Seeing what we mean: Co-experiencing a shared virtual world

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Abstract: The ability of people to understand each other and to work together face-to-face is grounded in their sharing of our meaningful natural and cultural world. CSCL groups—such as virtual math teams—have to co-construct their shared world with extra effort. A case study of building shared understanding online illustrates these aspects: Asking each other questions is one common way of aligning perceptions. Literally looking at the same aspect of something as someone else helps us to see what each other means. The co-constructed shared world has social and temporal as well as objective dimensions. This world grounds communicative, interpersonal, and task-related activities for online groups, making possible group cognition that exceeds the limits of the individual cognition of the group members.

The Shared World of Meaning

We all find others and ourselves within one world. We learn about and experience the many dimensions of this world together, as we mature as social beings. Infants learn to navigate physical nature in the arms of caregivers, toddlers acquire their mother tongue by speaking with others, adolescents are socialized into their cultures, and adults master the artifacts of the built environment designed by others. The world is rich with socially endowed meaning, and we perceive and experience it as immediately meaningful. Because we share the meaningful world, we can understand each other and can work together on concerns in common. Our activities around our common concerns provide a shared structuring of our world in terms of implicit goals, interpersonal relations, and temporal dimensions. These structural elements are reflected in our language: in references to artifacts, in social positioning, and in use of tenses. All of this is understood the same by us unproblematically based on our lived experience of the shared world. Of course there are occasional misunderstandings, particularly across community boundaries, but these are exceptions that prove the rule of shared understanding in general.

The "problem" of establishing intersubjectivity is a pseudo-problem in most cases. Human existence is fundamentally intersubjective from the start. We understand the world as a shared world and we even understand ourselves through the eyes of others and in comparison with others (Mead, 1934/1962). Rationalist philosophy—from Descartes to cognitive science—has made this into a problem by focusing on the mind of the individual as if it were isolated from the world and from other people. That raises the pseudo-problem of epistemology: how can the individual mind know about states of the world and about states of other minds? Rationalist philosophy (as described by Dreyfus, 1992) culminated in an information-processing view of human cognition, modeled on computer architecture: understanding is viewed as primarily consisting of a collection of mental representations (or propositions) of facts stored in a searchable memory.

Critiques of the rationalist approach (e.g., Dreyfus, 1992; Schön, 1983; Suchman, 1987; Winograd & Flores, 1986) have adopted a phenomenological (Heidegger, 1927/1996; Husserl, 1936/1989; Merleau-Ponty, 1945/2002), hermeneutical (Gadamer, 1960/1988), or ethnomethodological (Garfinkel, 1967) approach, in which understanding is grounded in being-in-the-world-together, in cultural-historical traditions, and in tacit social practices. This led to post-cognitive theories, with a focus on artifacts, communities-of-practice, situated cognition, distributed cognition, group cognition, activity, and mediations by actor-networks. Human cognition is recognized to be a social product (Hegel, Marx, Vygotsky) of interaction among people, over time, within a shared world. Knowledge is no longer viewed as primarily mental representations of individuals, but includes tacit procedural knowledge (Polanyi, 1966), designed artifacts (Hutchins, 1996), physical representations (Latour, 1992), small-group processes (Stahl, 2006), embodied habits (Bourdieu, 1972/1995), linguistic meanings (Foucault, 2002), activity structures (Engeström, Miettinen & Punamäki, 1999), community practices (Lave, 1991), and social institutions (Giddens, 1984). The critique of human thought as purely mental and individual is now well established for embodied reality. But what happens in virtual worlds, where the physical world no longer grounds action and reflection? That is the question for this paper.

Constructing a Shared Virtual World

The problem of shared understanding rises again—and this time legitimately—within the context of computer-supported collaborative learning (CSCL). That is because when students gather in a CSCL online environment, they enter a virtual world, which is distinct from the world of physical co-presence. They leave the world of nature, of physical embodiment, of face-to-face perception. They enter a world that they have not all grown into together. But this does not mean that "shared understanding" is just a matter of overlapping opinions of mental models for online groups either.

In the Virtual Math Teams (VMT) Project, we have been studying how students interact in a particular CSCL environment designed to support online discourse about mathematics. In this paper we will illustrate some of our findings about how interaction in the VMT environment addresses the challenge of constructing a shared virtual world, in which small groups of students can productively engage in collaborative mathematics.

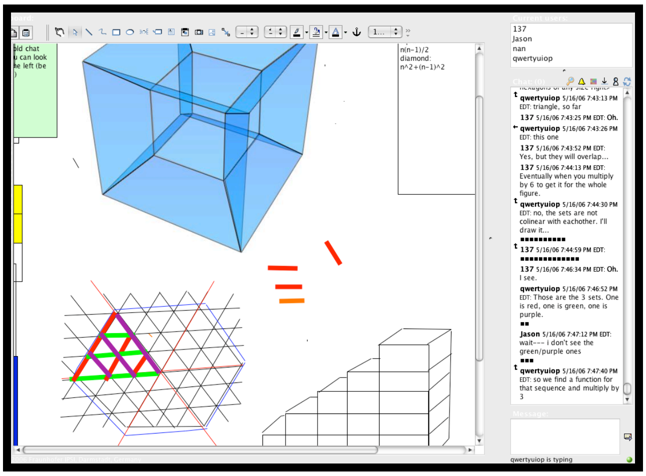

Figure 1. The VMT environment during Session 3.

This paper will present a case study of Session 3 of Team C in the VMT Spring Fest 2006. Here, students aged 12-15 from different schools in the US met online for four hour-long sessions. Neither the students nor the researchers knew anything about the students other than their login user names and their behavior in the sessions. A researcher joined the students, but did not engage with them in the mathematics. Between sessions, the researchers posted feedback in the shared whiteboard of the environment. The VMT Project is described and discussed in (Stahl, 2009); its theoretical motivation is presented in (Stahl, 2006). The VMT environment is shown in Figure 1. The complete chat log of Session 3 of Team C is given in the Appendix of the online version of this paper (http://GerryStahl.net/pub/cscl2011.pdf) and a Replayer version can be obtained from the authors.

In the next sections, we illustrate the following aspects of building shared understanding: (a) Asking each other questions is one common way of resolving or avoiding troubles of understanding and aligning perceptions. (b) Literally looking at the same aspect of something as someone else helps us to see what each other means. (c) The co-constructed shared world has social and temporal as well as objective dimensions. (d) This world grounds communicative, interpersonal, and task-related activities for online groups.

Questioning to Share Understanding

We have analyzed how questions posed in the VMT environment often work to initiate interactions that resolve troubles of understanding and deepen shared understanding (Zhou, 2009; 2010; Zhou, Zemel & Stahl, 2008). This is in contrast to the rationalist assumption that questions are requests for propositional information. We will here review a number of questions from Session 3 of Group C and indicate how they lead to shared understanding. Unfortunately, due to space limitations, we will not be able to provide the full context for these questions or a detailed conversation analysis.

The question by Qwertyuiop in Log 1 serves a coordination function, making sure that all the students have read the feedback to Session 2 before any work begins in the new Session. This is an effort, taking the form of a question, to maintain a shared experience by having everyone take this first step together.

Log 2 is part of a complicated and subtle process of co-constructing shared understanding. It is analyzed in detail in (Çakir, Zemel & Stahl, 2009). The student named 137 has attempted to construct a grid of triangles in the whiteboard (similar to those in the lower left corner of Figure 1). He (or she) has failed (as expressed by the ironic "Great"), and has erased the attempt and solicited help by posing a question. Qwertyuiop requests clarification with another question and then proceeds to draw a grid of triangles by locating and then tweaking three series of parallel lines, following much the same procedures as 137

Log 1.

Chat Index	Time of Posting	Author	Content
685	19:06:34	qwertyuiop	has everyone read the green text box?
686	19:06:44	Jason	one sec
687	19:06:45	137	Yes...
688	19:07:01	Jason	alright im done

Log 2.

694	19:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?
695	19:11:51	qwertyuiop	just a grid?
696	19:12:07	137	Yeah...
697	19:12:17	qwertyuiop	ok...

Log 3.

698	19:14:09	nan	so what's up now? does everyone know what other people are doing?
699	19:14:25	137	Yes?
700	19:14:25	qwertyuiop	no-just making triangles
701	19:14:33	137	I think...
702	19:14:34	Jason	yeah
703	19:14:46	nan	good:-)
704	19:14:51	qwertyuiop	triangles are done

did. Qwertyuiop's understanding of 137's request is based not only on the "Yeah…" response to his/her "just a grid?" question, but also on the details of the sequentially unfolding visual presentation of 137's failed drawing attempt.

In Log 3, the moderator, Nan, asks a question to make visible in the chat what members of the group are doing. Qwertyuiop is busy constructing the requested grid in the whiteboard and the others are presumably watching that drawing activity and waiting for its conclusion. The students do not seem to feel that there is a problem in their understanding of each other's activities. However, due to the nature of the virtual environment—in which the attentiveness of participants is only visible through their chat and drawing actions—Nan cannot know if everyone is engaged during this period of chat inaction. Her question makes visible to her and to the students the fact that everyone is still engaged. The questioning may come as a minor interference in their group interaction, since Nan's questioning positions her as someone outside the group ("everyone"), exerting authority by asking for an accounting, although it is intended to increase group shared understanding ("everyone know what other people are doing").

See What I Mean

Studies of the use of interactive whiteboards in face-to-face classrooms have shown that they can open up a "shared dynamic dialogical space" (Kershner et al., 2010) as a focal point for collective reasoning and co-construction of knowledge. Similarly, in architectural design studios, presentation technologies mediate shared ways of seeing from different perspectives (Lymer, Ivarsson & Lindwall, 2009) in order to establish shared understanding among design students, their peers, and their critics. Clearly, a physical whiteboard that people can gather around and gesture toward while discussing and interpreting visual and symbolic representations is different from a virtual shared whiteboard in an environment like VMT.

We have analyzed in some detail the intimate coordination of visual, narrative and symbolic activity involving the shared whiteboard in VMT sessions (Çakir, 2009; Çakir, Stahl & Zemel, 2010; Çakir, Zemel & Stahl, 2009). Here, we want to bring out the importance of literally looking at some mathematical object together in order to share the visual experience and to relate to—intend or "be at"—the object together. People often use the expression "I do not see what you mean" in the metaphorical sense of not understanding what someone else is saying. In this case study, we often encounter the expression used literally for not being able to visually perceive a graphical object, at least not being able to see it in the way that the speaker apparently sees it.

While empiricist philosophy refers to people taking in uninterpreted sense data much like arrays of computer pixels, post-cognitive philosophy emphasizes the phenomenon of "seeing as." Wittgenstein notes that one sees a wire-frame drawing of a cube not as a set of lines, but as a cube oriented either one way or another (Wittgenstein, 1953, sec. 177). For Heidegger, seeing things as already meaningful is not the result of cognitive interpretation, but the precondition of being able to explicate that meaning further in interpretation (Heidegger, 1927/1996, pp. 139f). For collaborative interpretation and mathematical deduction, it is clearly important that the participants see the visual mathematical objects as the same, in the same way. This seems to be an issue repeatedly in the online session we are analyzing as well.

137 proposes a mathematical task for the group in line 705 of Log 4. This is the first time

Log 4.

705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance

Log 5.

714	19:18:53	137	How do you color lines?
715	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:12	Jason	it's the fifth one from the right
717	19:19:20	137	Thanks.
718	19:19:21	Jason	there ya go :-)
719	19:19:48	137	Er... That hexagon.

Log 6.

720	19:20:02	Jason	so... should we try to find a formula i guess
721	19:20:22	Jason	input: side length; output: # triangles
722	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.
723	19:20:48	137	Like this?
724	19:21:02	qwertyuiop	yes
725	19:21:03	Jason	yup
726	19:21:29	qwertyuiop	side length is the same...
727	19:22:06	Jason	yeah
728	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
729	19:22:19	137	Each one has 1+3+5 triangles.

that the term, "hexagonal array," has been used. Coined in this posting, the term will become a mathematical object for the group as the discourse continues. However, at this point, it is problematic for both Qwertyuiop and Jason. In line 706, Qwertyuiop poses a question for clarification and receives an affirmative, but minimal response. Jason, unsatisfied with the response, escalates the clarification request by asking for help in seeing the diagram in the whiteboard *as* an "hexagonal array," so he can see it *as* 137 sees it. Between Jason's request in line 709 and acceptance in line 710, Qwertyuiop and 137 work together to add lines outlining a large hexagon in the triangular array. Demonstrating his ability to now see hexagons, Jason thereupon proceeds with the mathematical work, which he had halted in the beginning of line 709 in order to keep the group aligned. Jason tentatively proposes that every hexagon "has at least 6 triangles" and he makes this visible to everyone by pointing to an illustrative small hexagon from the chat posting, using the VMT graphical pointing tool.

In Log 5, 137 asks the group to share its knowledge about how to color lines in the VMT whiteboard. Jason gives instructions for 137 to visually locate the appropriate icon in the VMT interface. Demonstrating this new knowledge, 137 changes the colors of the six lines outlining the large hexagon, from black to blue, making the outline stand out visually (see Figure 1). 137 thereby finally clarifies how to look at the array of lines *as* a large hexagon, a task that is more difficult than looking at the small hexagon that Jason pointed to. In this excerpt, the group shares their working knowledge of their virtual world (the software functionality embedded in it), incidentally to carrying out their task-oriented discourse within that world.

In Log 6, Jason proposes a specific mathematical task for the group to undertake, producing a formula for the number of triangles in an hexagonal array of any given side length. (As we shall see below, the group uses the term "side length" as the measure of a geometric pattern at stage n.) Qwertyuiop responds to this proposal with the suggestion to "see" the hexagon (of any size) as a configuration of six triangular areas. (To see what Qwertyuiop is suggesting, look at Figure 1; one of the six triangular areas of the large hexagonal array has its "sticks" colored with thick lines. Looking at this one triangular area, you can see in rows successively further from the center of the hexagon a sequence of one small triangle, then three small triangles, then five small triangles.)

In line 723, 137 seeks confirmation that he is sharing Qwertyuiop's understanding of the suggestion. After posting, "Like this?" with a reference back to Qwertyuiop's line 722, 137 draws three red lines through the center of the large hexagon, dividing it visually into six triangular areas. Upon seeing the hexagon divided up by 137's lines, Qwertyuiop and Jason both confirm the shared understanding. Now that they are confident that they are all seeing the mathematical situation the same, namely *as* a set of six triangular sub-objects, the group can continue its mathematical work. Jason draws the consequence from Qwertyuiop's suggestion that the formula for the number of small triangles in a hexagon will simply be six times the number in one of the triangular areas of that hexagon, thereby subdividing the problem. 137 then notes that each of those triangular areas has 1+3+5 small triangles, at least for the example hexagonal array that they are looking at. The fact that the three members of the group take turns making the consecutive steps of the mathematical deduction is significant; it demonstrates that they share a common understanding of the deduction and are building their shared knowledge collaboratively.

The observation, "Each one has 1+3+5 triangles," is a key move in deducing the sought equation. Note that 137 did not simply say that each triangular area had nine small triangles. The posting used the symbolic visual representation, "1+3+5." This shows a pattern of the addition of consecutive odd numbers, starting with 1. This pattern is visible in the posting. It indicates that 137 is seeing the nine triangles *as* a pattern of consecutive odd numbers—and thereby suggests that the reader also see the nine triangles *as* such a pattern. This is largely a visual accomplishment of the human visual system. People automatically see collections of small numbers of objects as sets of that specific size (Lakoff & Núñez, 2000). For somewhat larger sets, young children readily learn to count the number of objects. The team has constructed a graphical representation in which all the members of the team can immediately see features of their mathematical object that are helpful to their mathematical task. The team is collaborating within a shared virtual world in which they have co-constructed visual, narrative, and symbolic objects in the chat and whiteboard areas. The team has achieved this shared vision by enacting practices specific to math as a profession for shaping witnessed events, such as invoking related math terms and drawing each others' attention to relevant objects in the scene (Goodwin, 1994). They have learned and taught each other how to work, discuss, and perceive as a group in this shared virtual world.

Dimensions of a Virtual World

There has not been much written about the constitution of the intersubjective world as the background of shared understanding, particularly in the CSCL online context. This is largely the result of the dominance of the cognitive perspective, which is primarily concerned with mental models and representations of the world; this rationalist view reduces the shared world to possible similarities of individual mental representations. Within the VMT Project, we have analyzed the dimensions of domain content, social interaction, and temporal sequencing in the co-construction of a virtual math team's world or joint problem space (Sarmiento & Stahl, 2008;

Sarmiento-Klapper, 2009a; Sarmiento-Klapper, 2009b). In this work, we have found the following conceptualizations to be suggestive and helpful: the joint problem space (Teasley & Roschelle, 1993) and the indexical ground of reference of domain content (Hanks, 1992); the social positioning of team members in discourse (Harré & Gillet, 1999) and their self-coordination (Barron, 2000); and the temporal sequentiality of discourse (Schegloff, 1977) and the bridging of temporal discontinuities.

In previous sessions, the group has tried to derive formulae for the number of two-dimensional objects (small squares or small triangles) in a growing pattern of these objects, as well as the number of one-dimensional sides, edges or "sticks" needed to construct these objects. A major concern in counting the number of sides is the issue of "overlap." In a stair-step two-dimensional pattern (like the 2-D version of the stair-step pyramid in the lower right section of Figure 1), one cannot simply multiply the number of squares by 4 to get the number of sides because many of the sides are common to two squares. In Session 1, Team C had seen that in moving from one stage to the next stage of the stair-step pattern most new squares only required two new sides.

In Log 7, Qwertyuiop moves on from the derivation of the number of triangles to that of the number of sides. He "bridges" back to the group's earlier in-sight that the addition of "each polygon corresponds to [an additional] 2 sides." In bridging to past sessions, we found, it is necessary to re-situate a previous idea in the current context. In line 731, Qwertyuiop is reporting that for their hexagon formula, such situating does not work—i.e., that the current problem cannot be solved with the same method as the previous problems. The group then returns to the formula for the number of triangles and efficiently solves it by summing the sequence of consecutive odd numbers using Gauss' technique—the sum of n consecutive odd integers is n(2n/2)—which they had used in previous sessions.

In Log 8, Qwertyuiop makes a particularly complicated proposal, based on a way of viewing the sides in the large hexagon drawing. He tries to describe his view in chat, talking about sets of collinear sides. Jason does not respond to this proposal and 137 draws some lines to see if he is visualizing what Qwertyuiop has proposed, but he has not. Qwertyuiop has to spend a lot of time drawing a color-coded analysis of the sides as he sees them. He has decomposed the set of sides of one triangular area into three subsets, going in the three directions of the array's original parallel lines. He can then see that each of these subsets consists of 1+2+3 sides. There are 3 subsets in each of the 6 triangular areas. Based on this and generalizing to a growing hexagonal array, which will have sums of consecutive integers in each subset, the team can derive a formula using past techniques. At some point, they will have to subtract a small number of sides that overlap between adjacent triangular areas. Qwertyuiop has proposed a decomposition of the hexagonal array into symmetric sets, whose constituent parts are easily visible. Thus, his approach bridges back to previous group practices, which are part of the shared world of the group—see the analysis of a similar accomplishment by Group B in (Medina, Suthers & Vatrapu, 2009). The hexagonal pattern, which Team C came up with on its own, turns out to be considerably more difficult to decompose into simple patterns that the original problem given in Session 1. It

Log 7.

731	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles
732	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?
733	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...
734	19:24:49	137	And there are n terms so... n(2n/2)
735	19:25:07	137	or n^2
736	19:25:17	Jason	yeah
737	19:25:21	Jason	then multiply by 6
738	19:25:31	137	To get 6n^2

Log 8.

742	19:25:48	qwertyuiop	an idea: Find the number of a certain set of colinear sides (there are 3 sets) and multiply the result by 3
746	19:26:36	137	As in those?
747	19:27:05	qwertyuiop	no-in one triangle. I'll draw it...
748	19:28:10	qwertyuiop	those
749	19:28:28	qwertyuiop	find those, and then multiply by 3
750	19:28:50	137	The rows?
751	19:30:01	qwertyuiop	The green lines are all colinear. There are 3 identical sets of colinear lines in that triangle. Find the number of sides in one set, then multiply by 3 for all the other sets.
752	19:30:23	137	Ah. I see.

Log 9.

804	19:48:49	nan	(we got a question for you from another team, which was posted in the lobby:
805	19:48:53	nan	Quicksilver 7:44:50 PM EDT: Hey anyone from team c, our team needs to know what n was in your equations last week
806	19:49:04	Jason	oh
807	19:49:15	137	The length of a side.
808	19:49:16	qwertyuiop	was n side length?
809	19:49:33	Jason	are you talking about the original problem with the squares
810	19:49:48	137	I think nan is.
811	19:49:58	qwertyuiop	i think it's squares and diamonds

strained the shared understanding of the group, requiring the use of all the major analytic tools they had co-constructed (decomposing, color-coding, visually identifying sub-patterns, summing series, eliminating overlaps, etc.).

In Log 9, the group work is interrupted by an interesting case of bridging across teams. At the end of each session, the teams had posted their findings to a wiki shared by all the participants in the VMT Spring Fest 2006. During their Session 3, Team B had looked at Team C's work on a pattern they had invented: a diamond variation on the stair-step pattern. In their wiki posting, Team C had used their term, "side length." Because members of Team B did not share Teams C's understanding of this term, they were confused by the equation and discussion that Team C posted to the wiki. Team B's question sought to establish shared understanding across the teams, to build a community-wide shared world. As it turned out, Team C had never completed work on the formula for the number of sides in a diamond pattern and Team B eventually discovered and reported the error in Team C's wiki posting, demonstrating the importance of community-wide shared understanding.

Grounding Group Cognition

CSCL is about meaning making (Stahl, Koschmann & Suthers, 2006). At its theoretical core are questions about how students collaborating online co-construct and understand meaning. In this paper, we conceptualize this issue in terms of online groups, such as virtual math teams, building a shared meaningful world in which to view and work on mathematical objects.

Log 10 illustrates a limit of shared understanding, closely related to the notion of a "zone of proximal development" (Vygotsky, 1930/1978, pp. 84-91). The original stair-step pattern consisted of one-dimensional sides and two-dimensional squares. In their Session 2, Team C had generalized this pattern into a three-dimensional pyramid consisting of cubes. Now Qwertyuiop proposes to further generalize into a mathematical fourth dimension and derive formulae for patterns of one, two, three, and four-dimensional objects. He had previously imported a representation of a four-dimensional hyper-cube (see the upper area of Figure 1) into the whiteboard for everyone to see.

Log 10.

20:12:22	qwertyuiop	what about the hypercube?
20:12:33	137	Er...
20:12:39	137	That thing confuses me.
20:13:00	137	The blue diagram, right?
20:13:13	qwertyuiop	can you imagine extending it it 4 dimensions, and a square extends into a grid?
20:13:17	qwertyuiop	yes
20:13:30	137	I didn't get that?
20:13:32	qwertyuiop	I'm having trouble doing that.
20:13:45	qwertyuiop	didn't get this?
20:13:50	137	Ya.
20:15:02	qwertyuiop	If you have a square, it extends to make a grid that fills a plane. A cube fills a space. A simaller pattern of hypercubes fills a "hyperspace".
20:15:19	137	The heck?
20:15:29	137	That's kinda confusing.
20:15:43	qwertyuiop	So, how many planes in a hyper cube latice of space n?
20:16:05	137	Er...
20:16:07	qwertyuiop	instead of "how many lines in a grid of length n"
20:16:17	qwertyuiop	does that make any sense?

At this point late in Session 3, Jason had left the VMT environment. Qwertyuiop was unable to guide 137 to see the drawing in the whiteboard as a four-dimensional object. Apparently, Qwertyuiop had been exposed to the mathematical idea of a fourth dimension and was eager to explore it. However, 137 had not been so exposed. They did not share the necessary background for working on Qwertyuiop's proposal. This shows that tasks for student groups, even tasks they set for themselves, need to be within a shared group zone of proximal development. The stair-step problem was in their zone—whether or not they could solve it themselves individually, they were able to solve it collectively, with enough shared understanding that they could successfully work together. Their three-dimensional pyramid turned out to be quite difficult for them to visualize in a shared way. Their diamond pattern seemed to be easy for them, although they forgot to work on some of it and posted an erroneous formula. The hexagonal array required them to develop their skills in a number of areas, but they solved it nicely. However, the hyper-cube exceeded at least 137's ability (or desire) to participate.

Rationalist philosophy reduces the complexity of social human existence to a logical, immaterial mind that thinks about things by representing them internally. It confuses the mind with the brain and conflates the two. It assumes that someone thinking about a hexagon or working on a math problem involving a hexagon must primarily be representing the hexagon in some kind of mental model. But one of the major discoveries of phenomenology (Husserl, 1936/1989) was that intentionality is always the intentionality of some object and that cognition takes place as a "being-with" that object, not as a mental act of some transcendental ego. As an example, we have seen that the members of Team C are focused on the graphical image of the hexagon in their virtual world on their computer screens. They reference this image and transform it with additional lines, colors, and pointers. They chat about this image, not about some personal mental representations. They work to get each other to see that image in the same way that they see it. This "seeing" is to be taken quite literally. Their eyes directly perceive the image. They perceive the image in a particular way (which may change and which they may have to learn to see). “Seeing” is not a metaphor to describe some kind of subjective mental process that is inaccessible to others, but a form of contact with the object in the world. Accordingly, we may say that shared understanding is a matter of the group members being-there-together at the graphical image in the whiteboard.

Being-there-together is a possible mode of existence of the online group. The “there” where they are is a multi-dimensional virtual world. This world was partially already there when they first logged in. It included the computer hardware and software. It included the VMT Spring Fest as an organized social institution. As they started to interact, the students fleshed out the world, building social relationships, enacting the available technology, interpreting the task instructions, and proposing steps to take together. Over time, they constructed a rich world, furnished with mathematical objects largely of their own making and supporting group practices that they had introduced individually but which they had experienced as a group.

Being-there-together in their virtual world with their shared understanding of many of this world’s features, the group was able to accomplish mathematical feats that none of them could have done alone. Each individual in the group shared an understanding of their group work at least enough to make productive contributions that reflected a grasp of what the group was doing. Their group accomplishments were achieved through group processes of visualization, discourse, and deduction. They were accomplishments of group cognition, which does not refer to anything mystical, but to the achievements of group interaction. The group cognition was possible because of, and only on the basis of, the shared understanding of the common virtual world. Shared understanding is not a matter of similar mental models, but of experiencing a shared world.

Of course, there are limits to group cognition, just as there are limits to individual cognition. We saw that Team C could not understand Qwertyuiop’s ideas about the fourth dimension. Without shared understanding about this, the group could not engage in discourse on that topic. Group cognition can exceed the limits of the individual cognition of the group members, but only by a certain amount. The individuals must be able to stretch their own existing understanding under the guidance of their peers, with the aid of physical representations, tools, concepts, scaffolds, and similar artifacts, whose use is within their grasp—within their zone of proximal development (Vygotsky, 1930/1978). We have seen that Team C was able to solve a complex mathematical problem that they set for themselves involving a hexagonal array by building up gradually, systematically, and in close coordination a meaningful virtual world.

An analysis of the log of the interaction in our case study has demonstrated much about the team’s group cognition. Their group work proceeded by contributions from different individuals, with everyone contributing in important ways. Their questions showed that their individual cognition was initially inadequate to many steps in the work; but their questions also served to expand the shared understanding and to ensure that each member shared an understanding of each step. Because the students demonstrated an understanding of the group work through their successive contributions, we can see not only that individual learning took place, but we can analyze the interactional processes through which it took place through detailed analysis of their chat and drawing actions.

As Vygotsky argued, not only does group cognition lead individual cognition by several years, but individual cognition itself develops originally as a spin-off of group cognition. Individuals can learn on their own, but the cognitive and practical skills that they use to do so are generally learned through interaction with others and in small groups. This is a powerful argument for the use of CSCL in education. It is incumbent upon CSCL research to further analyze the processes by which this takes place in the co-construction of shared understanding within co-experienced virtual worlds. As we have seen, participants in CSCL virtual environments co-construct worlds to ground their interactions. These virtual worlds exploit meaning-making, perceptual and referential practices learned in the physical social world.

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Appendix

Complete chat log of Session 3 of Group C of VMT Spring Fest 2006. A Replayer file of the entire Group C interaction, including whiteboard and chat is available on request from the authors.

Chat Index	Start Typing	Time of Posting	Author	Content	Refers to
662		16:08:08	jsarmi	joins the room	
663		17:20:42	nan	joins the room	
664		17:57:15	jsarmi	leaves the room	
665		19:01:25	Jason	joins the room	
666		19:02:22	137	joins the room	
667	19:02:30	19:02:37	nan	hi Jason and 137, welcome back	
668	19:02:49	19:02:49	Jason	hi	
669	19:03:05	19:03:06	137	Hi.	
670	19:02:56	19:03:09	nan	i'll be your facilitator tonight	
671	19:02:51	19:03:13	Jason	it looks like ssjnish is having connection problems again, even after i pointed him to an email on how to clear his Java cache	
672		19:03:45	qwertyuiop	joins the room	
673	19:04:07	19:04:13	nan	hi qwertyuiop	
674	19:04:23	19:04:23	qwertyuiop	hi	
675	19:04:24	19:04:36	nan	do any of you know if david is coming?	
676	19:04:33	19:04:41	137	So we do what we did last time again?	
677	19:04:46	19:04:47	nan	yes	[1] Ref to 676
678	19:04:42	19:04:52	137	I forgot to ask David at school.	[1] Ref to 675
679	19:04:59	19:05:04	137	I don't think he'd remember.	
680	19:04:48	19:05:11	nan	first take a few minutes to read the feedback posted on the whiteboard	
681	19:05:19	19:05:21	nan	no problem	[1] Ref to 678
682	19:05:23	19:05:27	nan	i guess we can start	
683	19:05:38	19:05:48	nan	david can join later when he comes	
684	19:05:53	19:05:54	137	Right.	
685	19:06:19	19:06:34	qwertyuiop	has everyone read the green text box?	
686	19:06:43	19:06:44	Jason	one sec	
687	19:06:43	19:06:45	137	Yes...	
688	19:07:00	19:07:01	Jason	alright im done	
689	19:06:27	19:07:02	nan	did you see some little squares aftermessage? i haven't seen thoese before, interesting	
690	19:07:10	19:07:11	qwertyuiop	yes	
691	19:07:07	19:07:12	Jason	yeah, they just indicate whiteboard activity	
692	19:07:31	19:07:32	137	Oh.	
693	19:07:22	19:07:40	nan	i see. i was on a leave for two weeks and this version is the latest	[1] Ref to 691
694	19:11:02	19:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?	
695	19:11:47	19:11:51	qwertyuiop	just a grid?	

696	19:12:04	19:12:07	137	Yeah...	
697	19:12:14	19:12:17	qwertyuiop	ok...	
698	19:13:40	19:14:09	nan	so what's up now? does everyone know what other people are doing?	
699	19:14:23	19:14:25	137	Yes?	
700	19:14:18	19:14:25	qwertyuiop	no-just making triangles	
701	19:14:31	19:14:33	137	I think...	[1] Ref to 699
702	19:14:32	19:14:34	Jason	yeah	
703	19:14:44	19:14:46	nan	good:-)	[1] Ref to 701
704	19:14:45	19:14:51	qwertyuiop	triangles are done	
705	19:14:46	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?	
706	19:15:22	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?	[1] Ref to 705
707	19:16:00	19:16:02	137	Ya.	[1] Ref to 706
708	19:16:13	19:16:15	qwertyuiop	ok...	
709	19:16:20	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...	
710	19:17:28	19:17:30	Jason	hmm.. okay	
711	19:17:42	19:17:43	qwertyuiop	oops	[1] Reference to whiteboard
712	19:17:35	19:17:44	Jason	so it has at least 6 triangles?	
713	19:17:55	19:17:58	Jason	in this, for instance	[1] Reference to whiteboard
714	19:18:48	19:18:53	137	How do you color lines?	
715	19:18:58	19:19:06	Jason	there's a little paintbrush icon up at the top	
716	19:19:06	19:19:12	Jason	it's the fifth one from the right	
717	19:19:19	19:19:20	137	Thanks.	
718	19:19:18	19:19:21	Jason	there ya go :-)	
719	19:19:44	19:19:48	137	Er... That hexagon.	
720	19:19:52	19:20:02	Jason	so... should we try to find a formula i guess	
721	19:20:13	19:20:22	Jason	input: side length; output: # triangles	
722	19:20:12	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.	
723	19:20:44	19:20:48	137	Like this?	[1] Ref to 722
724	19:21:01	19:21:02	qwertyuiop	yes	
725	19:21:00	19:21:03	Jason	yup	
726	19:21:23	19:21:29	qwertyuiop	side length is the same...	
727	19:22:05	19:22:06	Jason	yeah	
728	19:22:06	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon	
729	19:22:04	19:22:19	137	Each one has 1+3+5 triangles.	
730	19:22:17	19:22:23	Jason	but then we're assuming just regular hexagons	
731	19:21:53	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles	
732	19:22:43	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?	
733	19:23:43	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...	
734	19:24:22	19:24:49	137	And there are n terms so... n(2n/2)	
735	19:25:01	19:25:07	137	or n^2	[1] Ref to 734
736	19:25:17	19:25:17	Jason	yeah	
737	19:25:18	19:25:21	Jason	then multiply by 6	
738	19:25:26	19:25:31	137	To get 6n^2	[1] Ref to 737
739	19:25:21	19:25:39	Jason	but this is only with regular hexagons... is it possible to have one definite formula for irregular hexagons as well	
740	19:24:19	19:25:46	nan	(sorry to interrupt) jason, do you think you can ask ssjnish to check the email to see the instructions sent by VMT team, which might help?	
741	19:25:42	19:25:48	Jason	i'm not sure if its possible tho	
742	19:24:39	19:25:48	qwertyuiop	an idea: Find the number of a certain set of colinear sides (there are 3 sets) and multiply the result by 3	
743	19:25:57	19:26:03	Jason	i did--apparently it didn't work for him	[1] Ref to 740
744	19:26:05	19:26:13	Jason	or his internet could be down, as he's not even on IM right now	
745	19:26:10	19:26:13	nan	i see. thanks!	[1] Ref to 743
746	19:26:20	19:26:36	137	As in those?	[1] Ref to 742
747	19:26:46	19:27:05	qwertyuiop	no-in one triangle. I'll draw it...	[1] Ref to 746
748	19:28:09	19:28:10	qwertyuiop	those	

749	19:28:18	19:28:28	qwertyuiop	find those, and then multiply by 3	
750	19:28:48	19:28:50	137	The rows?	
751	19:29:01	19:30:01	qwertyuiop	The green lines are all colinear. There are 3 identical sets of colinear lines in that triangle. Find the number of sides in one set, then multiply by 3 for all the other sets.	
752	19:30:20	19:30:23	137	Ah. I see.	
753	19:31:00	19:31:07	137	Wait. Wouldn't that not work for that one?	
754	19:31:11	19:31:12	Jason	yeah	
755	19:31:12	19:31:15	Jason	beacuse that's irregular	
756	19:31:09	19:31:17	137	Or are we still only talking regular ones?	
757	19:31:20	19:31:22	137	About	
758	19:30:38	19:31:24	qwertyuiop	side length 1 = 1, side length 2 = 3, side length 3 = 6...	
759	19:32:32	19:32:50	137	Shouldn't side length 2 be fore?	[1] Ref to 758
760	19:32:52	19:32:53	137	*four	
761	19:33:06	19:33:10	qwertyuiop	I count 3.	[1] Ref to 759
762	19:33:20	19:33:25	137	Oh. Sry.	
763	19:33:24	19:33:30	qwertyuiop	It's this triangle.	[1] Reference to whiteboard
764	19:33:44	19:33:45	137	We	
765	19:33:47	19:33:54	qwertyuiop	I don't see the pattern yet...	[1] Ref to 758
766	19:33:50	19:34:01	137	We're ignoring the bottom one?	
767	19:34:11	19:34:29	qwertyuiop	no, 3 is only for side length 2.	[1] Ref to 766
768	19:34:36	19:34:52	137	And I think the'y;re all triangular numbers.	[1] Ref to 765
769	19:35:06	19:35:17	qwertyuiop	"triangular numbers"?	[1] Ref to 768
770	19:35:28	19:35:37	Jason	you mean like 1, 3, 7, ...	
771	19:35:39	19:35:39	Jason	?	
772	19:35:48	19:35:59	137	Like 1,3,6,10,15,21,28.	[1] Ref to 770
773	19:35:51	19:36:02	qwertyuiop	the sequence is 1, 3, 6...	[1] Ref to 770
774	19:36:02	19:36:30	137	Numbers that can be expressed as n(n+1)/2, where n is an integer.	
775	19:36:44	19:36:45	qwertyuiop	ah	
776	19:37:09	19:37:18	137	So are we ignoring the bottom orange line for now?	[1] Ref to 766
777	19:37:32	19:37:36	qwertyuiop	"green"?	[1] Ref to 776
778	19:37:44	19:37:48	137	THe short orange segment.	
779	19:37:49	19:38:05	137	PArallel to the blue lines.	
780	19:37:58	19:38:05	qwertyuiop	I don't think so...	
781	19:38:20	19:38:26	137	Wait, we are counting sticks right now, right?	[1] Ref to 780
782	19:38:35	19:38:48	qwertyuiop	yes-one of the colinear ets of sticks	
783	19:38:55	19:39:08	qwertyuiop	oops-"sets" not " ets"	
784	19:39:22	19:39:42	137	So we are trying to find the total number of sticks in a given regular hexagon?	[1] Ref to 782
785	19:39:50	19:40:18	qwertyuiop	not yet-we are finding one of the three sets, then multiplying by 3	[1] Ref to 784
786	19:40:25	19:40:40	qwertyuiop	that will give the number in the whol triangle	
787	19:40:34	19:40:51	137	Then shouldn't we also count the bottom line?	[1] Ref to 785
788	19:40:52	19:41:01	Jason	are you taking into account the fact that some of the sticks will overlap	[1] Ref to 786
789	19:41:25	19:41:41	137	Then number of sticks needed for the hexagon, right?	[1] Ref to 786
790	19:41:16	19:42:22	qwertyuiop	Yes. The blue and green/orange lines make up on of the three colinear sets of sides in the triangle. Each set is identical and doesn't overlap with the other sets.	[1] Ref to 788
791	19:42:50	19:42:50	Jason	ok	
792	19:43:03	19:43:11	Jason	this would be true for hexagons of any size right>	
793	19:43:09	19:43:13	qwertyuiop	triangle, so far	[1] Ref to 789
794	19:43:25	19:43:25	137	Oh.	
795	19:43:25	19:43:26	qwertyuiop	this one	[1] Reference to whiteboard
796	19:43:42	19:43:52	137	Yes, but they will overlap...	
797	19:43:59	19:44:13	137	Eventually when you multiply by 6 to get it for the whole figure.	
798	19:44:01	19:44:30	qwertyuiop	no, the sets are not colinear with eachother. I'll draw it...	[1] Ref to 796
799		19:44:59	137		[1] Ref to 798
800	19:46:22	19:46:34	137	Oh. I see.	
801	19:46:22	19:46:52	qwertyuiop	Those are the 3 sets. One is red, one is green, one is purple.	
802	19:47:04	19:47:12	Jason	wait--- i don't see the green/purple ones	

803	19:47:18	19:47:40	qwertyuiop	so we find a function for that sequence and multiply by 3	[1] Ref to 774
804	19:48:25	19:48:49	nan	(we got a question for you from another team, which was posted in the lobby:	
805	19:48:52	19:48:53	nan	Quicksilver 7:44:50 PM EDT: Hey anyone from team c, our team needs to know what n was in your equations last week	
806	19:49:04	19:49:04	Jason	oh	
807	19:49:12	19:49:15	137	The length of a side.	
808	19:49:10	19:49:16	qwertyuiop	was n side length?	
809	19:49:26	19:49:33	Jason	are you talking about the original problem with the squares	
810	19:49:44	19:49:48	137	I think nan is.	[1] Ref to 809
811	19:49:43	19:49:58	qwertyuiop	i think it's squares and diamonds	[1] Ref to 809
812	19:49:58	19:49:58	Jason	oh	
813	19:49:59	19:50:12	Jason	then if you look in the topic description, theres a column for N;	
814	19:50:12	19:50:14	Jason	thats what it is	
815	19:50:09	19:50:17	nan	ok, quicksilver said they got it	
816	19:50:22	19:50:25	Jason	so yes it is # sides	
817	19:50:21	19:50:26	nan	thanks guys	
818	19:51:11	19:52:19	qwertyuiop	what about: f(n)=2n-1 where n is side length	[1] Ref to 772
819	19:52:55	19:53:03	137	I don't think that works.	[1] Ref to 818
820	19:53:07	19:53:18	137	Howbout just n(n+1)/2	
821	19:53:37	19:53:41	Jason	for # sticks?	
822	19:53:38	19:53:48	qwertyuiop	that's number of sides for one set	[1] Ref to 820
823	19:53:50	19:53:51	qwertyuiop	?	
824	19:53:57	19:53:59	Jason	oh ok nvm	
825	19:54:26	19:54:29	137	Ya.	[1] Ref to 822
826	19:54:36	19:54:58	qwertyuiop	then x3 is 3(n(n+1)/2)	[1] Ref to 820
827	19:55:04	19:55:07	qwertyuiop	simplified to...	[1] Ref to 826
828	19:55:11	19:55:37	qwertyuiop	(n(n+1)1.5	
829	19:55:34	19:55:44	137	On second thought, shouldn't we use n(n-1) for these:	[1] Ref to 826
830	19:55:31	19:55:55	nan	just a kind reminder: Jason mentioned that he needs to leave at 7p central time sharp	
831	19:56:05	19:56:19	nan	rest of you can continue if you like	
832	19:56:19	19:56:25	137	Is that 5 pm PST?	
833	19:56:27	19:56:31	137	or 4pm?	
834	19:56:32	19:56:32	nan	yes	[1] Ref to 832
835	19:56:41	19:56:42	137	Ah.	
836	19:56:42	19:56:56	nan	which is a couple of min from now, right, Jason?	[1] Ref to 834
837	19:57:15	19:57:16	qwertyuiop	Jason?	
838	19:57:30	19:57:33	137	I think he left?	[1] Ref to 837
839	19:57:43	19:57:52	Jason	sorry i was away for a couple minutes	
840	19:57:58	19:58:02	Jason	yeah i'll need to go pretty soon	
841	19:58:23	19:58:25	qwertyuiop	back to this?	[1] Ref to 829
842	19:58:32	19:58:34	137	Ya	
843	19:58:39	19:58:49	qwertyuiop	why not n(n-1)?	[1] Ref to 829
844	19:58:39	19:58:50	Jason	you guys pretty much have the formula for this hexagon problem...	
845	19:58:57	19:59:28	qwertyuiop	We almost have it for the triangle. I don't know about the hexagon.	[1] Ref to 844
846	19:59:35	19:59:50	Jason	well that's just multiplied by a certain number for a hexagon, provided that it is regular	[1] Ref to 845
847	19:59:58	20:00:14	qwertyuiop	but the sides of the triangles making up the hexagon overlap	[1] Ref to 846
848	19:59:52	20:00:18	Jason	well i have to leave now; sorry for not participating as much as i wanted to, it's a pretty busy night for me with school and extracurricular stuff	
849	20:00:31	20:00:35	Jason	see you guys Thursday!	
850	20:00:44	20:00:48	nan	thanks for participating	[1] Ref to 849
851	20:00:53	20:00:57	nan	see you Thursday	
852	20:00:57	20:01:00	137	Cya/	
853		20:01:07	Jason	leaves the room	
854	20:01:19	20:01:31	137	Anyways, if we multiply the orange by 3, we get the:	
855	20:01:14	20:01:34	nan	do two of you want to continue working for a bit or stop here?	
856	20:01:40	20:01:44	nan	i guess that's the answer	[1] Ref to 854
857	20:01:47	20:01:48	nan	go ahead	

858	20:01:57	20:02:14	137	So then we add 12n for:	
859	20:01:28	20:02:15	qwertyuiop	actually, this doesn't complicate it that much. The overlaps can be accounted for with "-6n"	[1] Ref to 847
860	20:02:54	20:02:55	137	Oh.	[1] Ref to 859
861	20:02:56	20:03:07	137	I like addition more than subtraction.	
862	20:03:11	20:03:16	qwertyuiop	do you see why that works	[1] Ref to 859
863	20:03:18	20:03:18	qwertyuiop	?	
864	20:03:12	20:03:29	137	So: 9n(n+1)-6n.	
865	20:03:41	20:03:45	qwertyuiop	9, not 3?	
866	20:04:13	20:04:14	137	?	[1] Ref to 865
867	20:04:18	20:04:35	qwertyuiop	you have "9n(n..."	
868	20:04:37	20:04:47	qwertyuiop	not "3n(n..."?	
869	20:04:51	20:05:00	137	But we need to multiply by 6 then divide by 2	[1] Ref to 868
870	20:05:10	20:05:22	qwertyuiop	x6 and /2 for what?	[1] Ref to 869
871	20:05:44	20:05:47	137	FOr each triangle	
872	20:05:48	20:06:02	137	and /2 because it's part of the equation.	
873	20:06:03	20:06:06	137	of n(n+1)/2	
874	20:05:36	20:06:20	qwertyuiop	it's x3 for the 3 colinear sets, then x6 for 6 triangles in a hexagon... where's the 9 and 2?	
875	20:06:28	20:06:28	qwertyuiop	oh	[1] Ref to 872
876	20:06:35	20:06:38	137	So 18/2.	
877	20:06:42	20:06:50	137	A.K.A. 9	
878	20:06:48	20:07:08	qwertyuiop	(n(n+1)/2)x3x6	[1] Ref to 873
879	20:07:14	20:07:15	137	Yeah.	
880	20:07:20	20:07:27	qwertyuiop	which can be simplified...	
881	20:07:42	20:07:46	137	To 9n(n+1)	[1] Ref to 880
882	20:08:01	20:08:04	qwertyuiop	that's it?	[1] Ref to 881
883	20:08:10	20:08:12	137	-6n.	
884	20:08:17	20:08:24	137	So 9n(n+1)-6n	
885	20:08:20	20:08:34	qwertyuiop	i'll put it with the other formulas...	
886	20:09:39	20:09:47	qwertyuiop	number of triangles is...	
887	20:10:27	20:10:28	137	That.	[1] Ref to 738
888	20:10:37	20:10:43	137	6n^2	
889	20:11:25	20:11:26	qwertyuiop	oops	[1] Ref to 888
890	20:12:12	20:12:22	qwertyuiop	what about the hypercube?	
891	20:12:29	20:12:33	137	Er...	[1] Ref to 890
892	20:12:36	20:12:39	137	That thing confuses me.	[1] Ref to 891
893	20:12:56	20:13:00	137	The blue diagram, right?	
894	20:12:37	20:13:13	qwertyuiop	can you imagine extending it it 4 dimensions, and a square extends into a grid?	
895	20:13:16	20:13:17	qwertyuiop	yes	[1] Ref to 893
896	20:13:26	20:13:30	137	I didn't get that?	[1] Ref to 894
897	20:13:21	20:13:32	qwertyuiop	I'm having trouble doing that.	[1] Ref to 894
898	20:13:41	20:13:45	qwertyuiop	didn't get this?	[1] Ref to 894
899	20:13:49	20:13:50	137	Ya.	
900	20:13:57	20:15:02	qwertyuiop	If you have a square, it extends to make a grid that fills a plane. A cube fills a space. A simaller pattern of hypercubes fills a "hyperspace".	
901	20:15:17	20:15:19	137	The heck?	[1] Ref to 900
902	20:15:25	20:15:29	137	That's kinda confusing.	[1] Ref to 900
903	20:15:16	20:15:43	qwertyuiop	So, how many planes in a hyper cube latice of space n?	
904	20:16:04	20:16:05	137	Er...	[1] Ref to 903
905	20:15:48	20:16:07	qwertyuiop	instead of "how many lines in a grid of length n"	
906	20:16:11	20:16:17	qwertyuiop	does that make any sense?	
907	20:16:23	20:16:30	137	No. No offense, of course.	[1] Ref to 906
908	20:16:35	20:16:43	qwertyuiop	ok... let me think...	
909	20:16:58	20:17:19	qwertyuiop	Imagine our first problem with a grid of squares.	
910	20:17:29	20:17:31	137	Right.	[1] Ref to 909
911	20:17:23	20:18:07	qwertyuiop	The squares are 2 dimensional and they can be arranged in a grid to tessalate over a plane. The plane is also 2 dimensional.	
912	20:18:39	20:18:41	137	Right.	[1] Ref to 911

913	20:18:12	20:18:54	qwertyuiop	If you use 3 dimensional cubes, they can be arranged to fill a 3 dimensional space.	
914	20:19:08	20:19:17	137	And that structure's 4 dimensional?	[1] Ref to 913
915	20:18:56	20:19:25	qwertyuiop	If you have hypercubes, they can be arranged to fill a 4 dimensional "hyperspace"	
916	20:19:32	20:19:36	qwertyuiop	what's 4D?	[1] Ref to 914
917	20:19:45	20:19:46	137	?	[1] Ref to 916
918	20:19:43	20:20:04	nan	you may want to make your ideas available on the wiki before you go	
919	20:20:06	20:20:09	nan	which may take some time	
920	20:20:15	20:20:24	137	Actually, I only have around 10 minutes left.	[1] Ref to 919
921	20:19:51	20:20:29	qwertyuiop	you say "and that structure's 4 dimensional?" -what's "that"	[1] Ref to 914
922	20:20:33	20:20:34	nan	oh	[1] Ref to 920
923		20:20:35	137		[1] Ref to 921
924	20:20:36	20:20:43	137	The hypercube	
925	20:20:50	20:20:50	qwertyuiop	yes	[1] Ref to 924
926	20:21:00	20:21:02	137	Um...	
927	20:20:58	20:21:09	qwertyuiop	I have homework to do, too...	[1] Ref to 920
928	20:21:03	20:21:33	137	So how the heck are we supposed to calculate the number of four-dimentional figures?	
929	20:21:32	20:21:42	nan	do you want to stop here and start putting ideas on wiki?	
930	20:21:47	20:21:47	qwertyuiop	ok	
931	20:21:51	20:21:52	137	Sure.	[1] Ref to 929
932	20:22:02	20:22:09	qwertyuiop	resume from here next time?	[1] Ref to 928
933	20:22:16	20:22:17	nan	sure	
934	20:22:18	20:22:19	137	Ya.	
935	20:22:32	20:22:48	qwertyuiop	We have the 2 hexagon equations to put on the wiki.	
936	20:23:04	20:23:04	137	Right.	[1] Ref to 935
937	20:23:11	20:23:18	qwertyuiop	Where's the wiki again?	
938	20:23:27	20:23:30	nan	open "view topic"	[1] Ref to 937
939	20:23:23	20:23:31	137	Somewhere in the View topic button	
940	20:23:39	20:23:41	nan	there's link	
941	20:23:53	20:23:54	qwertyuiop	I see it.	
942		20:24:28	137	leaves the room	
943	20:24:57	20:25:02	qwertyuiop	i'll write it.	
944		20:25:05	qwertyuiop	leaves the room	
945		20:25:19	nan	leaves the room	