

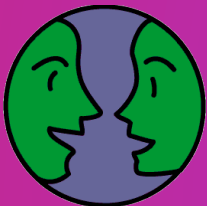
The Integration of Cognitive Levels

Gerry Stahl

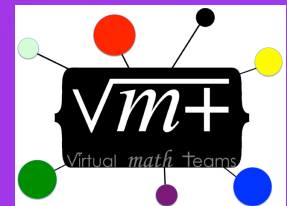
The Integration of Cognitive Levels

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- Main claim: a math topic can integrate individual skills, group collaboration & community knowledge.
- Illustrative resource: a challenging dynamic-geometry topic (pattern of sticks in a diamond pattern, segments in a hexagon array, inscribed polygons).
- Supporting data: Virtual Math Team session by teachers (100 hours) and students (800 hours).
- Current status: analysis underway of 8-session series; sessions to be repeated next year.



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Pattern of sticks in diamond

if so to see what you are
g. it seems that there are
y you are following each
clear that you are really in
pletely understand each
thually discover some more
sings in more detail - to be
u are in agreement.

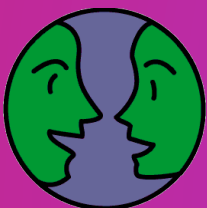
ou could revisit a problem
n before, in order to state
her groups in the wiki: (a) a
problem, (b) a solution and
the problem. Or you could
of these pattern problems,
of group C's diamond

iroust whatever most
hat enables you to improve
lity to work together. As you
es by pretty quickly, so it's
time for a complicated
ve and enjoy the session.

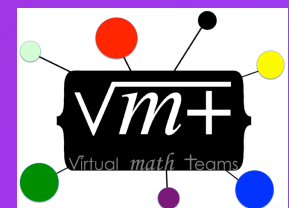
$$\sum_{i=1}^n 4i = 4n(n+1) + (n+1)^2$$

of sticks
 $(n^2 + (n-1)^2) \cdot 2 + n^2 \cdot 3 - 2$
of squares
 $n^2 + (n-1)^2$

Chat messages:
bwang@ 1/18/06 8:25:28 PM
IDT: let's check
bwang@ 1/18/06 8:25:55 PM
IDT: yes
bwang@ 1/18/06 8:26:00 PM
IDT: it actually is
Azmx 1/18/06 8:26:02 PM
IDT: we got it!
bwang@ 1/18/06 8:26:02 PM
IDT: omg
Azmx 1/18/06 8:26:04 PM
IDT: yay!
bwang@ 1/18/06 8:26:08 PM
IDT: I think we got it!!!!!!
Azmx 1/18/06 8:26:12 PM
IDT: WE DID IT!!!!
bwang@ 1/18/06 8:26:12 PM
IDT: and it is so simple
Azmx 1/18/06 8:26:14 PM
IDT: YAY!!!!
Azmx 1/18/06 8:26:16 PM
IDT: know
bwang@ 1/18/06 8:26:17 PM
IDT: lol
Azmx 1/18/06 8:26:18 PM
IDT: lol



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Move 1. Open the topic

Bwang: i think we are very close to

Azrx: We can solve on that topic.

Move 2. Decide to start

Bwang: well do you want to solve

Azrx: Alright.

Move 3. Pick an approach

Azrx: How do you want to approach

Bwang: 1st level have 1×4 ... 4th level

Move 4. Identify the pattern

Azrx: So it's a pattern of $+2$ s?

Bwang: yes

Move 5. Seek the equation

Bwang: what is it

Azrx: n^2 ... or $(n/2)^2$

Move 6. Negotiate the solution

Azrx: its n^2

Bwang: so that's wrong

Move 7. Check cases

Azrx: would be $4n^2$

Bwang: it actually is

Move 8. Celebrate the solution

Bwang: i think we got it!!!!!!!!!!!!

Azrx: WE DID IT!!!!!!!!

Move 9. Present a formal solution

Azrx: So you're putting it in the

Bwang: yes

Move 10. Close

Azrx: we should keep in touch

Bwang: yeah

- Longer sequence of interactions (adjacency pairs)
- Persistent co-attention to problem-solving process
- Shared understanding of problem & solution
- Community contributes problem & standards of math discourse
- Individual contributes computations, math content, candidate solutions
- Group organizes, negotiates, accepts

Segments of a hexagon array

are we still only talking regular ones?

137 5/16/06 7:31:22 PM EDT: About

qwertyuiop 5/16/06 7:31:24 PM EDT: side length 1 = 1, side length 2 = 3, side length 3 = 6...

137 5/16/06 7:32:50 PM EDT: Shouldn't side length 2 be fore?

137 5/16/06 7:32:53 PM EDT: "four

qwertyuiop 5/16/06 7:33:10 PM EDT: I count 3.

137 5/16/06 7:33:25 PM EDT: Oh. Sry.

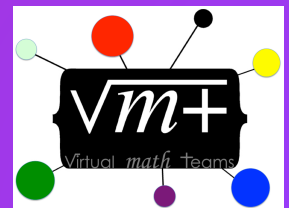
qwertyuiop 5/16/06 7:33:30 PM EDT: It's this triangle.

Message:

137, qwertyuiop are typing



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705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i <u>don't</u> really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance

- Being-there-together at the math object as seen by the group
- Wait!!! The group cannot continue until everyone sees the same
- The group has a shared understanding from the same perspective
- The shared view is locally created and repaired constantly
- The group develops group practices to see the same: deictic pointing, highlighting with colors, lines, arrows, names,
- Math: "hexagon"; group: inscription; individual: understanding

Inscribed polygons

Take turns dragging vertex A of Quadrilateral ABDC and vertex E of Quadrilateral EFGH.

Chat about dependencies you notice and what you wonder about this figure.

Construct a Quadrilateral inscribed in a Quadrilateral that behaves the same as this one.
Chat about how you are constructing and why.

Note that the Compass tool is available by pulling it down from the Circle tool in the tool bar.

Take Control | nobody has control | Polygon

Current users:

Chat (0):

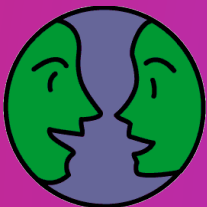
make the sides equal because the sides are their radius

fruitloops 3/4/13 4:02:39 PM
EST: point m is like point e because it moves around

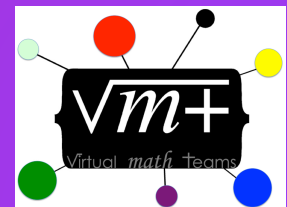
fruitloops 3/4/13 4:02:48 PM
EST: and its the same color

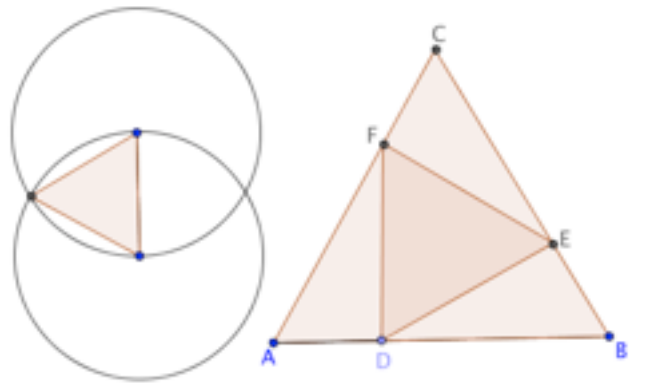
fruitloops 3/4/13 4:04:14 PM
EST: good!!

fruitloops 3/4/13 4:04:40 PM
EST: now hide the circles



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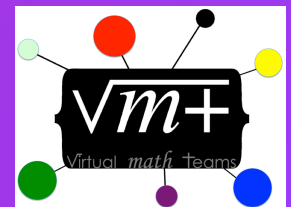
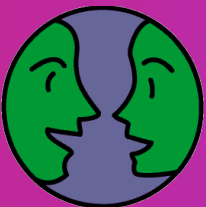


➤ **Community:** Euclid's 1st proposition (construct equilateral triangle), problem of inscribed triangles, definitions of regular polygons.

➤ **Individual:** perception of equal lengths, coordinated movements, explorative dragging, memory of similar problem solutions

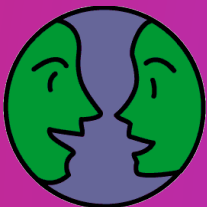
➤ **Small group:** group practices of taking turns, dragging, coloring, naming, discussing

➤ **Group cognition:** shared attention, collaborative discourse, joint solution

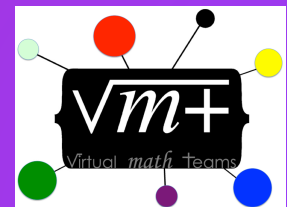


Further Reading

- [Stahl, G. \(2012\). Traversing planes of learning. *ijCSCL*. 7\(4\), 467-473.](#)
- [Stahl, G. \(2013\). Learning across levels. *ijCSCL*. 8\(1\), 1-12.](#)
- [Stahl, G. \(2013\). Transactive discourse in CSCL. *ijCSCL*. 8\(2\).](#)
- [Stahl, G. \(2013\). *Translating Euclid: Creating a human-centered mathematics*: Morgan & Claypool Publishers. Web: <http://gerrystahl.net/elibrary/euclid>.](#)
- [Stahl, G. \(2013\). Workshop presentation: The integration of cognitive levels. Presented at CSCL 2013. Web: <http://GerryStahl.net/pub/cscl2013levels.pdf>.](#)



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The Virtual Math Teams Trilogy

Group Cognition (2006)

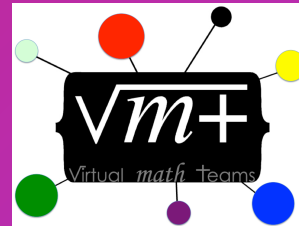


Computer Support for Building Collaborative Knowledge

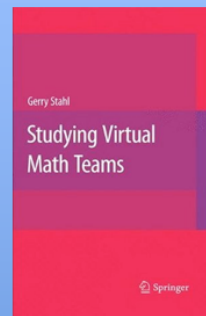
MIT Press, 510 pages
Available for Kindle

The theory of group cognition emerges from several studies of CSCL and CSCW technologies. Analysis of interaction. Theory of CSCL.

www.GerryStahl.net/elibrary/gc



Studying Virtual Math Teams (2009)

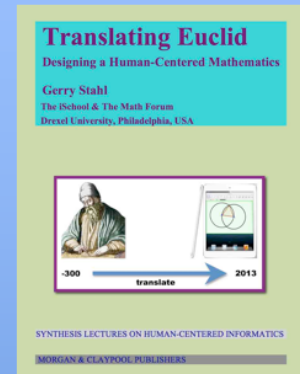


Springer Press, 626 pages
CSCL Book Series, paperback

Studies of the VMT Project technology, pedagogy, analysis, theory by team members and international collaborators

www.GerryStahl.net/elibrary/svmt

Translating Euclid (2013)



Creating a Human-Centered Mathematics

Morgan Claypool Publishers,
325 pages, e-book & paperback

Latest results of this design-based CSCL research from many perspectives.

www.GerryStahl.net/elibrary/euclid