

**How (not) to build a pyramid in the VMT environment:
(mis)understanding in collaborative problem solving**

Christian Greiffenhagen (University of Manchester)
Jacqueline Eke (Manchester Metropolitan University)

DRAFT

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VERSION

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INTRODUCTION: The trouble with pyramids

The research presented here takes as its materials, log files of the interaction between three participants who joined the virtual math teams (VMT) Spring Fest 2006 project as team B (see Stahl, 2006, forthcoming). The participants come together in an online forum for four one-hour sessions. The forum has the feature not only of ‘chat’ but also of a collaborative drawing tool or ‘whiteboard’. Participants are new to both the VMT project and the VMT environment (though probably not new to chat forums).

Session 1 of Spring Fest 2006 invites teams to collaborate on a math problem and the ‘task sheet’ also provides instructions for sessions 2 and 3.¹ Feedback on each session is added to the whiteboard ahead of participants entering the VMT environment for each next session. So, on entering the VMT environment to begin session 2, team B find feedback on their work in session 1.² Together, the feedback and the task sheet provide instructions for ongoing collaboration in the VMT environment (for each of the four sessions).

The focus of our analysis is on participants’ construction of a three-dimensional pyramid configuration as the basis for their mathematical problem and problem-solving, which happens across sessions 2 and 3. In this paper, we will only deal with (part of) what happens in session 2.

The ‘trouble’ with pyramids begins almost as soon as the proposal is made to use this ‘shape’ as the basis for mathematical work in session 2 of the data set. Trouble emerges as a result of one member trying to communicate to his/her team mates, the nature and character of the pyramid he/she has ‘in mind’.³ The team are eventually successful in providing a formula to express the pattern of growth for a three-dimensional, fully centred pyramid (of the Aztec kind), but this seems almost in spite of the troubles in ‘fixing’ a pyramid shape in their mutual sense.

Why this stretch of the data set? First, the work focused on the pyramid shape could be considered in abstract from the full data set while still providing some substantive coherence to the material under consideration. Second, and more interestingly, the material allows us to focus on the issue of ‘misunderstanding’. What would strike anyone on inspection of the data set is just how ‘muddled’ it seems. Of course, much work has been done with ‘chat’ materials which points to the way that participants to chat environments cannot rely, in the way that ordinary conversationists can, on next-turn sequentiality for issues of ‘sense’ (cf., Garcia & Jacobs, 1998, 1999; Herring, 1999; O’Neill and Martin; 2003; Zemel, 2005; Zemel & Cakir, forthcoming), so ‘muddled’ is an expectation in materials of this sort. The ‘muddles’ of interest though, are not merely the result of the non-sequentiality of chat turns, although

this is implicated in what is of concern. Rather, the interest is in what comes off as a great deal of ‘misunderstanding’ between the participants, but often unnoticed by them.

‘Misunderstanding’ is taken up in two ways; as a members matter, and as an analysts/educationists matter. Our research approach takes a principled position in its orientation to social phenomenon such that we are committed to the explication of participants’ work. It is the participants, team B and Gerry, who ‘produce’ the orderliness (and disorderliness) of the four one-hour long sessions which make up the data set. Our task is to explicate or describe the work that just this gang of people do, using just the tools they have at hand to do it. In doing this, we are oriented to the ways the participants themselves orient to ‘trouble’ and ‘misunderstanding’ and our analyses describe this. But we are also interested in ‘misunderstanding’ in another way. As analysts (and educators) we find ourselves identifying places where ‘misunderstanding’ occurs but which members themselves have not ‘found’. We are in a ‘privileged’ position where we can say that these moments of ‘disattended misunderstanding’ have (serious) consequences for members’ future mutual understanding/misunderstanding. How then, as researchers committed to explicating members’ work, should we deal with this?

We take a broad-brush approach to our materials at this time, keeping our focus general. This provides a detailed enough description of the materials to allow us to raise a number of issues for closer discussion.

ANALYSIS: Building (and sustaining) a pyramid in the VMT environment

1. Looking for and finding a new problem: 3-d figures, a row of blocks, or maybe a pyramid

To begin our consideration of the ‘pyramid data’, we must set it within the context of its production as a proposal for team B’s mutual, collaborative mathematical work. In their second session, having established that they had each read the feedback comments, the participants spend approximately nine minutes attending to the issues raised in the feedback before orienting to a new topic for mathematical consideration (which is the final issue raised in the feedback).

331	Aznx	So let's brainstorm through some problems that we think are challenging.	10.05.2006	07.09.33
332	Quicksilver	yes...new topic	10.05.2006	07.09.40
333	bwang8	ok	10.05.2006	07.09.42

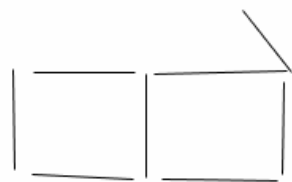
Aznx makes an initial, candidate proposal to move on from their previous activity: “So let’s brainstorm some problems that we think are challenging” [331]. The proposal is accepted and formulated for its ‘upshot’ by Quicksilver: “yes ... new topic” [332], and Bwang’s simple agreement “okay” arrives at line 333. Just as earlier chat (not presented as part of this paper) oriented to feedback comments, so too does Aznx’s proposal. The problem is to be one that they can discover themselves and that will be interesting to them. Aznx’s analysis of the feedback instruction (and the task sheet) is embodied then in his suggestion to “brainstorm some problems” that “we think are challenging”.

334	Quicksilver	3-d figures?	10.05.2006	07.10.20
335	Aznx	I think we should discuss on the different methods.	10.05.2006	07.11.06
336	Aznx	So that we can easily apply our thoughts quickly when seeing a problem.	10.05.2006	07.11.24
337	Quicksilver	Yes....but we must find a question or problem to investigate	10.05.2006	07.11.30
338	Aznx	Yeah.	10.05.2006	07.11.37
339	Aznx	I think we should start off with a conjecture, that we need to prove.	10.05.2006	07.11.50
340	Aznx	Not a hard one, but one that can be challenging.	10.05.2006	07.12.03
341	Quicksilver	Maybe a row of blocks	10.05.2006	07.12.17
342	Quicksilver	likethis	10.05.2006	07.12.27

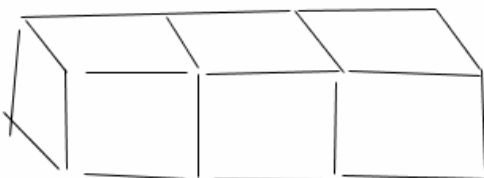
A first candidate ‘new topic’ is proposed by Quicksilver “3-d figures?” [334] and this is followed immediately by additional proposals from Aznx [335, 336/339]. Quicksilver rejects the first of Aznx’s proposals directly, arguing that it cannot be achieved without there first being “a question or problem to investigate” [337]. During the course of this exchange, indeed, on posting the proposal to investigate “3-d figures”, Quicksilver begins drawing on the whiteboard using his/her existing two-dimensional drawing of adjacent squares as a base to create a block of three, three-dimensional cubes (Figure 1). The two-dimensional adjacent squares were initially drawn by Quicksilver to illustrate his/her (earlier) description of the pattern of growth of sticks in session 1’s task.



(i)



(ii)



(iii)

Figure 1: Temporal development of an existing two-dimensional figure to a ‘three-dimensional’ row of blocks

A couple of things are notable about this proposal. First, Quicksilver commits to his/her proposal of using three-dimensional figures for their investigation *by* engaging in the whiteboard work of rendering a three-dimensional object visible. By line 341/342, Quicksilver is in a position to pursue the three-dimensional figure proposal by simultaneously proposing *and* presenting an object in the category of “3-d figures”, namely “a row of blocks.” Indeed, Quicksilver leaves nothing to chance in getting his/her proposal ‘on the table’ using as he/she does, the available tools of shading and text-to-whiteboard pointer (Figure 2; cf. Cakir *et al.*, 2009, p. 128). Second, Quicksilver’s proposal “maybe a row of blocks” can be (but is not

necessarily) seen and heard then as a candidate for his/her initial proposal of “3-d figures” – that is, “a row of blocks” can be heard as a proposed a member of the category device ‘types of 3-d figures.’⁴

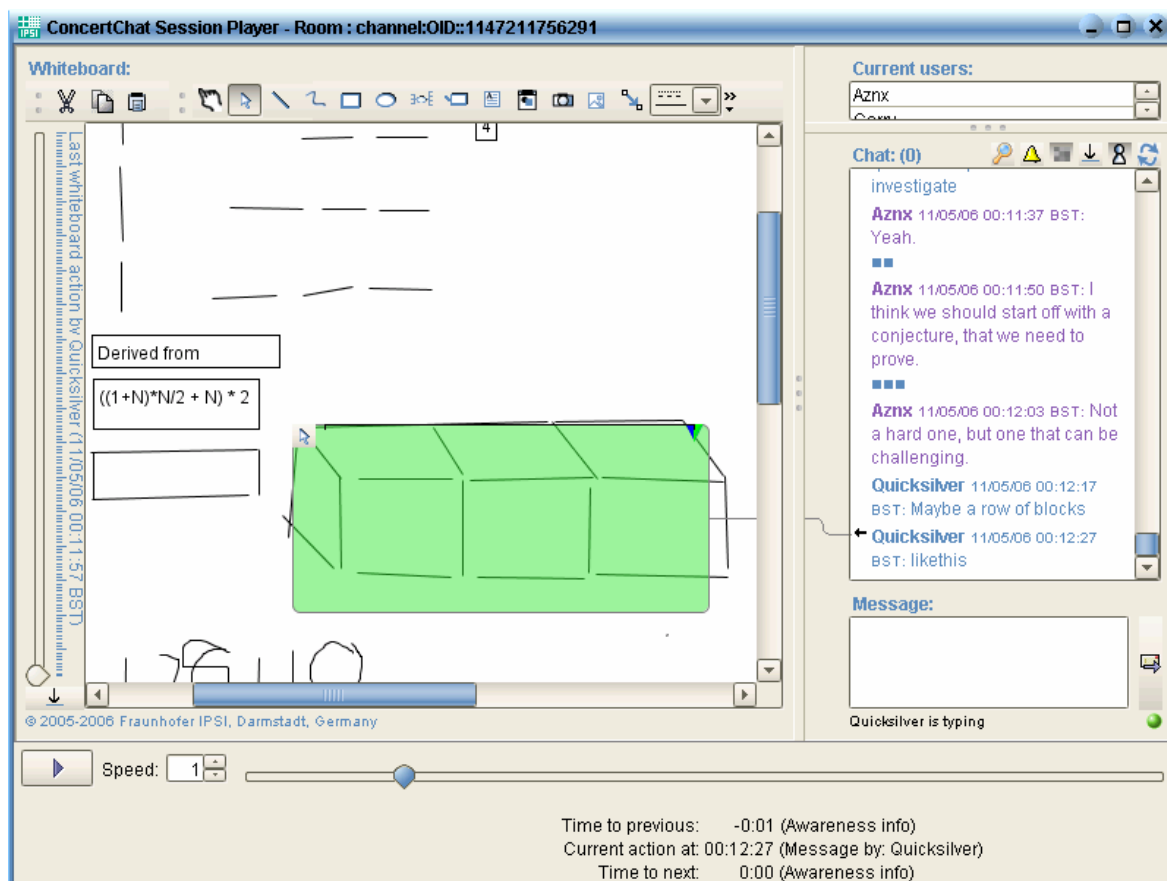


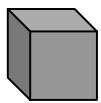
Figure 2: The referent of Quicksilver’s “like this”

After Quicksilver has produced this proposal, Aznx asks for clarification about the nature of the problem to be explored, “What about them?” In asking this question, Aznx orients to the way that having a shape is only one half the work of finding a problem to work on.

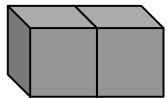
343	Aznx	What about them?	10.05.2006	07.12.44
344	Quicksilver	The amount of sticks may increase in a pattern?	10.05.2006	07.12.59
345	bwang8	The problem from yesterday, but only 3-d	10.05.2006	07.13.01
346	Quicksilver	yea i guess	10.05.2006	07.13.07
347	Quicksilver	not that good	10.05.2006	07.13.09

In response to Aznx’s request for clarification [343], Quicksilver explicates his/her proposed problem to work on such that “the amount of sticks [in the row of blocks] may increase in a pattern” [344]. It is notable that at no other time do team B raise issue with what kind of problem a drawing of this kind might be said to ‘represent’. That is, with the exception of this one instance, participants can see *in* the drawing both that the drawing is meant to stand for a sequence or pattern of sorts, and that the task will be of finding the pattern of growth of the number of sticks and blocks. As is common with many classroom tasks, the team retains from session 1 the *kind* of problem that they will be working on (“Can your group see a pattern of growth for the number of sticks and squares”), while change the shape to a more complex or difficult one.

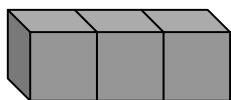
Taken together, Bwang is able to find in this proposal and the whiteboard object pointed-to at line 342, that it is the same problem the team tackled in session 1, but in three dimensions rather than two [345]. Thus for Bwang, the drawing, together with the explication of the problem is enough to identify in it a pattern or sequence similar to that of their initial task (see endnote 1), “but only 3-d”, that is, a sequence of the kind represented in Figures 3 and 4.



Step 1

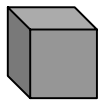


Step 2

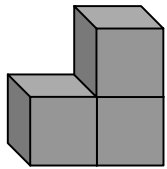


Step 3

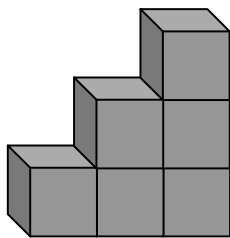
Figure 3: One possible sequence that Bwang might see in Quicksilver’s proposal



Step 1



Step 2



Step 3

Figure 4: Another possible sequence

Bwang’s evaluation of the proposed problem is taken up by Quicksilver who agrees with Bwang’s formulation (seeing the problem as a three-dimensional version of the problem in session 1) and formulates the upshot of this for the problem as “not that good” [346-347] displaying his/her reading of Bwang’s evaluation as a negative one. It is on the back of this mutual evaluative work that Quicksilver now proposes a pyramid as a new shape for the focus for their mathematical investigation [348]:

348	Quicksilver	maybe a pyramind	10.05.2006	07.13.18
349	bwang8	Yeah	10.05.2006	07.13.24
350	Quicksilver	although that's hard to draw	10.05.2006	07.13.30
351	bwang8	pyramid is good	10.05.2006	07.13.35
352	Aznx	Yeah, I liked that.	10.05.2006	07.13.36
353	Quicksilver	but we should be able to manage	10.05.2006	07.13.36
354	Quicksilver	e	10.05.2006	07.13.36
355	Quicksilver	side view	10.05.2006	07.14.25

Quicksilver qualifies the pyramid proposal as “hard to draw”, but both Bwang and Aznx seem to approve of the proposal. After line 354 Quicksilver begins to draw on the whiteboard, announcing at line 355, again using the shading and text-to-whiteboard pointer, that this drawing is a “side view” (Figure 5):

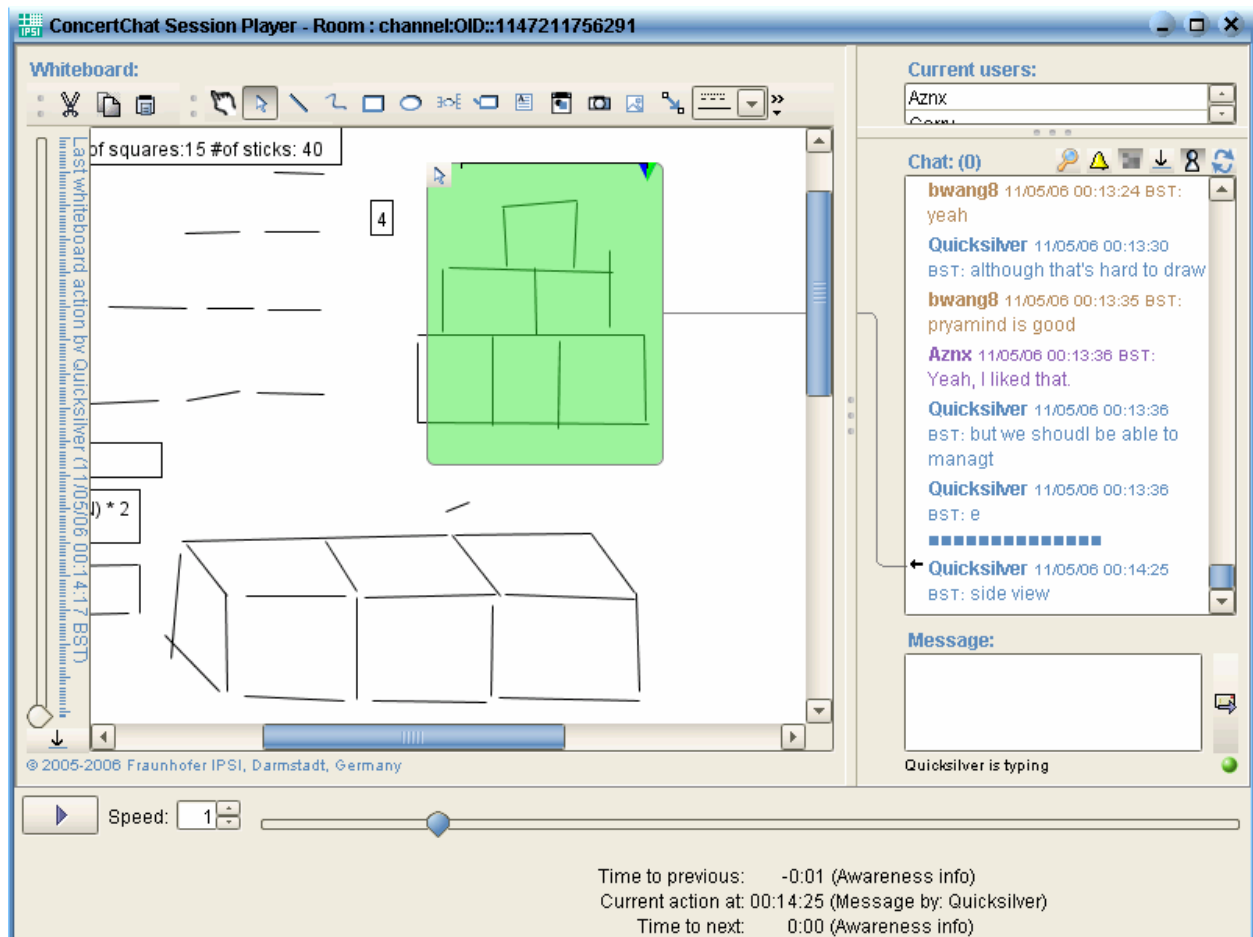


Figure 5: Quicksilver’s first pyramid drawing: ‘side view’

2. Is a pyramid “the same as yesterday’s problem”?

Let us take a closer look at Quicksilver’s drawing. There is nothing inherent in the drawing itself to indicate that this “side view” is to be seen as three-dimensional and Bwang does not appear to see it as three-dimensional. Instead, Bwang once again sees in Quicksilver’s drawing, the same problem as the one investigated in session 1: “isn’t this the same as yesterday problem” [356]:

356	bwang8	isn't this the same as yesterday problem	10.05.2006	07.14.56
357	Quicksilver	Really?	10.05.2006	07.15.03
358	Aznx	Except it's 3-D.	10.05.2006	07.15.10
359	Quicksilver	no it's three d	10.05.2006	07.15.12
360	bwang8	ok	10.05.2006	07.15.16
361	Aznx	So there would be more sticks	10.05.2006	07.15.16
362	Aznx	and blocks	10.05.2006	07.15.19
363	Quicksilver	and i was thinking of like 9 bricks on the bottom and 4 in the middle and 1 on top	10.05.2006	07.15.30

Bwang's reading of the "side view" pyramid drawing as "the same as yesterday's problem" makes sense if he/she has heard Quicksilver's proposal of a pyramid as just *another* candidate shape, that is, another of the type "new topic" [332], rather than another of the type "3-D figures"[334]. Neither Quicksilver nor Aznx have heard the pyramid proposal in this way though, and this is made clear in their response to Bwang's evaluation. Their exclamations at lines 358/359 "Except it's 3-D" and "no it's three d" embody the way in which they have both conceived of the pyramid proposal as a *further* candidate within the type of object '3-d figures'. For Quicksilver and Aznx, when the "row of blocks" proposal was thrown out, the property of three-dimensionality was preserved.

What is noteworthy here is that in the case of this "side view", the drawing and its description is 'not enough', that is, it does not 'speak for itself' unlike Quicksilver's earlier drawing of the 'row of blocks'. It seems that both Quicksilver and Aznx are able to see this drawing as representing a three-dimensional pyramid – are able to make a transition of the kind rendered in Figure 6 below – but they are able to do this only as a result of connecting the drawing to their prior and ongoing chat. Bwang, on the other hand, is able to see the drawing as representing a sequence or pattern only *by* disattending to prior ongoing chat. That is, seeing the drawing not as a three-dimensional shape but as a two dimensional shape 'ignores' certain features of prior chat. The issue of participants 'missing' some features of their ongoing interaction is taken up in the discussion.

With the three-dimensionality aspect of the problem (the pyramid) repaired at lines 358 and 359, both Aznx and Quicksilver add further description, 'filling out' the sense of the pyramid, such that the pyramid now has nine bricks on the bottom tier, four in the middle, and one on the top layer [363] and therefore as has more sticks and blocks than yesterday's problem [361, 362].

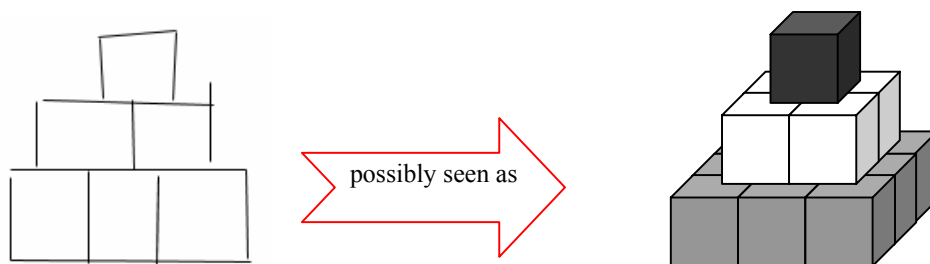


Figure 6: Authors' representation of seeing a 'full, centred pyramid' [cf., 363]

This first 'trouble' with the shape of the pyramid appears then to be resolved relatively easily. This is achieved first through the clarification of the three-dimensional rather than two-dimensional character, and second, through the additional descriptions of more sticks and blocks and the specific numbers of blocks in each tier which provide a resource for 'filling out' or 'imagining' the pyramid shape (as we have done in Figure 6).

3. An unusual task – finding a question *and* its solution

It's perhaps worth noting at this point, the rather unusual nature of the task the team are engaged in. It is not usual for students to define the task they will work on. As Sarmiento (forthcoming, p. 91) puts it "One could argue that the task presented here does not properly specify a problem yet. The 'problem' at hand is, rather, to create a problem." Obviously, the drive behind the VMT project and the Virtual Math Forum is to consider ways of engaging students in mathematical work that inspires and motivates a passion for mathematics and it is this drive that makes the task sensible. But at the practical level of participation, team B must work on both the question *and* its solution, collaboratively, to achieve a mutual sense of both and in order to achieve the goal set in the feedback and task sheet.

Having agreed to use a pyramid shape as the basis for their problem, and having come to a mutual sense, or view of the nature of the pyramid (a three-dimensional pyramid in three tiers, with nine blocks, four blocks, and one block in each tier respectively), team B turn their attention to a solution. It is worth noting here that the team have not explicitly agreed the exact nature of the question or problem they will tackle in relation to the pyramid. As noted in section 1 above, the closest the team have come to specifying the exact nature of their task is Quicksilver's proposal in relation to the three-dimensional figure of a row of blocks which was stated as "the amount of sticks may increase in a pattern?" (line 344). As analysts we assume here that the team will stick to this problem and look for a pattern of growth in the sticks that make up the pyramid shape.

A full minute and fifteen seconds (which feels like a long time) pass before a next interaction. Are the members of team B doing anything ‘off-screen’? Finally, Aznx asks how they should tackle the problem and this prompts a series of proposals.

364	Aznx	So, how should we approach this?	10.05.2006	07.16.45
365	Aznx	What can we use that we already know?	10.05.2006	07.16.54
366	Quicksilver	Layer by layer shown in a chart?	10.05.2006	07.16.57
367	bwang8	well we can divide it into a front and a back	10.05.2006	07.17.01
368	Aznx	I'd suggest yesterday's problem.	10.05.2006	07.17.02
369	bwang8	Yeah	10.05.2006	07.17.10
370	bwang8	using the formula from yesterday's problem	10.05.2006	07.17.22
371	bwang8	we can figure the front and back easily	10.05.2006	07.17.32
372	Quicksilver	This	10.05.2006	07.17.36
373	bwang8	we just need to find the center	10.05.2006	07.17.43

Quicksilver suggests [366] using a chart to present their calculations for each ‘layer’ (in the same way that the task was set up in session 1). Bwang suggests [367] adopting the strategy which produced the solution for the task in session 1, where the two-dimensional squares were pulled apart horizontally and vertically. Bwang suggests using the formula that this strategy produced, to calculate the ‘front’ and ‘back’ sticks which make up the three-dimensional blocks in the pyramid as these remain the same as for the two-dimensional squares of their first task. He/she proposes then, that only the ‘centre’ sticks (the new element to the problem) would need to be found.

That the team monitor one another’s posts for their sense is nicely illustrated in Quicksilver’s “this” [372], together with the referencing tool of the VMT environment, which clarify Bwang’s post at line 370, and require participants to ‘see’ the pointed-to formula for a sense of his/her own post: “this”. It constitutes a nice example of collaborative effort, where one participant (Quicksilver) clarifies the proposal of another participant (Bwang).

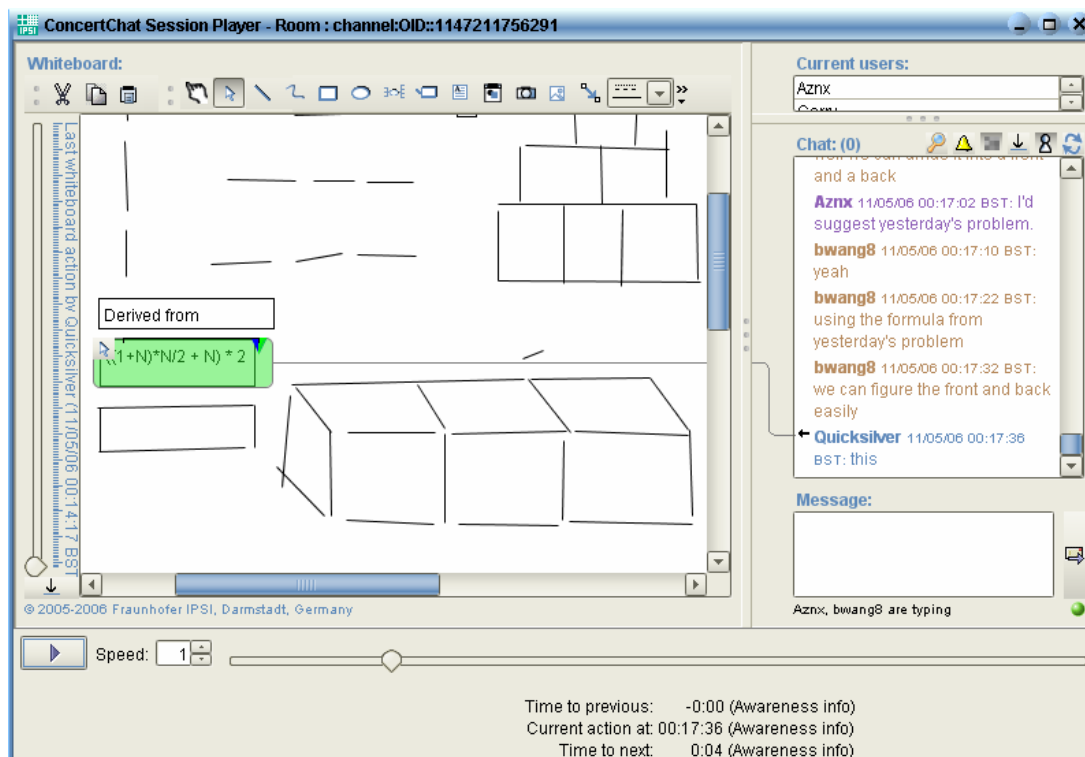


Figure 7: Quicksilver’s analysis of Bwang’s proposal to use “yesterday’s formula”

On posting “we just need to find the centre” [373], Bwang begins to modify Quicksilver’s earlier drawing of a three-dimensional row of blocks (Figure 2) to express what he/she means by “the centre” (Figure 8). The ‘scribbles’ on the sticks connecting what Bwang describes as the ‘front’ and ‘back’ (of each of the three three-dimensional bricks) are what he/she is referring to as “the centre [sticks].”

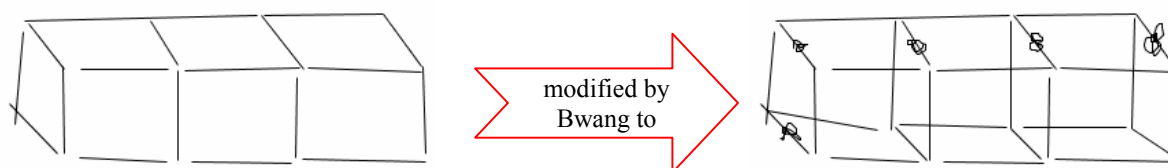


Figure 8: Bwang identifies what is meant by ‘centre’ by modifying Quicksilver’s drawing

4. What shape does ‘pyramid’ refer to?

Bwang’s drawn explication of “the centre [sticks]” receives an exclamatory response from Quicksilver at line 374 “Oh!! Wait ... Your thinking of the kind of pyramid that is flat on one whole edge”:

374	Quicksilver	Oh!! Wait...Your thinking of the kind of pyramid that is flat on one whole edge	10.05.2006	07.18.13
375	Quicksilver	I mean like a real pyramid that each layer is completely centered	10.05.2006	07.18.32
376	Azrx	Draw it.	10.05.2006	07.18.44

Quicksilver seems to see in Bwang’s proposal of a strategy – a proposal that is expressed both in the chat [367, 371, 372] and on the whiteboard (Figure 8) – a display of the kind of pyramid that Bwang is working on. That is to say, Bwang’s formulation of a strategy simultaneously displays Bwang’s understanding of the problem.

Quicksilver’s exclamation is an evaluation of one aspect of Bwang’s problem-solving, namely the shape which it appears, at least to Quicksilver, to be a solution to. That is, Quicksilver’s evaluative interjection is not concerned with the strategy Bwang has adopted, but rather, the shape of the pyramid that can, at least so far, be discerned from Bwang’s description of how they can use the formula from session 1 to help solve their problem here in session 2. Thus, Quicksilver’s evaluation of Bwang’s problem-solving, finds that ‘pyramid’ is, here, a term that can describe more than a single structural form.

It is noteworthy that the characterization of the pyramid Quicksilver attributes to Bwang is much less ‘definite’ than the characterization of Quicksilver’s own pyramid. Quicksilver has both specified the number of blocks [363] and the shape of the pyramid as ‘centred’ [375]. In contrast, Quicksilver’s characterization of the pyramid he/she believes Bwang to have in mind, gives no indication of the number of blocks (is it the same as Quicksilver’s having 1, 4, 9, or perhaps fewer, maybe 1, 2, 3?), only a very vague and indeterminate indication of the shape, namely that it is “flat on one whole edge” [376]. Figure 9 is an authors’ rendition of the structural form Quicksilver might be attributing to Bwang.

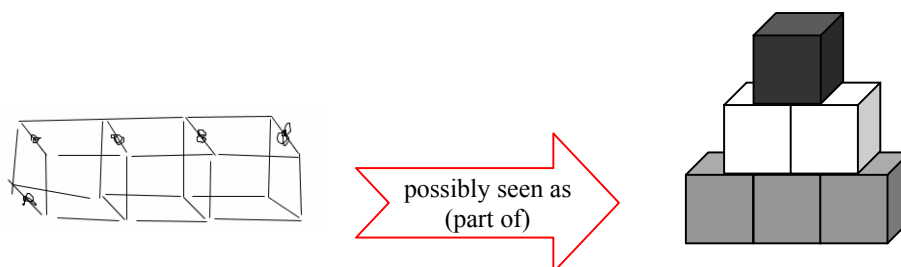


Figure 9: Authors’ representation of ‘wall pyramid’ – the pyramid possibly to be found in Bwang’s strategy

Finding this shape or structural form in Bwang’s description of a strategy to solve their problem is warrantable on two grounds. Firstly, Bwang speaks of “a front and a back” [367, 370] and it is difficult for Quicksilver (or indeed analysts) to find how ‘front’ and ‘back’ would apply to the full, Aztec-type pyramid shape that he/she has in mind. After all, a pyramid with the configuration of nine blocks on the bottom tier, four in the middle and one on the top, would look the same from all angles – a front and back are not then ‘sensible’ descriptions of such a structure. Secondly, the shape makes sense if we read Bwang as seeing the two images drawn by Quicksilver as juxtaposed (see Figure 10) (where perhaps Bwang has not seen/read line 363 specifying the number of blocks in each tier of the pyramid).

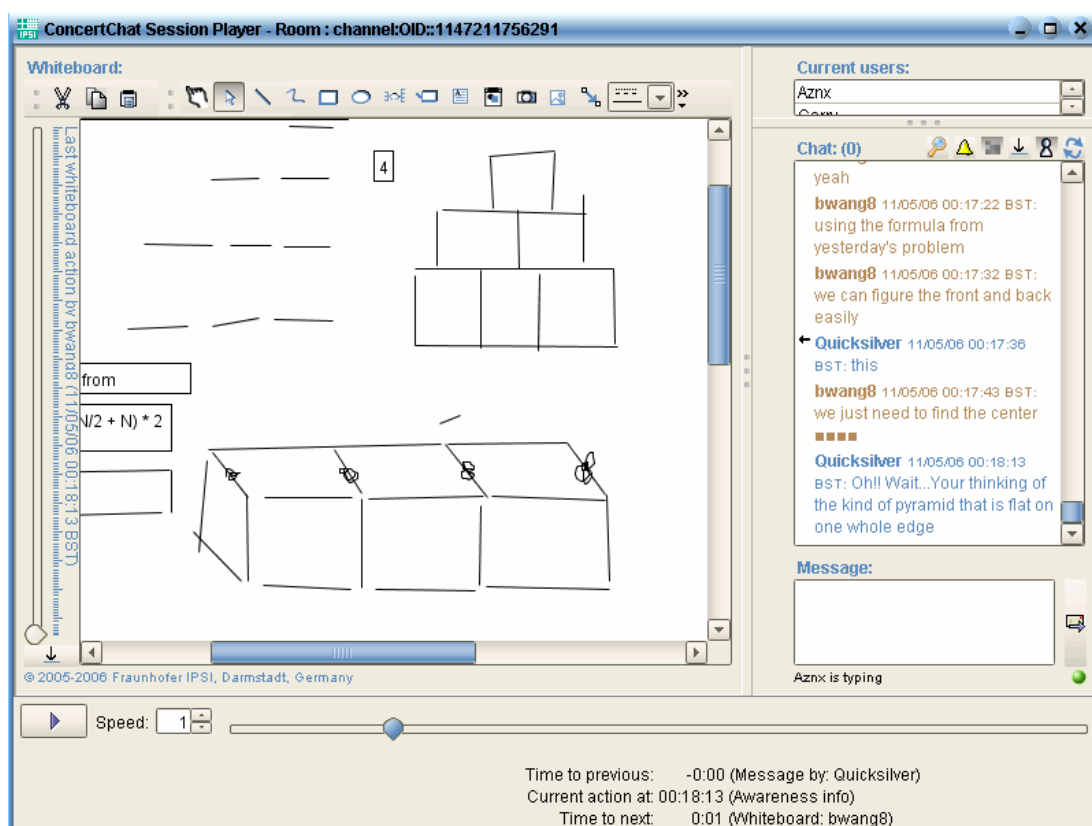


Figure 10: juxtaposition of Quicksilver’ two drawings in the whiteboard

In summary, Bwang has started to propose a way of solving ‘a’ problem [367, 370, 371, 373]. Quicksilver does not see that Bwang’s strategy *will* solve the problem of a ‘fully centred’ pyramid – but rather a different shape altogether. Consequently, Quicksilver goes back to the problem of finding/formulating the *problem* (while Bwang is already working on the solution).

In an attempt to (re)align their mutual sense of the pyramid problem, Quicksilver provides further descriptive elaboration on the ‘shape’ of ‘his/her’ pyramid namely that it is “a real pyramid that each layer is completely centred” [375]. On Aznx’s request, Quicksilver begins drawing on the whiteboard producing a progression of squares, each next one larger than, and surrounding its prior. On completion of the third square, Quicksilver adds a line that creates a division between the third, outer square and the second, inner, or middle square. At this point, line 381, Bwang posts “o ic” [381] suggesting that he/she has seen, within the progression of the drawing-so-far, it’s intended complete form (Figure 11):

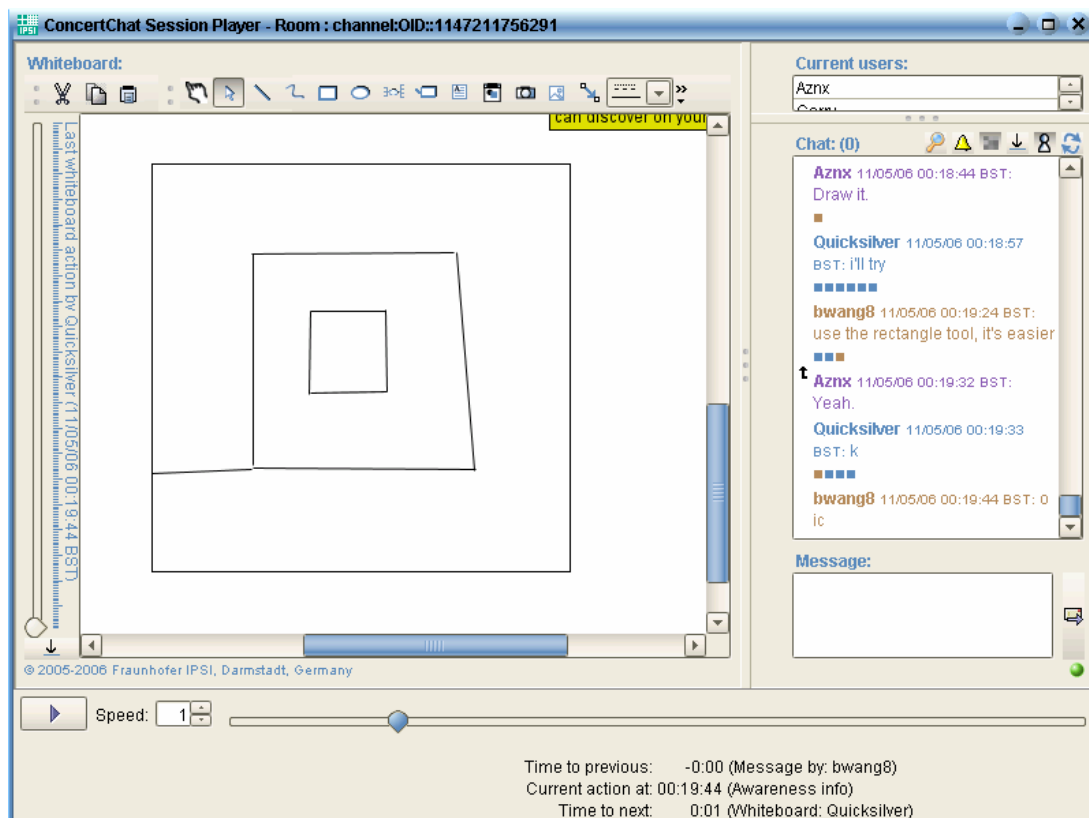


Figure 11: Seeing what is intended in a drawing

Quicksilver further specifies the sense of the new drawing by stating that it represents a “top view” [382]. Quicksilver thus explicitly orients to the inherent ambiguity of using this ‘two-dimensional’ drawing tool to represent three-dimensional shapes. The first drawing of the pyramid (Figure 5) was explicitly characterized as a “side view”, while this drawing is characterized as a “top view”.

This further worded clarification from Quicksilver, “top view ... something like that” [382-383] receives re-confirmation of understanding from Bwang and Quicksilver pursues the issue of understanding by asking Aznx directly whether he/she also understands: “do u aznx?” [385].

377	Quicksilver	i'll try	10.05.2006	07.18.57
378	bwang8	use the rectangle tool, it's easier	10.05.2006	07.19.24
379	Aznx	Yeah.	10.05.2006	07.19.32
380	Quicksilver	K	10.05.2006	07.19.33
381	bwang8	o ic	10.05.2006	07.19.44
382	Quicksilver	top view	10.05.2006	07.19.49
383	Quicksilver	something like that	10.05.2006	07.20.16
384	bwang8	i c what you mean	10.05.2006	07.20.19
385	Quicksilver	do u aznx?	10.05.2006	07.20.27
386	Aznx	Since we're both in Spanish class, are you saying something what the Aztecs made?	10.05.2006	07.20.30

It is well documented (e.g., Sacks, 1992, Vol. II, p. 141, p. 252; McHoul & Watson, 1982; Walker, 1995, p. 103) that a way of demonstrating understanding is to do something with the point at issue such as reformulate it and here Aznx appeals to his/her and Quicksilver's common experience and knowledge gained from “Spanish class”, referring to Quicksilver's pyramid as the sort of pyramid the Aztecs made. This gives Quicksilver a resource to evaluate whether Aznx does or does not understand the pyramid shape as conceived by him/herself. It seems now that all members of team B hold a mutual understanding or sense that their mathematical problem is to be a full, centred, Aztec-type pyramid (see authors' Figure 6). Approximately ten minutes after the team agreed to ‘brainstorm’ ideas for a new topic or problem, the sense of a problem (or at least a problem shape) appears now to be ‘in place’.

Several issues are of note in this stretch of team B's collaboration. Firstly, participants must use a two-dimensional tool to draw their three-dimensional shape. While Quicksilver was able to draw-in the three-dimensionality of his/her ‘row of blocks’ proposal, the possibility of doing so for the proposed pyramid was ‘qualified’ from the start and Quicksilver sought to provide for the three-dimensionality through worded description coupled with two-dimensional drawing. The first of Quicksilver's drawings seemed insufficient to provide Bwang with the resources for seeing the ‘side view’ as a three dimensional Aztec-type pyramid. Quicksilver's ‘top view’ on the other hand, seems to have been ‘enough’ to clarify this.

Secondly, the team seems to orient to seeing a pattern or sequence in a single drawing. In session 1, the task was set in regard to a sequence of growth. The first three steps in the sequence were drawn out separately. In contrast to the task sheet team B draw and orient to a single drawing, or single form, to find in it, a sequence.

It has frequently been observed that there is nothing ‘in’ a finite number of steps that ‘determines’ how the sequences should be continued. This observation is often attributed to Wittgenstein’s (1953, §§185-242) ‘number series’ example (see Kripke [1982] for a ‘skeptical’ interpretation of this and Baker & Hacker [1984, 1985] for a ‘non-skeptical’ one). However, regardless of this as an ‘in principle’ problem, at least in the first session participants had no problem in agreeing what kind of sequence the first three steps were meant to stand for.

In this session, the situation is slightly more complicated. Rather than drawing at least a few (e.g., three) steps in the sequence, participants only draw *one*. This one drawing is taken to stand for the whole sequence. In other words, participants need to see the sequence in *one* step. Why don’t participants draw steps of the sequence? It might be that they simply see no need to do so, i.e., that the current drawing is ‘good enough’ to see the sequence they want to work on. This might especially suit patterns where the sequence is perhaps the most basic progression possible: by adding one more, e.g., the ‘row of blocks’ progresses as the natural numbers: 1, 2, 3, ... Alternatively, it might also be that doing so, using the available tools, would take too much time and effort. It is much more difficult, for example, to draw the first three pyramids using the whiteboard tool than it is to do so using pen and paper.

5. “This prob is kinda hard” or this problem is *too* hard?

Now that Aznx and Bwang seem to understand what shape Quicksilver is talking about, Bwang provides an evaluation of that shape.

387	bwang8	this prob is kinda hard	10.05.2006	07.20.30
388	Aznx	The pyramid?	10.05.2006	07.20.41
389	Quicksilver	I don't know!!!	10.05.2006	07.20.43
390	Quicksilver	Probably	10.05.2006	07.20.46
391	Aznx	Lol	10.05.2006	07.20.49
392	Aznx	I get it.	10.05.2006	07.20.51
393	Quicksilver	Let's go back to original idea: the flat face	10.05.2006	07.21.02
394	Quicksilver	then we can try and get this from that	10.05.2006	07.21.08
395	Quicksilver	So we are going back to the flat faced one? Agree?	10.05.2006	07.22.06
396	Aznx	Agree.	10.05.2006	07.22.39
397	bwang8	we can first figure out the bottom level	10.05.2006	07.22.46
398	Quicksilver	In that case, someone should post the problem on the wiki.	10.05.2006	07.22.49
399	bwang8	Agree	10.05.2006	07.22.50
400	Quicksilver	Yes	10.05.2006	07.22.59
401	Quicksilver	the amount of sticks	10.05.2006	07.23.03
402	Aznx	How about	10.05.2006	07.23.21
403	Quicksilver	How many blocks do we want though?	10.05.2006	07.23.34

Bwang's comment "this prob is kinda hard" [387] seems to be heard as a complaint that the pyramid problem is *too* hard. First Aznx questions if the comment refers to their 'pyramid problem' and Quicksilver sees in the content of the exchange that Bwang's comment "probably" does. On the basis of this analysis, Quicksilver proposes that they "go back to original idea: the flat face?" and asks for agreement. In sum, Bwang provides an evaluation of Quicksilver's proposal, which is taken by Quicksilver to be a *negative* evaluation, thereby calling for an alternative proposal.

Quicksilver's proposal is, on closer inspection, a *strange* one as it implies, in its suggestion "to go back", that there are, or have been, *two* pyramid problems proposed, the current one to which Bwang's evaluation of it being 'kinda hard' is pointed (and which we take to refer to Quicksilver's 'full, centred' pyramid and which is, common-sensically, a rather complex shape), *and* an "original" one. But what is the "original idea" that Quicksilver proposes they should go back to?

Quicksilver gives more specification to what he/she means by characterizing the "original idea" as "the flat face". This could suggest that Quicksilver is operating with two possible pyramid shapes, those identified and thus differentiated in 374/5 as a 'pyramid that is flat on one whole edge' attributed by

Quicksilver to be the sort of pyramid Bwang was thinking of and ‘a real pyramid that each layer is completely centered’ and which refers to the pyramid shape quicksilver him/herself had/has in mind. We could, therefore, hear Quicksilver’s utterance in 393 as suggesting the team abandon work on Quicksilver’s own pyramid and instead take up the one that Quicksilver thinks Bwang was thinking of. The latter, it has to be remembered, has never been given a very definite form and this perhaps accounts for Quicksilver’s later utterance, “How many blocks do we want though?” [403]. It might be said then that Quicksilver is not sure about the specifics of his/her alternative proposal.

In any case, when neither Aznx nor Bwang reply to Quicksilver’s post in 393/4, and after waiting for almost a minute, Quicksilver *repeats* his suggestion at 395, this time explicitly asking for agreement (As Stahl [2006, p. 449] remarks: “To get a response to a proposal, one must elicit at least an affirmation or recognition.”) Quicksilver seems to read both Aznx’s “agree” at line 396 *and* Bwang’s comment “we can first figure out the bottom level” at line 397 as explicit agreement of the proposal. That Quicksilver has heard it this way is strongly suggested in his/her next action (a) prefaced as it is “in that case”, and (b) requesting that someone should put the problem on the wiki; something to do only *when* they have mutually agreed on the problem they are going to investigate. It seems then that all three participants have agreed on a problem to work on: “the flat faced one”.

Closer inspection of just what Bwang’s agreement is tied to throws doubt upon the extent to which a sense of what has been agreed *to* is shared by all participants. Looking closely, Bwang’s agreement at 399 could have one of three referents: (a) a delayed, explicit agreement of Quicksilver’s proposal to go back to the “flat faced one”, (b) agreement that someone should put the problem on the wiki, or (c) an invitation to Quicksilver and Aznx to agree that they “can first figure out the bottom level” of the full, centred, Aztec pyramid. While neither Quicksilver, Aznx nor Bwang topicalise this, i.e. find ‘trouble’ here, later interaction in the session does point to this being a pivotal moment in the dissolution of mutual understanding between Bwang on the one hand and Quicksilver and Aznx on the other.

It is worth noting here that while Quicksilver appears to accept Aznx and Bwang’s agreements [396/399], the agreements themselves do not *display* what they understand to be agreeing *to*. That is, unlike Quicksilver’s (earlier) demonstration of his/her analysis of Bwang’s suggestion to use the formula from session 1 (see Figure 7), or Aznx’s appeal to a shared experience of ‘Spanish class’ to show Quicksilver that he/she does in fact understand the shape of pyramid being proposed, here neither Bwang nor Aznx’s agreements contain anything more than a ‘claim’ to understanding.

6. And now for something completely different

During the exchange outlined above [387 – 401], Aznx is ‘filling in’ Quicksilver’s “top view” pyramid drawing – the ‘unfinished’ drawing in which Bwang found the full centred Aztec pyramid in-and-as-of the progression of its drawing (Figure 11). Aznx begins this ‘filling in’ at lines 392 and continues through 393 and 394. There are occasional contributions to what has the feel of ‘doodling’ from both Quicksilver and Bwang. The resulting drawing has lost its resemblance to a pyramid viewed from above, and now looks more like a checkerboard, or “board format” as Aznx refers to it (Figure 12):

402	Aznx	How about	10.05.2006	07.23.21
403	Quicksilver	How many blocks do we want though?	10.05.2006	07.23.34
404	Aznx	Instead of a triangular format of the sticks, we do the one you jsut made: the board format?	10.05.2006	07.23.42
405	Quicksilver	what do u mean?	10.05.2006	07.24.18
406	Aznx	Look at my arrow.	10.05.2006	07.24.42
407	Quicksilver	Ok	10.05.2006	07.24.42

With this ‘new’ drawing in place on the whiteboard, Aznx proposes an alternative problem to the “flat faced one”. That he/she employs the preface, or perhaps, ‘misplacement marker’ (Schegloff & Sacks, 1973) “how about” [402] embodies Aznx’s own analysis that the proposal ‘cuts in’ or ‘interrupts’ an otherwise coherent topical sequence, i.e. that it is a change of topic.

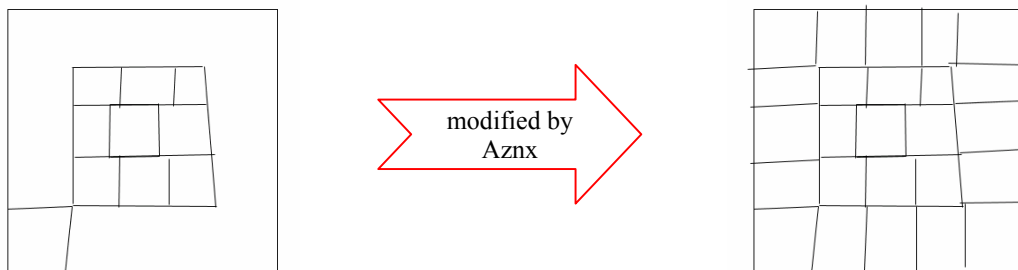


Figure 12: from pyramid “top view” to “the board format”

Quicksilver has problems seeing what Aznx’s proposal [404] could be speaking of: “what do u mean?” [405]. In response, Aznx tries to make explicit the referent for his remark in 404 by drawing an arrow on the whiteboard and inviting Quicksilver to “look at” it (Figure 13). It is noticeable that while Quicksilver

and Bwang have ‘found’ the shading and text-to-whiteboard referencing tool for expediting actions of this sort, Aznx does not appear to have done so.

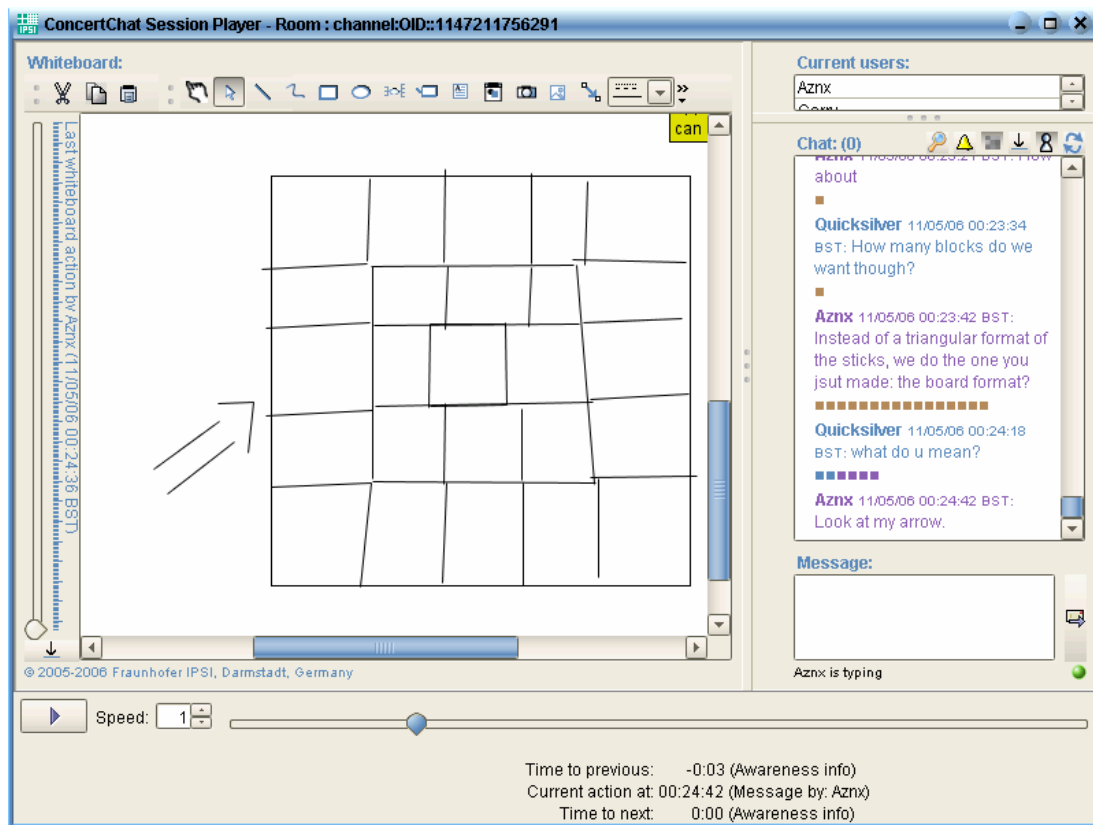


Figure 13: “Look at my arrow”

It becomes apparent in their exchange at 417/418/420/421 that Aznx seems to have abandoned Quicksilver’s proposal to work on 3-d figures [334] and instead appears to be thinking of *any* kind of sequence, for example ones that modify the one from session 1 (endnote 1). The original sequence grew as a ‘triangle’, whereas Aznx seems to suggest one that grows as a ‘board’.

408	Aznx	So you start off with one block.	10.05.2006	07.24.49
409	Quicksilver	And that's a top view right	10.05.2006	07.24.52
410	Aznx	Yes.	10.05.2006	07.25.00
411	Quicksilver	Well there's a problem	10.05.2006	07.25.04
412	Aznx	So, the first one has 1 block.	10.05.2006	07.25.34
413	Aznx	and four sticks	10.05.2006	07.25.41
414	Quicksilver	first block	10.05.2006	07.25.48
415	Aznx	The second one has 5 blocks.	10.05.2006	07.25.51
416	Aznx	Wait	10.05.2006	07.25.59
417	Quicksilver	no it is 3	10.05.2006	07.26.00
418	Quicksilver	D	10.05.2006	07.26.02
419	Aznx	You're doing it wrong.	10.05.2006	07.26.03
420	Quicksilver	3d	10.05.2006	07.26.04
421	Aznx	You want to do 3-D?	10.05.2006	07.26.12

Quicksilver tries to find a three-dimensional shape in Aznx's drawing. In 409, Quicksilver asks whether the drawing constitutes a "top view", which only makes sense in relation to a two-dimensional drawing that is meant to be read as a representation of a three-dimensional figure (rather than a two-dimensional drawing in itself). It is only later [417, 418] that Quicksilver sees the need to re-iterate the point that they, according to Quicksilver, have agreed to work on a three-dimensional pattern.

Why is Aznx surprised by Quicksilver's persistent recourse to three-dimensionality? For Quicksilver three-dimensionality is a 'fixed' feature throughout. The issue seems to be one of finding a suitable three-dimensional shape, one that provides challenge, but doesn't make the problem 'too hard'. Until now, for Aznx too, the three-dimensionality has been a 'fixed' feature in the search for a problem. However, over the duration of time between Quicksilver taking Bwang's evaluation of the full, centred pyramid as too hard and the point at which Aznx has 'finished' Quicksilver's 'top view' drawing of the full, centred pyramid, Aznx loses three-dimensionality as an essential feature of any problem they might take up.

Over the course of their work of 'finding a problem', team B makes a variety of proposals. So far we have seen three-dimensional figures proposed. A row of blocks was proposed *as* a candidate three-dimensional figure and this was rejected as being the same as session 1's problem only in three dimensions rather than two. A pyramid was proposed and accepted. The three-dimensionality of the pyramid had to be repaired for Bwang who hadn't seen this feature in the two-dimensional 'side view'. That is, Bwang did not carry forward the feature of three dimensionality from the 'row of blocks' to the 'pyramid'. In response to

Bwang’s remark that the problem (the full, centred pyramid) is hard, Quicksilver proposes the team swap this pyramid form for another, ‘the flat faced one’. In the course of an exchange between Quicksilver and Aznx in which finding a problem again becomes the focus, Aznx now drops the feature of three-dimensionality. A similar ‘preservation’ and ‘transformation’ can be seen in the way the team orient to drawings and objects on the whiteboard. Quicksilver’s sketch of the ‘top view’ of the full, centred pyramid is a case in point. The drawing starts its life intended to clarify for Bwang, the ‘full’ and ‘centred’ character of the pyramid (as opposed to ‘flat faced-ness’). Once this clarification was achieved, the drawing was abandoned leaving it open for Aznx later to transform it from a three-dimensional pyramid into a two-dimensional ‘board’. That is to say that Aznx preserves Quicksilver’s drawing, but transforms its intended interpretation.

7. “What are we doing?”

Quicksilver closes down this exchange with Aznx over the form and character of the problem through an appeal to Bwang.

422	Quicksilver	Bwang8, what are we doing?	10.05.2006	07.26.27
423	bwang8	?	10.05.2006	07.26.30
424	bwang8	you are trying to find a pattern	10.05.2006	07.26.41
425	bwang8	divide them up into levels	10.05.2006	07.26.53
426	Quicksilver	Oh.....	10.05.2006	07.27.01
427	Quicksilver	so that is the bottom level	10.05.2006	07.27.05
428	Quicksilver	I get it	10.05.2006	07.27.06
429	bwang8	oops	10.05.2006	07.27.42
430	bwang8	lol	10.05.2006	07.27.45
431	Quicksilver	what?	10.05.2006	07.27.52
432	bwang8	the last level have 9	10.05.2006	07.27.55
433	Quicksilver	yeah	10.05.2006	07.28.07
434	bwang8	so we will just have to figure out how many sticks make up 3 by 3 blocks	10.05.2006	07.28.28
435	Aznx	Yes.	10.05.2006	07.29.06
436	Aznx	After that, we go up to Nth step.	10.05.2006	07.29.15
437	Quicksilver	Yes	10.05.2006	07.29.20

From the time that Bwang announced “this prob is kinda hard,” approximately six minutes have passed to the point at which Quicksilver now asks Bwang what they’re doing. During this time, Bwang has been (almost completely) absent from text postings. It becomes apparent in what follows Quicksilver’s appeal,

that Bwang has been working on the problem ‘off-screen’. When asked “what are we doing?” Bwang’s response is not in terms of the continuing trouble of ‘fixing’ a problem (or problem shape) to work on (he/she does not go back to stipulating the shape to be addressed), but rather, in terms of the solution to a problem. Bwang’s explication of what they are doing “you are trying to find a pattern ... divide them up into levels ... the last level have 9 ... so we will just have to figure out how many sticks make up 3 by 3 blocks” *tells* Quicksilver and Aznx what *finding a solution* entails or consists in.

So, while Aznx and Quicksilver have been working on finding a ‘problem’ (pervious section), Bwang has been working on finding a ‘solution’. In what follows, Bwang explicates the steps for solving the problem. Inspection of Bwang’s math, provides the resources to find that he/she is working on the full, Aztec pyramid as had been agreed and thus provides us with warrant for returning to his/her posting “this prob is kinda hard” as merely the evaluative exclamation of someone actually at work on the problem, and not as a complaint that the problem was *too hard*; the hearing of which led to Quicksilver and Aznx’s abandonment of the full pyramid proposal, agreement to work on “the flat faced one” and eventually to Aznx’s proposal for an entirely new shape, “the board format”. The mishearing of Bwang’s exclamation then, is what has provided for a fairly extended sequence of ‘trouble’ (at least for Quicksilver and Aznx). That is, the misunderstanding of Bwang’s exclamation as a complaint has produced the environment which, for Quicksilver and Aznx, makes sensible their next actions – a continued search for a problem.

DISCUSSION

We described at the start of this paper, how on first inspection of the VMT materials, we raised constant questions and experienced endless frustration. As analysts the material presents the most formidable data we have yet encountered. The materials are massively complex just in terms of the availability of media through which participants can communicate. That is, the juxtaposition of whiteboard tool and chat demand multi-orientational viewing, at least on the part of the analyst. But our frustrations didn't emerge just from the perspective of the analyst. As educators too, we experienced great frustration. Through early readings of the materials we found ourselves almost exclaiming out loud 'what are you doing?' and 'why haven't you settled that?' We were struck then by a sense of 'disorderliness' with participants occasionally appearing to be aligned in their understanding of what they were doing, but more often than not, apparently not aligned. What features of the team B materials produce these grossly observable features? The following discussion picks out five features in and through which the messy complexity of the materials could be said to be organised.

1. the problem of finding a problem

In Session 1, team B had been given a fairly typical classroom-type problem that they were asked to solve (see task sheet in endnote 1). In Session 2, they are given another problem. It is typical for these kinds of situations that there is a progression in the difficulty of the given problems. Thus it is to be expected that the problem in Session 2 is going to be more complicated, more difficult, more complex than the problem given to them in Session 1. However, the problem that they are given in Session 2 is not only more difficult, it is in fact a different *kind* of problem. While in Session 1, they were asked to find the solution to a specified problem, in Session 2, the team is asked to 'find a problem' (cf., Sarmiento, forthcoming, p. 91):

“Mathematicians do not just solve other people's problem – they also explore little worlds of patterns that they define and find interesting. Think about other mathematical problems related to the problem with the sticks. [...]” (from task sheet; see endnote 1)

In Session 2, the students are not just expected to find the solution to a problem given to them by the instructors, but rather they are faced with the problem of finding a problem – to which they then have to find the solution to.

It is unusual, in our experience, for students in schools or universities, to be asked to define the problem that they want to work on. That is to say, students are more familiar with the kind of problem given to

them in Session 1. This kind of problem does not need a ‘solution’, but rather needs a ‘decision’. That is to say, ‘find a problem to work on’ is not asking for a solution to be discovered, but rather is asking the group to agree on something to work on – and the team are certainly trying to do so:

334	Quicksilver	3-d figures?	10.05.2006	07.10.20
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And a bit later:

348	Quicksilver	maybe a pyramid	10.05.2006	07.13.18
349	bwang8	yeah	10.05.2006	07.13.24
350	Quicksilver	although that's hard to draw	10.05.2006	07.13.30
351	bwang8	pyramid is good	10.05.2006	07.13.35
352	Azrx	Yeah, I liked that.	10.05.2006	07.13.36

Quicksilver makes a candidate proposal [348], which is quickly agreed to by both Bwang [351] and Azrx [352].

It is noteworthy that almost all their effort is spent on finding a *shape* to work on (“a row of blocks”, “a pyramid”, a “board format”), while how the shape will constitute a problem remains predominantly implicit. However, given the kind of problem that they worked on in Session 1 and the instruction for Session 2 to “Think about other mathematical problems related to the problem with the sticks” (see task sheet in endnote 1), it is enough for participants to propose a shape to work on. Participants are able to see *in* the proposed shape an example of “some problems that we think are challenging” [331].

Furthermore, participants are able to see in *one* drawing a whole *sequence* of drawings. That is to say, they can use a particular case (typically $n = 3$) to represent the whole sequence (cf., Koschmann, Stahl, Zemel’s contribution to this symposium, p. 8, footnote 3) – although this can be a source of ambiguity. However, this is not always without problems.

In sum the new problem is treated as a progression to the problem of Session 1: it preserves the task of seeing “a pattern of growth for the number of sticks and squares” (task sheet; endnote 1) and thereby orients to this feature of classroom education; but it transforms the particular sequence of shapes that they will investigate.

2. ambiguity in the pyramid problem

Having agreed on a problem to work on, the team is ready to move forward to solving the agreed-to-problem. In fact, Bwang immediately starts to express a way of solving the problem:

367	bwang8	well we can divide it into a front and a back	10.05.2006	07.17.01
370	bwang8	using the formula from yesterday's problem	10.05.2006	07.17.22
371	bwang8	we can figure the front and back easily	10.05.2006	07.17.32

In proposing a solution strategy, Bwang is simultaneously displaying an understanding of the agreed-to-problem. This in turn allows Quicksilver to see *in* Bwang's proposed strategy (expressed in both the chat and whiteboard) the shape that Bwang seems to be working on, which for Quicksilver is a different one to the one that he/she had in mind:

374	Quicksilver	Oh!! Wait...Your thinking of the kind of pyramid that is flat on one whole edge	10.05.2006	07.18.13
375	Quicksilver	I mean like a real pyramid that each layer is completely centered	10.05.2006	07.18.32

It is very much in the way that Bwang proposes to solve 'the' problem, that Quicksilver realises that working on "a pyramid" does not specify 'a' problem, but at least 'two' possible problems. That is to say, Quicksilver realises that there are *different* shapes that could be described as "a pyramid". "3D pyramid" turns out to be underspecified – and that is something that is *discovered* by participants, at least by Quicksilver.

Researchers are fond of saying things like: 'nothing is ever definite' or to speak of the 'indeterminacy of meaning', but typically these are 'in principle' problems. One of the fundamental contributions of ethnomethodology to the analysis of social action was to point out that participants are not interested in an 'absolute', but in an 'for-all-practical-purposes' definiteness of sense (cf., Garfinkel, 1967; Garfinkel and Sacks, 1970; Heritage and Watson, 1980). What is noteworthy about this episode is that participants (at least Quicksilver) discover that "pyramid" is *not* definite enough for *their purposes* (in contrast to, for example, their use of the word "block", which is used to a 2-D square or a 3-D cube, but where such 'in principle' ambiguity never becomes a problem for the team).

The proposal to work on "a pyramid" turns out to be an underspecified problem and participants discover that there are a variety of "pyramid-like" structures, so they need to decide which one they are going to work on. However, doing so turns out not to be without further troubles. Why might that be?

Quicksilver's "Your thinking" [374] and "I mean" [375] is a perspicuous example of signalling that he/she is explicating how he/she analyses what Bwang is saying and doing. As Harvey Sacks once remarked (1992, Vol. 1, p. 720), "I mean" is a common way of signalling 'explication' and a common way in which participants can repair other participants' understanding of something done before. In ordinary conversation, we frequently have instances of "You're thinking of *X*, I mean *Y*" **[is there a CA paper which deals explicitly with this???]**, where such 'repair' (Schegloff *et al.*, 1977) is typically done very quickly, i.e., without the need for *further* explication.

In contrast, Quicksilver's utterances [374, 375] require *further* work, requested immediately by Aznx: "Draw it" [376]. Furthermore, Quicksilver has considerable difficulty in doing explicating what he/she means and despite all his/her work ambiguity remains. One of the reasons for this can be found in the constraints of the technology in 'talking about' a three-dimensional pyramid.

3. constraints of the technology: drawing 3D figures

Quicksilver proposes to work on "3-d figures" [334], more specifically, "a pyramid" [348]. However, the whiteboard of the VMT is a 'two-dimensional' drawing tool (in contrast to other drawing packages). Furthermore, it does not have any 'pre-specified' 3D-shapes. It has a 'rectangle', but not a 'cube' drawing tool (in contrast to Microsoft Word, which under 'AutoShapes' has a cube, which we used to draw our 'interpretations' of what the drawings produced by Team B 'mean'). The team, in particular Quicksilver, progressively discover ways of using the available resources of the whiteboard to draw / represent a three-dimensional pyramid.

The first drawing that is meant to stand for or represent a three-dimensional figure, namely "a row of blocks", is treated as unproblematically *as* standing for a three-dimensional shape:

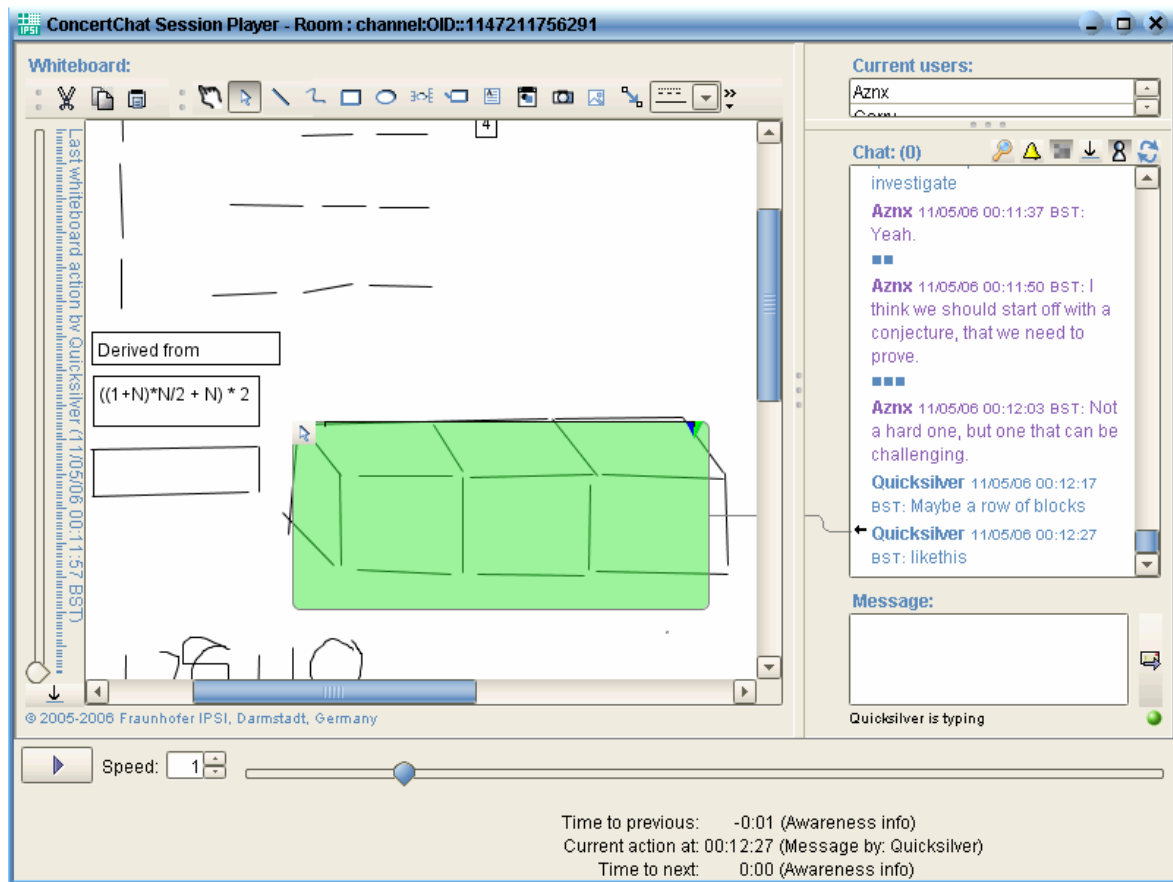


Figure 14: Quicksilver’s “row of blocks”

Quicksilver simply uses the references tool of the VMT environment to specify which drawing is meant to exemplify “a row of blocks”. Quicksilver does not provide any further descriptive elaboration of how this drawing is meant to stand for a three-dimensional shape – nor is one requested by the other participants. In contrast, the first drawing that is meant to represent a pyramid, immediately gets an additional description from Quicksilver: “side view” [355].

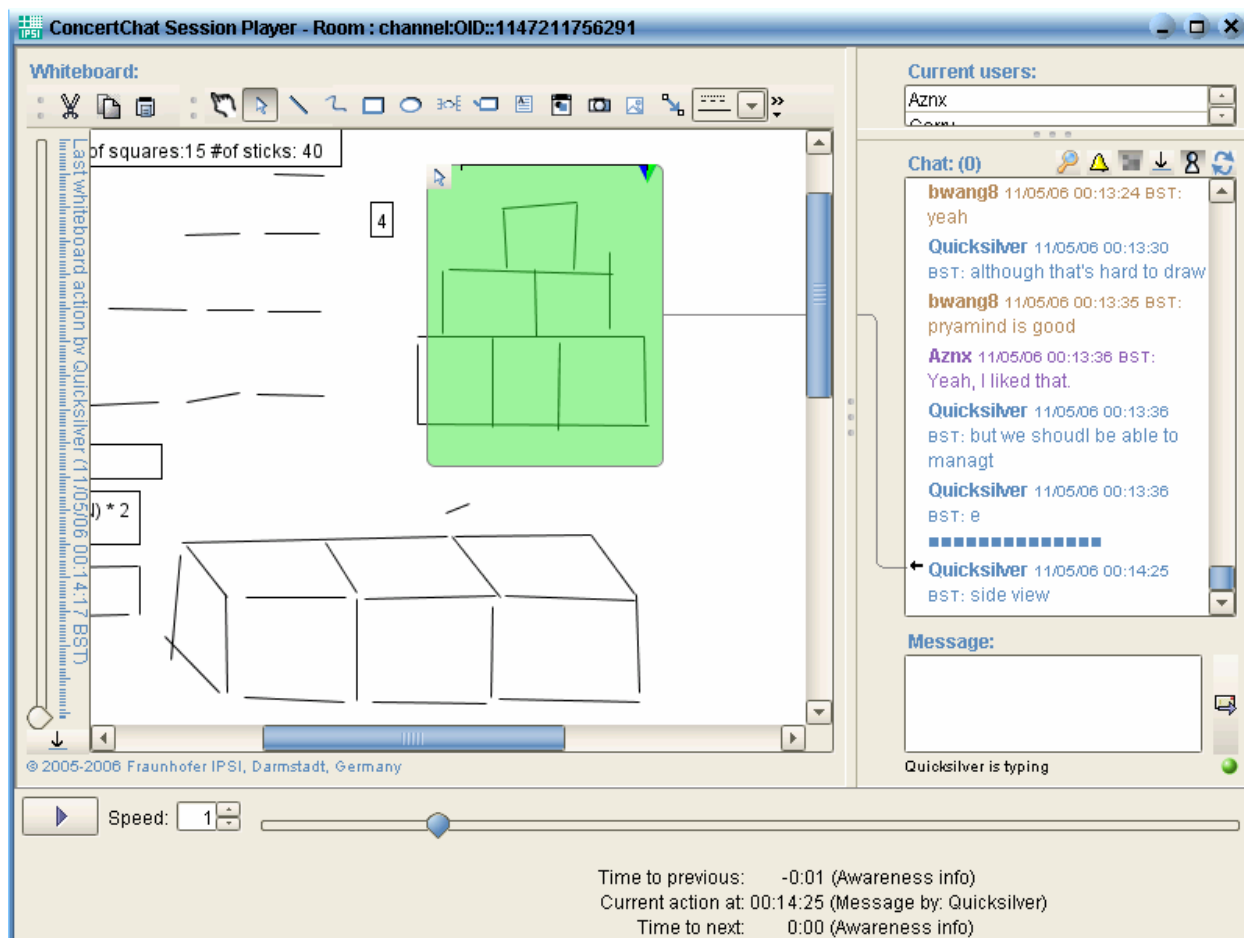


Figure 15: Quicksilver’s “side view”

For Quicksilver, it seems that the drawing of a “row of blocks” ‘speaks for itself’. Given the conventions that we are accustomed to as members of society, it is immediately seeable *as* a three-dimensional shape and therefore not in need of any further descriptive elaboration. In contrast, the first drawing of the pyramid is treated by Quicksilver as not as ‘self-explicating’ in that way and in need of the label “side view”.

Taken out of context, the drawing of the pyramid could easily be seen as a two-dimensional shape (e.g., as an altered version of the shape given to them in Session 1: rather than ‘justified’ squares, the whiteboard drawing has ‘centered’ squares). Quicksilver orients to the possibility of an ambiguity of the drawing by clarifying that this is meant to constitute a “side view” – where ‘side view’ is a term explicitly associated with three-dimensional (rather than two-dimensional) shapes.

Quicksilver gets progressively more sophisticated in using the available resources to represent the three-dimensional pyramid. Once Quicksilver realises that he/she is thinking of a different pyramid-shape than Bwang [362, 363] and is asked by Aznx to “draw” the one that he/she is thinking of [364], Quicksilver makes successive attempts to draw something that would represent a “top view” [382]:

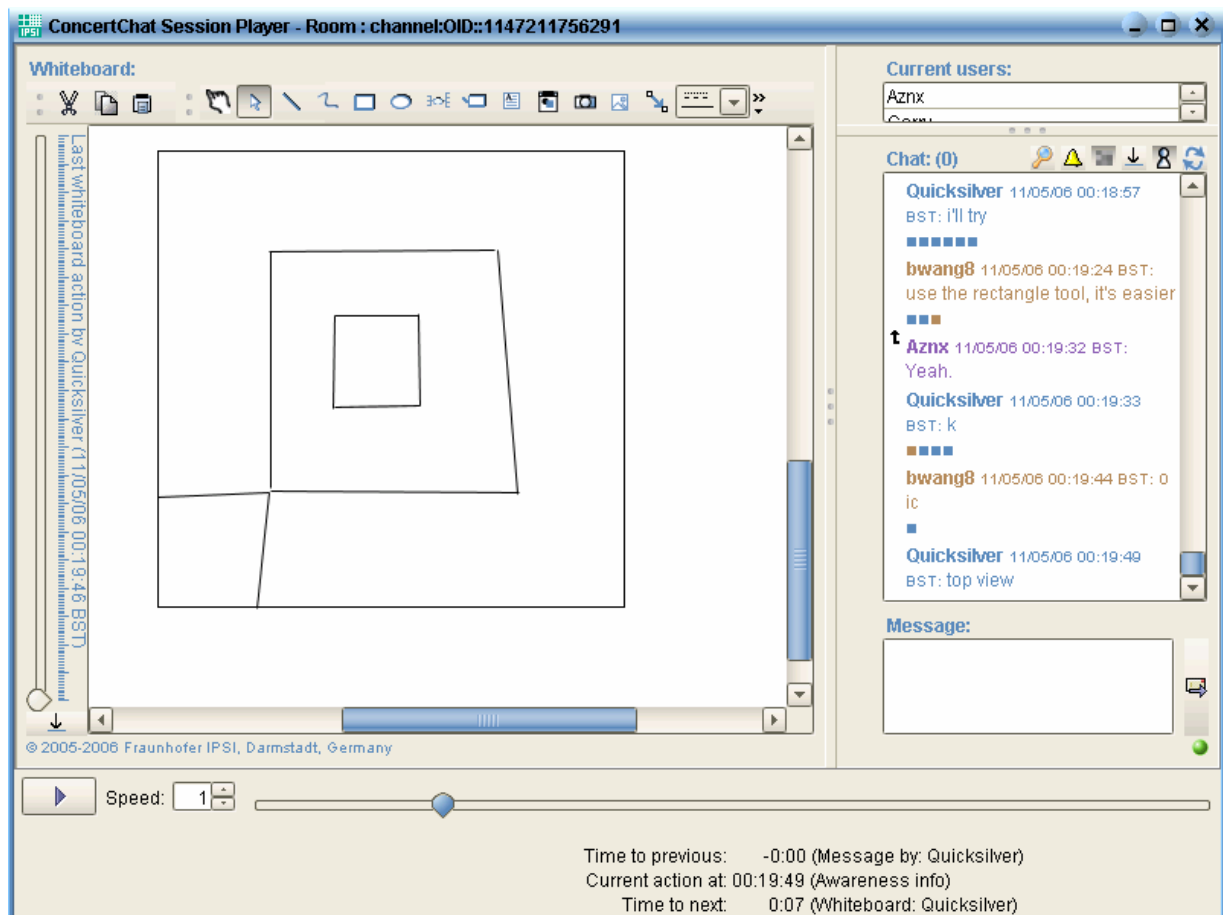


Figure 16: Quicksilver’s “top view”

During the exchange with Aznx, Quicksilver tries to find yet another way to represent the fact that the drawing is meant to represent a three-dimensional shape, namely through the use of colours (in fact, so did we: the figures that we used to represent the kind of pyramid the team might be talking about uses different shades of grey in an attempt to represent the three-dimensional nature of the two-dimensional drawing).

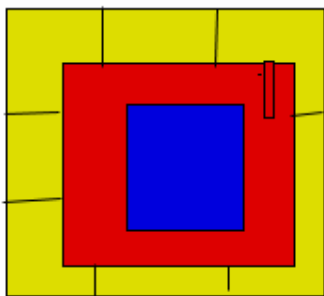


Figure 17: Quicksilver's 'coloured pyramid'

In sum, Quicksilver gets progressively more sophisticated in his/her attempts to represent a '3D pyramid', e.g., by explicating that a particular drawing might be a "side view" or a "top view" of the pyramid, by using the rectangle tool to speed up the drawing (suggested by Bwang in 378), and using colours.

This points to another problem that the team are faced with in these sessions. Whereas they can be said to be 'experts' in face-to-face problem-solving, they are novices in using the VMT environment to solve mathematical problems (although they might be familiar with using chat in general).

Early studies of chatroom conversation pointed to possible misinterpretations of the placements of turns (Garcia and Jacobs, 1998, 1999; Herring, 1999). However, as Hutchby (2001, p. 191) points out, the separation between 'serial' and 'sequential' order in chatrooms is often only a problem for novices, i.e., not inherently or ongoingly problematic. In fact, team B seems to be quite 'expert' in 'chatting'. In contrast, how to use the chat and the whiteboard to 'talk about' a three-dimensional pyramid *is* novel for participants. Furthermore, participants do not necessarily acquire the expertise at the same pace (as Zhou [forthcoming, p. 146] puts it: "In a peer group engaged in math problem solving, competence—either in doing math, in being a member, or in other matters—is not always equally distributed among participants in an interaction."). For example, both Bwang and Quicksilver learn to use the 'referencing tool' in Session 2, while Aznx draws an arrow on the whiteboard to make reference to the shape he/she is talking about.

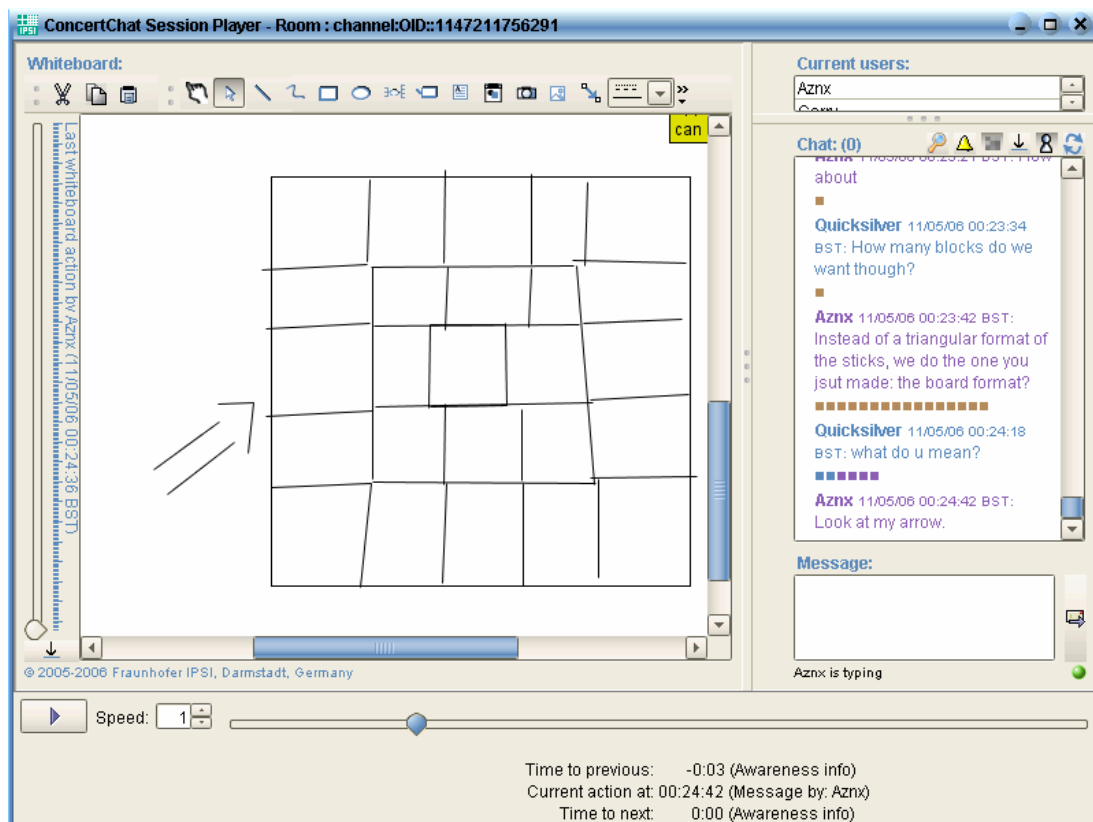


Figure 18: Aznx's 'arrow'

In sum, the team does not have the technical language to discriminate different pyramid shapes (in fact, neither did we: we spent a lot of time figuring out how to *label* the different possible pyramid shapes that the team might have 'in mind'). Quicksilver is not able to *describe* the difference between his/her pyramid and the pyramid that Quicksilver thinks Bwang has in mind.

4. constraints of the technology: complexity and awareness

Not only is it difficult to use the resources of the VMT environment to represent three-dimensional shapes, but use of the VMT environment is in itself rather complex:

- it contains a chatroom *and* a whiteboard, i.e., constitutes a 'dual interaction space' (Dillenbourg, 2005; Cakir *et al.*, 2009)
- the chatroom does not 'hold-in-view' that many prior turns (at least in many configurations);

- not all of the whiteboard is necessarily visible, i.e., there can be a need to ‘scroll’ (and therefore no reciprocity of perspective for participants – although that does not appear to be a problem in this episode);
- participants may not necessarily oriented to the screen at all times – but may, on occasion, work ‘offline’ using ‘scratch paper’ and pencil (for example, Aznx remarks at 906: “You can do it on your own scratch piece of paper =P”).

All of this means that “members’ incessant monitoring of [their] shared understandings or expectancies” (Heritage and Watson, 1980, p. 245), typical of face-to-face interaction, is *relaxed* in various ways.

Both chat and whiteboard require participants to *look*, i.e., rely predominantly on their visual sense, which means that participants have to constantly glance from one to the other (compare with a configuration where the participants would be ‘talking’ using their aural sense, while still using a shared drawing tool; in that configuration, their eyes could be focussed on the whiteboard at all times).

The complexity of the environment then, means that not every ‘turn’ within the VMT environment is necessarily ‘seen and noticed’ by all participants. Zemel & Cakir (forthcoming, p. 265) put it like this:

Participants appear to orient to the fact that simply posting a text message or a graphical artifact may not always be adequate to assure that other users will ‘see’ it or give it the consideration that the author might hope for. Because a participant’s attention may not be given to that part of the interface displaying a newly posted text or graphical artifact, the producer of a text or graphical artifact cannot be sure that any given recipient is aware of a posted text or artifact unless an explicit response to that posting is produced and displayed. While graphical displays in the whiteboard are viewable by any participant, such displays need not necessarily be designed or produced to solicit responses from others, and they are typically not treated that way (though on occasion they are).

This issue gets intensified if we take into account that it is very likely that participants are also working ‘offline’ using pen and paper (again, Aznx remarks in 906: “You can do it on your own scratch piece of paper =P”), which also requires them to look at what they are doing. In fact, the situation is perhaps not a ‘dual’ interaction space, but (at least) a ‘triple’ interaction space. Furthermore, while working ‘offline’ participants are unlikely to be aware of everything that happens in the chat and on the whiteboard (and part of what has been done in the chat may have scrolled by the time they look again). Of course, in many situations, people can ‘switch off’ and focus on something different. However, here it might be that focussing on the task may entail that one has to stop to focus on the chat.

A good example of this is the exchange between Aznx and Quicksilver about a “board format” [402-421], during which Bwang does neither post anything on the chatroom nor on the whiteboard. When Quicksilver decides to elicit Bwang’s help to intervene between himself /herself and Aznx, Bwang does not display any awareness of what Aznx and Quicksilver have just been talking about:

422	Quicksilver	Bwang8, what are we doing?	10.05.2006	07.26.27
423	bwang8	?	10.05.2006	07.26.30
424	bwang8	you are trying to find a pattern	10.05.2006	07.26.41

Bwang does not display any awareness of the fact that Aznx has just proposed a different problem to work on, while Quicksilver has tried to stick with a ‘3D pyramid’. For Bwang, it seems clear that they have decided to work on (Quicksilver’s) pyramid and that they have therefore moved to the next stage of their task, i.e., to find a solution, which consists of “trying to find a pattern” [424].

5. problems of misunderstanding?

As we said in the introduction, this episode struck us as interesting because it seemed that participants were constantly ‘misunderstanding’ each other. At least that was our first impression of what was happening in this episode. But is this actually the case?

What we can say is that this episode is a perspicuous illustration of the difference between what Sacks (1992, Vol. II, p. 252) has termed ‘claimed’ versus ‘proved’ understanding or agreement:

Proved relationships are attended by parties as systematically different than other sores of relationships, e.g., *claimed* relationships. Things like, e.g., at the end of some first story a recipient says ‘I know just what you mean.’ Period. We can say that that’s a claimed understanding as compared to having some way to produce some materials that *exhibit* an understanding. And there are other sorts of things that have a similar sort of attended differentiation. For example, if someone tells a story that has a point, or states a certain opinion, then a routine thing that somebody who receives that story or opinion does, is to say ‘I agree.’ Again, we could say that that’s a ‘claimed’ agreement. And people are perfectly well aware that though he said that, he might not mean it, and in any event so far as they know, he’s just saying it. [...]

In our view, in this episode it is predominantly Quicksilver who is trying to *prove* or *exhibit* his/her understanding of what they are doing. For example, it is Quicksilver who sees in what Bwang is doing that Bwang and himself/ herself might not be ‘on the same page’, and it is Quicksilver who makes this explicit:

374	Quicksilver	Oh!! Wait...Your thinking of the kind of pyramid that is flat on one whole edge	10.05.2006	07.18.13
375	Quicksilver	I mean like a real pyramid that each layer is completely centered	10.05.2006	07.18.32

It is also Quicksilver who, after hearing Bwang’s characterisation of Quicksilver’s pyramid as “kinda hard” [387], interprets this as a negative evaluation, calling for an alternate proposal. Quicksilver explicitly tries to secure common acceptance of this alternate proposal [395]:

393	Quicksilver	Let’s go back to original idea: the flat face	10.05.2006	07.21.02
394	Quicksilver	then we can try and get this from that	10.05.2006	07.21.08
395	Quicksilver	So we are going back to the flat faced one? Agree?	10.05.2006	07.22.06
396	Azrx	Agree.	10.05.2006	07.22.39
397	bwang8	we can first figure out the bottom level	10.05.2006	07.22.46
398	Quicksilver	In that case, someone should post the problem on the wiki.	10.05.2006	07.22.49
399	bwang8	agree	10.05.2006	07.22.50

In contrast to such ‘proved’ understanding or agreement, both Bwang and Azrx often provide only ‘claimed’ ones. Thus in response to Quicksilver’s attempt to explicitly try to secure a common understanding [393-395], both Azrx and Bwang “agree” [396, 399] – but they thereby do not display what they agree *to*.

As analysts, we have been continuously frustrated in not being able to determine what *exactly* a particular utterances means (and were thus frequently hoping that participants would ask one another to clarify their utterances). For example:

- what kind of sequences does Bwang see in Quicksilver’s “row of blocks”?
- what pyramid does Bwang have ‘in mind’ when he/she suggests to “divide it into a front and a back” [367] so “just need to find the center” [373]?
- what pyramid does Quicksilver refer to as “the flat faced one” [395]?

What is remarkable is that although we, as analysts / educationalists, are sometimes frustrated, this does not seem to capture how the team experiences this episode. This is not, we would argue, because they ‘really know’, but rather because they seem to be less concerned with getting a more definite understanding of each other’s utterances. This might point to differences in what lessons educationalists want participants to see or learn, and what participants themselves want to take away from these sessions.

The moderator, like us, picks up on the fact that the team is not always trying to understand what other team members are up to:

“It seems that there are times when you say you are following each other, but it is not clear that you are really in agreement or completely understand each other. You might actually discover some more math if you state things in more detail – to be completely sure you are in agreement.” (feedback at the beginning of Session 4)

The team seems to be orienting to getting the job done (they are certainly predominantly ‘on task’, in contrast to many classroom situations where during group work students may often be found to be ‘off task’), but perhaps they are not so much reflecting on what this might involve. In particular, that in this situation what ‘the job / the task’ *is* has to be collaboratively agreed upon. And as part of that, it may be necessary to realize that other team members may have a different conception of the task (here: the shape of the pyramid) than yourself.

Would that not be the lesson that we hope students to take away?

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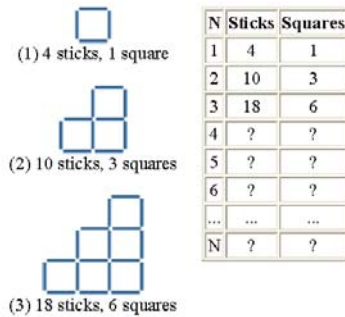
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ENDNOTES

¹ Spring Fest 2006 ‘task sheet’ is presented below:

VMT Spring Fest

Here are the first few examples of a particular pattern or sequence, which is made using sticks to form connected squares:



Scroll down to see instructions for each Session.

Session I

1. Draw the pattern for $N=4$, $N=5$, and $N=6$ in the whiteboard. Discuss as a group: How does the graphic pattern grow?
2. Fill in the cells of the table for sticks and squares in rows $N=4$, $N=5$, and $N=6$. Once you agree on these results, post them on the [VMT Wiki](#)
3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the [VMT Wiki](#).

Session II and III

1. Discuss the feedback that you received about your previous session.
2. **WHAT IF?** Mathematicians do not just solve other people's problems - they also explore little worlds of patterns that they define and find interesting. Think about other mathematical problems related to the problem with the sticks. For instance, consider other arrangements of squares in addition to the triangle arrangement (diamond, cross, etc.). **What if** instead of squares you use other polygons like triangles, hexagons, etc.? Which polygons work well for building patterns like this? How about 3-D figures, like cubes with edges, sides and cubes? What are the different methods (induction, series, recursion, graphing, tables, etc.) you can use to analyze these different patterns?
3. Go to the [VMT Wiki](#) and share the most interesting math problems that your group chose to work on.

² At the beginning of the second session, team B finds the following feedback on their first session on the whiteboard:

“VMT Feedback

We were very interested in the approach that divided the figure into the horizontal lines and the vertical lines and the quickness with which formulas fell out of that approach. It seemed as though

you also were paying attention to each other's work and quickly reached agreement. You handled the technology of the chat environment and the wiki easily.

We also noticed two places in the chat where some kinds of conversation did not happen. There was a point where 44 was posted as the number of sticks and 40 was offered as a correction. There was no discussion of how 44 was calculated. At another moment, Quicksilver posted an explanation of the pattern of growth that was not discussed.

There was a sense in which you indicated that your work was done when you had at least one answer for the questions in the problem. For the next step we will encourage you to think more about the different approaches and the problems that you can discover on your own and that are interesting to pursue.”

³ By speaking of a participant having something ‘in mind’ we do not mean that this something is therefore essentially private, subjective, or mental. Rather, what a participant has ‘in mind’ is very much observable to other participants. Although, on occasion there might be a discrepancy between what a participant has ‘in mind’ and what other participants think that participant has ‘in mind’, that this difference again becomes visible and observable (as it does in these sessions).

⁴ McHoul & Watson (1982) is an example of the adaptation of Sacks’s membership categorisation activities to non-persons.