how to reconstitute and represent the response structure that drives group interaction. At this level, we analyze a proposal-response pair that is typical in math chats, and look at the referencing patterns that determine chat threading when this pair is successfully completed and when it fails (see Section 7). Often, math proposals involve deictic references to math objects. Accomplishing such references without physical gestures can be challenging; they require support from special software functionality (see Section 8).

In the most successful VMT chats, meaning is created at the group unit of analysis rather than by particular individuals or isolated minds. The recognition that collaborative groups constitute themselves interactionally and that their sense making takes place at the group unit of analysis has fundamental methodological implications for the study of collaboration. The field of computer-supported collaborative learning (CSCL) was founded a decade ago to pursue the analysis of group meaning making (Stahl, Koschmann, & Suthers, 2006). We view the research described here as a contribution to that CSCL tradition.

3. Research methodology for recording social practices and group cognition

The VMT service and its technological infrastructure have been systematically designed as an experimental testbed for studying group cognition. The chat room is itself persistent and the drawings and text messaging can be replayed for researchers with their original sequentiality. There is no need to involve videotaping and transcription, with their methodological complexities, in order to capture what takes place and make it available for detailed study. While many things are not captured that may take place for individual participants at their distributed physical locations, most of what enters into the group interactions and is necessary for its analysis is readily available to researchers. For instance, the students know little about each other from outside of the chat room. Communication cues that are hard to specify in face-to-face communication have been largely excluded from the text-based interaction.

In chat settings, participants exchange textual postings. This is the sole visible basis for interaction, communication, mutual understanding and collaborative knowledge building within a simple chat environment. The VMT chat environment now supplements this with some social awareness features and with a shared whiteboard for drawing geometric figures, but for the moment let us consider a generic chat room. In addition to the content of the typed postings, their order, sequentiality and timing typically play a significant role in how the postings are understood (O'Neill & Martin, 2003). The participants log in with a chat “handle” that is associated with their postings; the wording of this handle may imply

something about the person so named. The postings by a given participant are linked together as his (or hers?) via the handle. Furthermore, we assume that the participants come to the chat room with specific expectations and motivations—in our case, because it is part of the Math Forum site and may be recommended by a teacher, parent or friend. Thus, there is an open-ended set of factors that may enter the chat from its socio-cultural context. There is also more-or-less shared language (e.g., English and basic math terminology) and culture (e.g., contemporary teen subculture and classroom math practices) that can play a role in the chats.

The current VMT environment is quite complex. In addition to the chat window, there is a shared whiteboard for drawing diagrams, geometric figures, tables of numbers and text boxes (see Figure 1 in Section 8). The chat and text boxes support mathematical notation. Both the chat and the whiteboard are persistent and their history can be scrolled by the users. There are social awareness messages indicating who is typing and drawing at the moment or entering and leaving. Many students participate in multiple sessions, and in these cases Math Forum staff provide feedback in the chat room between sessions, that the students can read later. Recently, we have added a wiki, where students from different teams can post results and respond to what others have discovered.

To support our research, we now have a replay facility in which we can view the whole interaction process in real time or fast-forward and step through the interaction with the display of the chat, drawing and awareness notices all coordinated. This gives us a tool for analysis that is analogous to digital video for face-to-face interaction, but without all the complications of lighting, camera angles, transcription and synchronization. Moreover, there is nothing going on “off camera” that affects the interaction because everything that was visually shared by the participants is replayed for us.

To study what takes place among students in chat rooms, we hold interaction analysis *data sessions* (Jordan & Henderson, 1995). These are meetings in which a number of researchers collaboratively take a careful look at chat logs and discuss what is taking place. Focus is directed toward brief extracts that present interactions of analytic interest to the research group. The chat log reveals to the researchers what was visible to the student participants. The researchers can take into account the institutional context in which the chat took place when it is made relevant within the chat. As members of the broader society to which the students also belong, the researchers share to a large extent a competent understanding of the culture and language of the chat. Thus, they are capable of making sense of the chat because they see the same things that the participants saw and can understand them in similar ways. Moreover, by repeatedly studying the persistent log of the chat and by bringing their analytic skills to it, researchers who have made themselves familiar with this genre can make explicit many aspects of the interaction that were taken for granted by participants in the flow of the moment. By working as a group, the researchers can minimize the likelihood of idiosyncratic analyses. We also work individually, studying transcripts and writing about them, but we periodically bring our analyses to the group for feedback and confirmation.

We have adapted the scientific methodology of conversation analysis (Sacks, 1992) to the micro-analysis of online, text-based, mathematical discourse (see next Section). We adopt an ethnomethodological focus on the methods that participants use to make shared sense of what they are jointly doing (Garfinkel, 1967). Later in this chapter, we summarize some of our preliminary findings about how small groups make sense collaboratively in settings like VMT. For instance, we distinguish between expository and exploratory modes of narrative, showing alternative ways individual and group knowledge can be intertwined. The negotiation of communication genres like these involve the constitution of the group as such.

Ethnomethodology provides a further theoretical justification for the ability of researchers to produce rigorous analyses of recorded interactions. This has to do with the notion of

accountability (Garfinkel, 1967; Livingston, 1987). When people interact, they typically construct social order (such as conducting a fun chat or developing a math solution) and may produce social objects (like textual postings). These objects are accountable in the sense that they were tacitly designed to reveal their own significance. A brief text, for instance, is written to be read in a certain way; its choice of wording, syntax, references and placement in the larger chat are selected to show the reader how to read it (Livingston, 1995). The account that a chat posting gives of itself for the other students in the chat can also be taken advantage of by the researchers. The researchers in a data session discuss the log in order to agree on the accounts of the postings, individually and in their interactive unity.

The social structure and the accountability of human interactions make it possible for researchers to draw generalized understandings from the analysis of unique case studies. Interactions in the VMT setting and elsewhere are extremely dependent upon the specific circumstances of the interactional context that they sequentially build and the physical or socio-cultural context that they repeatedly index. So the data of student interactions is not reproducible and cannot be compared under conditions of experimental control. However, the social structures that people construct during their interactions necessarily have a generality. Otherwise, if every event had a completely unique significance, people would not be able to understand each other. Shared understanding is the basis for human interaction and it relies upon the generality of the structures that are interactionally created. These structures may vary within limits from culture to culture and in reaction to different mediational circumstances. Students in an English-language chat in Singapore might interact differently from adults in an asynchronous discussion forum in Scotland. But experienced researchers can make sense of events in both contexts by taking into account the differences. As the analyses of interactions in VMT have shown us, there are basic patterns that students repeatedly call upon to discuss math; at the same time, even minor changes in technology support may cause participants to invent new forms of meaning making in reaction to the affordances and barriers that they enact in their online environments.

The VMT service is being developed through a design-based research approach to co-evolve the software, pedagogy, mathematics and service through an iterative process of trial, analysis and design modification. The software started with off-the-shelf chat systems and now involves development of a custom research prototype. The pedagogy started with principles of mathematics education and computer-supported collaborative learning and is now incorporating efforts to build a user community engaged in discussing math and facilitating collaborative practices. The math problems started out using the same Problems-of-the-Week offered to individuals and are now providing opportunities for groups to explore open-ended mathematical worlds as well as to work on issues that the participants generate themselves. The service started as occasional offerings and is now gearing up for continuous availability supported by as-needed monitoring and feedback.

As the trials progress, we analyze the resultant logs in the ways indicated in this paper and use our results to inform our redesign of the software, pedagogy, mathematics and service. Thereby, ethnomethodologically-informed interaction analysis provides the analytic component of design research, a component that is not often specified in discussions of design-based research (Koschmann, Stahl, & Zemel, 2006). In this sense, the usage of our insights into how students interact in chat is at odds with the usual practices of ethnomethodology and conversation analysis, which claim not to impose researcher or designer interests on their data. While we try to understand what the student participants are up to in their own terms and how they are making sense of the activity structure that we provide for them, we are doing this in order to motivate our subsequent design decisions. Our goal is not just to understand the student meaning-making processes, but to use that

understanding to modify the VMT service to allow groups to engage more effectively in math discourse.

4. Social practices for discussing math online

In order to understand the experience of people and groups collaborating online in VMT, we look in detail at the captured interactions. We conceptualize the patterns of interaction that we observe as *member methods* or *social practices*. This is a concept that we take from ethnomethodology (Garfinkel, 1967). Ethnomethodology is a phenomenological approach to sociology that tries to describe the methods that members of a culture use to accomplish what they do, such as how they carry on conversations (Sacks, Schegloff, & Jefferson, 1974) or how they “do” mathematics (Livingston, 1986). In particular, the branch of ethnomethodology known as conversation analysis (Sacks, 1992) has developed an extensive and detailed scientific literature about the methods that people deploy in everyday informal conversation and how to analyze what is going on in examples of verbal interaction.

Methods are seen as the ways that people produce social order and make sense of their shared world. For instance, conversation analysis has shown that there are well-defined procedures that people use to take turns at talk. There are ways that people use to determine when they can speak and how they can signal that others may take a turn at conversation (Sacks *et al.*, 1974).

We adopt the general approach of conversation analysis, but we must make many adaptations to it given the significant differences between our chat logs and informal conversation. Our data consists of chat logs of student messages about mathematics. The messages are typed, not spoken, so they lack intonation, verbal stress, accent, rhythm, personality. The participants are not face-to-face, so their bodily posture, gaze, facial expression and physical engagement are missing. Only completed messages are posted; the halting process of producing the messages is not observable by message recipients (Garcia & Jacobs, 1998, 1999). The messages are displayed in a particular software environment and the messages are designed by their posters to be read and responded to in that environment (Livingston, 1995; Zemel, 2005). The textual messages are persistent and may be read or ignored at will, and may be re-read later—although they scroll off-screen after several other postings appear. Several participants may be typing messages at the same time, and the order of posting these messages may be unpredictable by the participants (Cakir *et al.*, 2005). Consequently, messages do not necessarily appear immediately following the messages to which they may be responding. In addition to these features of chat, our logs are concerned with mathematics and are created within educational institutional contexts—such as the Math Forum website and sometimes school-related activities or motivations. Thus, the chats may involve building mathematical knowledge, not just socializing and conversing about opinions or everyday affairs.

These differences between our chats and normal conversation mean that the rules of turn-taking, etc. have all been transformed. What remains, however, is that people still develop methods for creating and sustaining social order and shared meaning making. Chat participants are skilled at creating and adapting sophisticated methods that accomplish their tasks in these unique environments. It is the analyst’s job to recognize and describe these methods, which are generally taken for granted by the participants.

Among the student chat methods of interest to us are the interactional means that the students use:

- To introduce each other
- To adapt to institutional settings

- To socialize; to have fun; to flirt
- To get to know each other better
- To establish interpersonal relations or roles
- To form themselves into groups
- To define a problem to work on
- To start working on a problem
- To agree on how to proceed
- To bring in math resources
- To clarify a point
- To make a proposal
- To tell a story
- To justify a claim
- To negotiate a decision
- To reference an object
- To count items together
- To step through an analysis
- To agree on solutions
- To stop problem solving

5. Expository and exploratory discourse

Although our ethnomethodological chat analysis methodology modeled on conversation analysis has so far yielded the most insight into the VMT data, we are pursuing a variety of approaches including coding (Strijbos & Stahl, 2005), statistical comparisons (Zemel, Xhafa, & Cakir, 2005) and ethnographic investigations (Sarmiento, Cakir, & Stahl, 2006). These independent approaches can shed important light on the data and inform each other. Ethnographic analyses of the socio-cultural context, such as the classroom experiences of individual chat participants or their other activities in the Math Forum community help to clarify the personal motivations and the math resources that students bring into the chat (Renninger & Shumar, 1998).

In our project, a statistical analysis led to an interesting conversation analytic result. A statistical comparison of codes between chats in which students had time to work on math problems individually prior to the chats (condition A) and those where they first saw the problem in the collaborative chat context (condition B) led to a puzzling anomaly (Zemel, Xhafa, & Stahl, 2005). While most of the chats in both conditions were clustered together, one chat from each condition clustered more with the chats from the other condition. A conversation analysis of the two anomalous chats led to a distinction between *expository narrative* and *exploratory inquiry* (Mercer & Wegerif, 1999). In conversation analytic terms, this is largely a difference in turn-taking methods. In exposition, one person makes a bid to “tell a story” about how they solved a problem. The other group members offer the expositor an extended turn at talking (or posting). The expositor dominates the discourse, providing a sequential account across several unusually long turns. The other group members listen (read) attentively, provide brief encouraging exclamations, pose questions and provide an audience. In a math problem-solving session, there may be multiple expositions concerning subsequent parts of the problem solution, possibly by different people. In exploratory inquiry, the turns are more equally shared as the group collectively investigates the problem and co-constructs a solution path. The steps in exploration may each involve several participants, with one person proposing a move and others agreeing, making the move or challenging it. The distinction of exposition versus exploration parallels that between cooperation (people dividing up tasks to reach a common goal) and collaboration (people working together on each task) (Dillenbourg, 1999).

The statistical quandary was resolved by noticing that the anomalous chat from condition A consisted largely of collaborative exploration despite the fact that the students may have had a chance to produce their own solutions in advance. In the anomalous chat from condition B, the students took time in the chat to first work out at least partial solutions on their own before contributing to the chat; they then provided expositions on what they found. These examples demonstrate that external conditions do not mechanically determine the

methods that people use to interact. In fact, it is common for students in a chat to alternate between cooperative expository and collaborative exploratory sequences of interaction.

6. The group of individuals

The difference between cooperative exposition and collaborative exploration in math problem solving chats is related to the difference between individual solution and group solution. A given math chat log can be ambiguous as to whether it should be analyzed as a set of contributions from individual thinkers or whether it should be analyzed as a group accomplishment. Often, it is helpful to view it both ways and to see an intertwining of these two perspectives at work (Stahl, 2006a).

We tried a session where we had students solve standard math problems individually and then had the same students solve the same problems in VMT chat groups. In the group that we tracked, the group not only correctly solved all the problems that were solved by any one member of their group individually, but also solved some that no one did by themselves.

When we first looked at the log of a complicated problem that no individual in the group had been able to solve, it appeared that one student, Mic, who seemed particularly weak in math was clowning around a lot and that another, Cosi, managed to solve the problem herself despite this distraction in the chat room coming from Mic.

In thinking about why Cosi could solve this problem in the group context but not alone, we noticed that she was not simply solving the problem as one would in isolation (e.g., setting up algebraic equations), but was interacting with the group effort. In particular, Dan, Mic and Hal had set up a certain way of thinking about the problem and of exploring possible solutions, although none of them was able to carry out the approach. Cosi was reflecting on the group approach and repairing problems in its logic. The numbers, words and considerations that she used were supplied by the group and its context of on-going interactive activities and shared meanings.

If we combine the sequence of proposals from Mic, Dan, Hal and Cosi, they fit together much like the cognitive process of an individual problem solver. Clearly, Cosi made some contributions to the group that were key to the group solution. They were acknowledged as such. Cosi was termed “very smart”—although this could equally well be said of the group as a whole. While no individual in the group could completely see how to solve the problem, everyone contributed to exploring it in a way that rather efficiently led to a solution. In particular, Mic used clowning around as an extremely effective way to facilitate the group process. By joking and laughing a lot, the group relieved some of the pressure to solve a problem that was beyond any individual’s reach and to open a social space in which ideas could be put forward without fear of being harshly judged. Through non-threatening forms of critique and repair, the group solved the problem.

Attributing the solution to the group rather than to the sum of the individuals in the group can be motivated by seeing that the construction of mathematical meaning in the solution process was done *across* individuals. That is, meaning was created by means of interactions among individual contributions (postings) to the chat—such as through what are called *adjacency pairs* in conversation analysis—more than by individual postings construed as expressing personal mental representations.

7. Math proposal adjacency pairs

In an early chat of the VMT project, we observed a repeated pattern of interaction that we have since found to be common in math chats (Stahl, 2006d). Here is an excerpt from that chat (line numbers added; handles anonymized):

17. Avr (8:23:27 PM): i think we have to figure out the height by ourselves
18. Avr (8:23:29 PM): if possible
19. pin (8:24:05 PM): i know how
20. pin (8:24:09 PM): draw the altitude'
21. Avr (8:24:09 PM): how?
22. Avr (8:24:15 PM): right
23. Sup (8:24:19 PM): proportions?
24. Avr (8:24:19 PM): this is frustrating
25. Avr (8:24:22 PM): I don't have enough paper

In this log we see several examples of a three-step pattern:

- A proposal bid is made (by Avr in lines 17 and 18) for the group to work on: (“I think we have to”).
- The bid is taken up by someone else (Pin in line 19) on behalf of the group: (“I know how”).
- There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step (such as Pin’s new proposal bid in line 20).

The third step in this pattern initiates a repeat of the three-step process:

- A proposal bid is made (by Pin in line 20) for the group to work on: (“Draw the altitude”).
- An acceptance is made by someone else (Avr in line 22) on behalf of the group: (“Right!”).
- There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step (such as Sup’s new proposal bid in line 23).

But here the pattern breaks down. It is unclear to us as analysts what Sup’s proposal bid, “Proportions?” is proposing. Nor is it responded to or taken up by the other group members as a proposal. Avr’s lines 24 and 25 ignore it and seem to be reporting on Avr’s efforts to work on the previous proposal to draw the altitude.

Breakdown situations are often worth analyzing carefully, for they can expose in the breach practices that otherwise go unnoticed, taken for granted in their smooth execution. Our analysis of Sup’s “failed proposal” (in Stahl, 2006d) helps to specify—by way of counter-example—conditions that promote successful proposals in math chats: (a) a clear semantic and syntactic structure, (b) careful timing within the sequence of postings, (c) a firm interruption of any other flow of discussion, (d) the elicitation of a response, (e) the specification of work to be done and (f) a history of helpful contributions. In addition, there are other interaction characteristics and mathematical requirements. For instance, the level of mathematical background knowledge assumed in a proposal must be compatible with the expertise of the participants and the computational methods must correspond with their competencies.

We call the three-step pattern described above a *math proposal adjacency pair*. It seems to be a common interaction pattern in collaborative problem solving of mathematics in our chats. We call this a form of “adjacency pair” in keeping with conversation analysis terminology (Duranti, 1998; Schegloff, 1991), even though in chat logs the two parts of the pair may not appear adjacent due to the complexities of chat postings: e.g., line 22 responds to line 20, with line 21 intervening as a delayed response to line 19. As we see in other chats, however, not all student groups adopt this method of making math proposals.

8. Deictic referencing and threading

The more we study chat logs, the more we see how interwoven the postings are with each other and with the holistic Gestalt of the interactional context that they form. There are many ways in which a posting can reference elements of its context. The importance of such indexicality to creating shared meaning was stressed by Garfinkel (1967). Vygotsky also noted the central role of pointing for mediating intersubjectivity in his analysis of the genesis of the infant-and-mother’s pointing gesture (1930/1978, p. 56). Our past analysis of face-to-face collaboration emphasized that spoken utterances in collaborative settings tend to be elliptical, indexical and projective ways of referencing previous utterances, the conversational context and anticipated responses (Stahl, 2006b, chapter 12). So we provide support for pointing in chat.

We recently adapted VMT-Chat (Mühlpfordt & Stahl, 2007; Wessner *et al.*, 2006), a chat environment that not only includes a shared whiteboard, but has functionality for referencing areas of the whiteboard from chat postings and for referencing previous postings (see Figure

The screenshot displays the VMT-Chat interface. On the left, a whiteboard shows a grid with a rectangle ABCD. A staircase path is drawn from point A to point B. A red diagonal line represents the hypotenuse of a right triangle with AB as the hypotenuse. The area of this triangle is labeled as $2\sqrt{3}$. The chat log on the right shows a conversation about the area of a circumscribed circle and the perimeter of a section. Line 12 of the chat is selected.

WhiteboardChat: (VMT0512G4)

Whiteboard:

Reference [tools] [100%]

1. What is the shortest path along the grid between the two points?
 2. How many possible routes are there from point A to point B?
 3. What is the maximum distance from point A to B if you can only travel on each POINT once?
 4. How many ways are there to get from A to B in rectangle ABCD?
 5. Make a right triangle with AB as the hypotenuse. What is the area of the circumscribed circle?
 6. Can you go off the edge and come back somewhere else?

Extras

7. What is the shortest path along the grid between any two points $A(x1, y1)$, $B(x2, y2)$?
 8. How many shortest paths are there from A to B and how does this vary with changes in the positioning of A relative to B?
 9. Suppose the right and left edges of the grid are connected. How does that change the distances between points?

Chat: (207)

ImHere (May 12, 2005 8:42 PM): what is the area of this shape?
 Jason (May 12, 2005 8:42 PM): which shape?
 ImHere (May 12, 2005 8:43 PM): woops
 ImHere (May 12, 2005 8:43 PM): ahhi
 Jason (May 12, 2005 8:42 PM): kinda like this one?
 Jason (May 12, 2005 8:42 PM): the one highlighted in black and dark red?
 ImHere (May 12, 2005 8:43 PM): between th stairs and the hypotenuse
 Jason (May 12, 2005 8:43 PM): oh
 Jason (May 12, 2005 8:43 PM): that would be a tricky problem, each little "sector" is different
 Jason (May 12, 2005 8:43 PM): this section...
 ImHere (May 12, 2005 8:44 PM): perimeter is $12\sqrt{3}$
 Jason (May 12, 2005 8:43 PM): is smaller than this section
 ImHere (May 12, 2005 8:44 PM): assume those lines are on the blocks
 Jason (May 12, 2005 8:44 PM): the staircase lines?
 ImHere (May 12, 2005 8:45 PM): yeah
 Jason (May 12, 2005 8:44 PM): they already are on the blocks

Message:

Figure 1. Screen view of VMT-Chat with referencing. Line 12 of the chat is selected.

1). The shared whiteboard is necessary for supporting most geometry problems. Sharing drawings is not enough; students must be able to reference specific objects or areas in the drawing. The whiteboard also provides opportunities to post text where it will not scroll away. The graphical references (see the blue line from a selected posting to an area of the drawing) can also be used to reference one or more previous postings from a new posting, in order to make the threads of responses clearer in the midst of “chat confusion” (Fuks, Pimentel, & de Lucena, 2006).

In one of our first chats using VMT-Chat, the students engaged in a particularly complex interaction of referencing a figure in the whiteboard whose mathematics they wanted to explore (Stahl, 2006c). Here is the chat log from Figure 1 (graphical references to the whiteboard are indicated by “[REF TO WB]” in the log.):

1 ImH: what is the area of this shape? [REF TO WB]
 2 Jas: which shape?
 3 ImH: woops
 4 Imh: ahh!
 5 Jas: kinda like this one? [REF TO WB]
 6 Jas: the one highlighted in black and dark red?
 7 ImH: between th stairs and the hypotenuse
 8 Jas: oh
 9 Jas: that would be a tricky problem, each little “sector” is different
 10 Jas: this section [REF TO WB]
 11 ImH: perimeter is $12\sqrt{3}$
 12 Jas: is smaller than this section [REF TO WB]
 13 ImH: assume those lines are on the blocks
 14 Jas: the staircase lines?
 15 ImH: yea
 16 Jas: they already are on the blocks

Line 1 of the chat textually references an abstract characteristic of a complex form in the whiteboard: “*the area of this shape.*” The software function to support this reference failed, presumably because the student, ImH, was not experienced in using it and did not cause the graphical reference line to point to anything in the drawing. With line 5, Jas provides a demo of how to use the referencing tool. Using the tool’s line, a definite textual reference (“*this one*”) and the use of line color and thickness in the drawing, lines 5 and 6 propose an area to act as the topic of the chat. Line 7 makes explicit in text the definition of a sub-area of the proposed area. Line 8 accepts the new definition and line 9 starts to work on the problem concerning this area. Line 9 references the problem as “that” and notes that it is tricky because the area defined does not consist of standard forms whose area would be easy to compute and add up. It refers to the non-uniform sub-areas as little “sectors”. Line 10 then uses the referencing tool to highlight (roughly) one of these little sectors or “sections”. Line 12 continues line 10, but is interrupted in the chat log by line 11, a failed proposal bid by ImH. The chat excerpt continues to reference particular line segments using deictic pronouns and articles as well as a growing vocabulary of mathematical objects of concern: sectors, sections, lines, blocks.

Progress is made slowly in the collaborative exploration of mathematical relationships, but having a shared drawing helps considerably. The students use multiple textual and graphical means to reach a shared understanding of mathematical objects that they find interesting but hard to define. In this excerpt, we start to get a sense of the complex ways in which brief

textual postings weave dense webs of relationships among each other and with other elements of the collaborative context in order to co-construct understanding of meaningful artifacts in a virtual social world.

9. Small groups and social practices

In order to accomplish their work and learning, people often form themselves into small groups and dyads (Engeström, Y. & Toiviainen, H., this volume; Wegerif & De Laat, this volume). In the VMT project, we look at the ways in which people form effective groups to accomplish the object of their activity. For instance, one person might engage in explanatory narrative and engage others as audience or might offer a proposal on how to proceed and elicit a group acceptance or challenge. One might joke or tease in order to engage and sustain others in a joint effort. Alternatively, one might point to an object and construe it as a problem to be understood and solved by a group. Once a group is constituted, similar social practices are used to make shared sense of the joint activities and available artifacts.

Social practices are typically established, shared and taken for granted within a culture. When innovative technologies and pedagogies open up new circumstances, participants in the new environments must often make sense of their unaccustomed situation and develop creative practices, leading their fellow participants to adopt the proposed practices. In the VMT project, we put students into virtual worlds where the usual institutional practices of school, business or face-to-face sociality are largely missing. We observe how they manage and we respond to their difficulties with new designs to support their activity.

Of course, our approach represents only one possible research perspective. It is also possible to focus on individual trajectories (psychology) or on larger social constructs (e.g., activity structures). In certain respects, social practices and group cognition provide a foundation upon which the other views can be built, e.g., through internalization or structuralization. For the VMT project, this approach—concerned with the social practices of group cognition—has proven useful in providing guidance and understanding for the iterative design process.

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