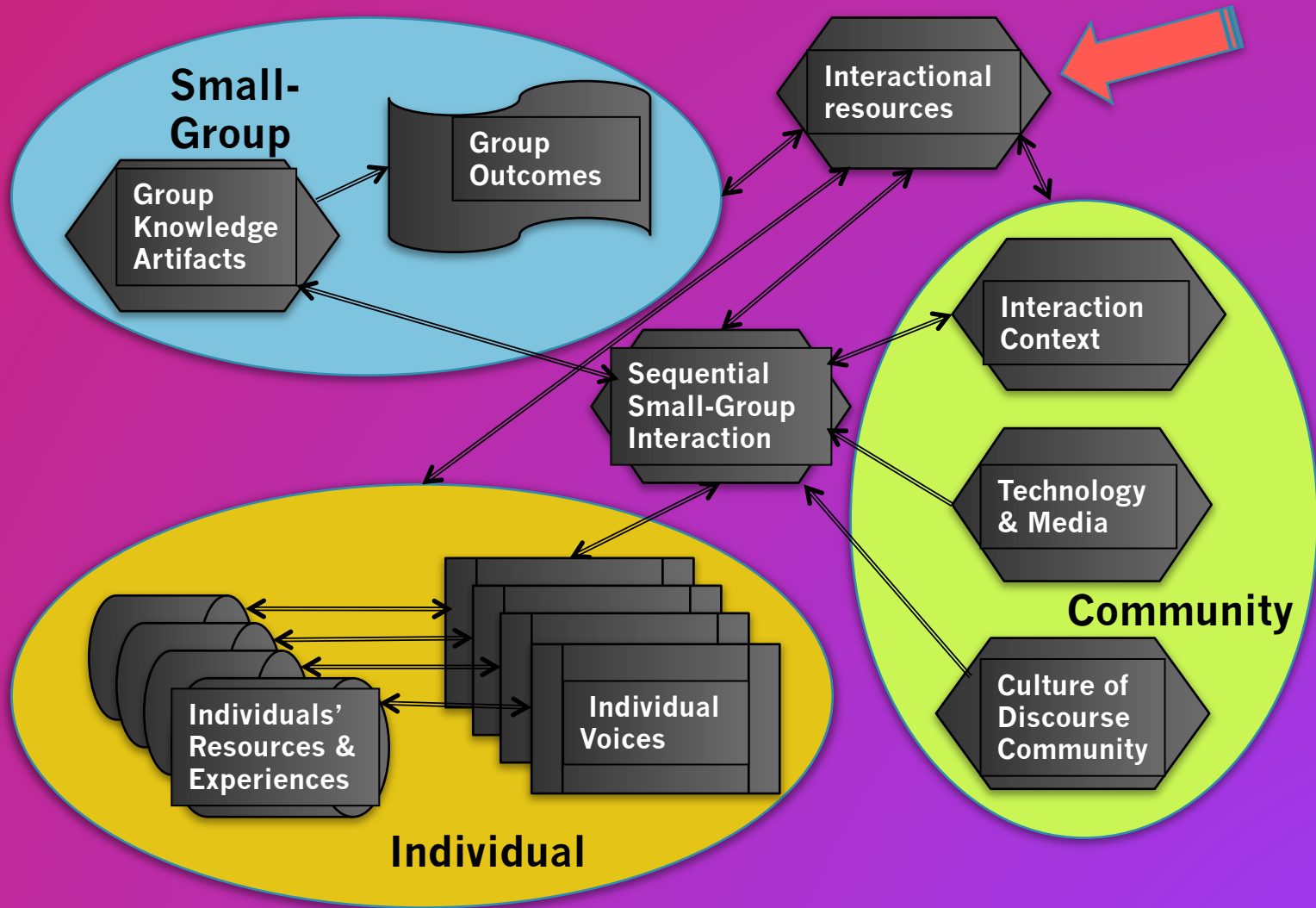


Resources for Connecting Levels of Learning

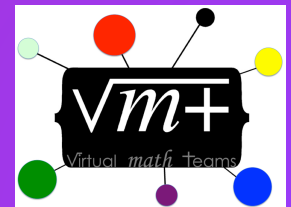
Gerry Stahl
Diler Öner





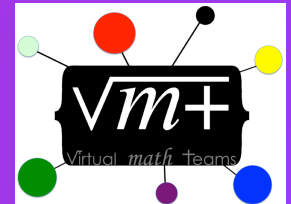
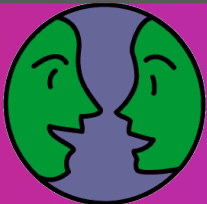
Levels of analysis connected by interactional resources

- In a series of sessions, a particular math topic connects group work to traditional mathematical content and elicits individual contributions; it guides & structures both small-group interaction and the group's trajectory of work
- Interactional resource = problem of inscribed triangles
- Individual level = 2, 3 or 4 participants
- Small-group level = dyad or triad or quad
- Community level = school math practices and geometry content – introduced in the classroom by the teacher and in the culture by enculturation

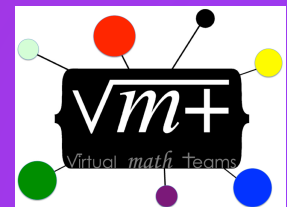
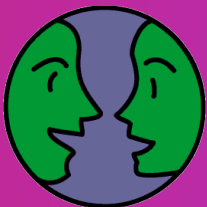


Öner (2013) contrasted social/collaborative/relational resources with content-related resources:

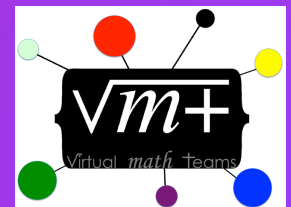
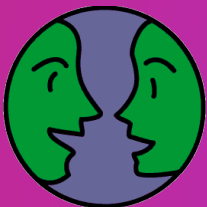
- Text chat versus shared-whiteboard graphics:
[Çakir, Zemel & Stahl, 2009](#)
- Building a joint problem space (JPS) versus solving a problem:
[Roschelle & Teasley, 1995](#)
- A relational space versus a content space: [Barron, 2000](#)
- Diachronic content versus temporal dimensions of the JPS:
[Sarmiento & Stahl, 2008](#)
- Project discourse versus mathematical discourse: [Evans et al., 2011](#)
- Spatio-graphical observation (SG) versus technical reflection (T):
[Laborde, 2004](#)



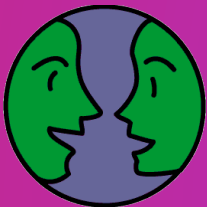
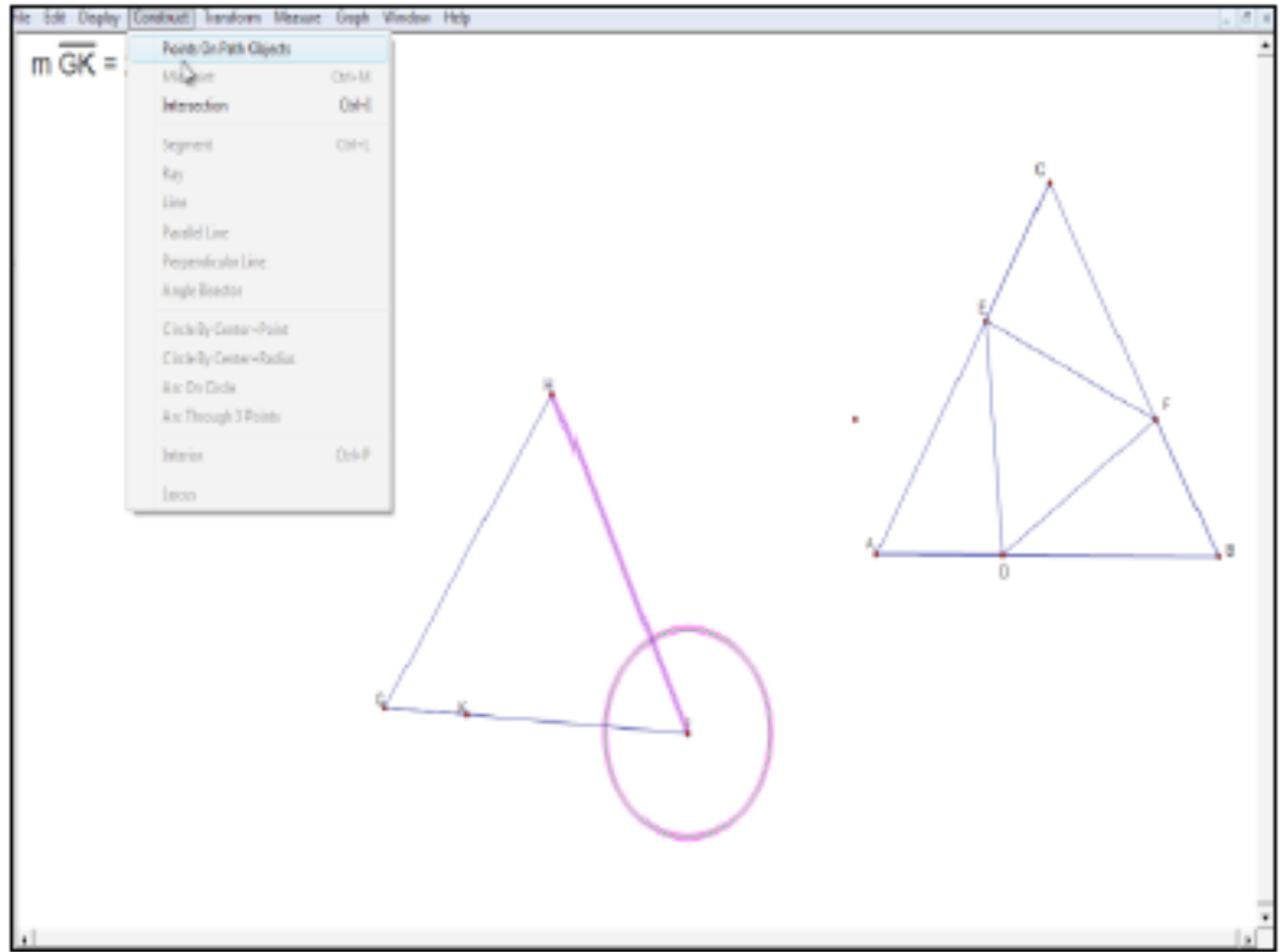
- Öner's (2013) analysis focuses on small-group processes (discourse, relationships, observation, shared attention) vs. math practices (defining problem, solution steps, solution path)
- Stahl (2013) construes the math problem as an integrating resource:
- dragging to discover vs. constructing to create dependencies



- In 4 experiments with a variety of groups, all successful groups use dragging to explore for constraints, exploratory construction to search for solution, design of dependencies;
- The inscribed-triangles problem is a resource that guides the group processes of meaning making and those of math solving



➤ Öner (2013) : F2F with Geometer's Sketchpad and grad students



➤ VMT with GeoGebra with researchers

The screenshot displays the GeoGebra interface with a complex geometric construction. The construction features two circles at the bottom, centered at points A and B. A horizontal line segment AB is labeled 'a'. A point G lies on the line segment AB. A point D is located above the line AB. A point C is located above D. A point J is located on the right circle. A point K is located on the line segment AB. A point L is located on the line segment AD. A point M is located on the line segment CD. A point N is located on the line segment BC. A point P is located on the line segment AC. A point Q is located on the line segment AB. A point R is located on the line segment AB. A point S is located on the line segment AB. A point T is located on the line segment AB. A point U is located on the line segment AB. A point V is located on the line segment AB. A point W is located on the line segment AB. A point X is located on the line segment AB. A point Y is located on the line segment AB. A point Z is located on the line segment AB. The construction is overlaid on a grid. The interface includes a menu bar (File, Edit, View, Perspectives, Options, Tools, Window, Help), a toolbar with various geometric tools, and a status bar at the bottom with buttons for 'Refresh View', 'Take Control', 'History', and 'nobody has control'. On the right side, there is a chat window with a list of messages.

Chat (0)

4/10/12 3:01:40 PM EDT: that's we can come back to that if you want to explain what you did

4/10/12 3:02:26 PM EDT: loretta, did you create A and B to have equal radii>

4/10/12 3:02:27 PM EDT: ?

4/10/12 3:02:31 PM EDT: I abandoned the center, and worked with the lengths of the sides

4/10/12 3:02:57 PM EDT: used the compass tool to measure the distance from D to C

4/10/12 3:03:08 PM EDT: and then found that distance from each of the other vertices

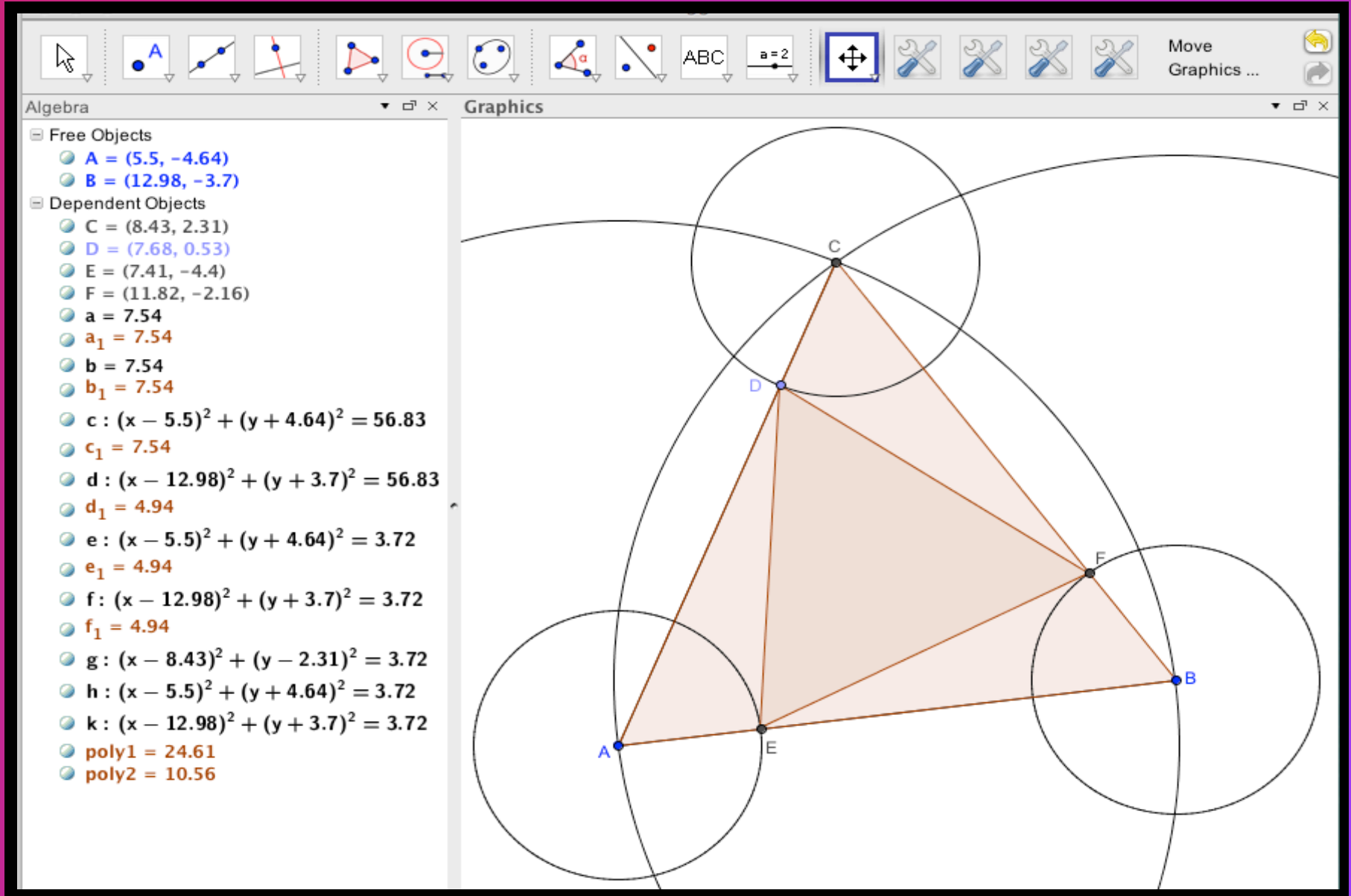
4/10/12 3:03:15 PM EDT: Are you ready to summarize your response to the original questions?

4/10/12 3:03:24 PM EDT: using the fact that all equilateral triangles are similar

4/10/12 3:03:30 PM EDT: questions?

Message:

➤ VMT with math teachers



➤ VMT triangles and squares with middle school students

The screenshot shows a geometry software interface with a menu bar (File, Edit, View, Options, Tools, Window, Help) and a toolbar. The toolbar includes icons for various geometric tools, with the 'Move Graphics View' tool highlighted. The main workspace contains a yellow instruction box with the following text:

Take turns dragging vertex A of Quadrilateral ABDC and vertex E of Quadrilateral EFGH.

Chat about dependencies you notice and what you wonder about this figure.

Construct a Quadrilateral inscribed in a Quadrilateral that behaves the same as this one.

Chat about how you are constructing and why.

Note that the Compass tool is available by pulling it down from the Circle tool in the tool bar.

Below the text are two diagrams. The left diagram shows a square with vertices labeled B, F, A, G, D, H, C. The right diagram shows a similar construction with circles around the vertices. At the bottom of the software window, there is a status bar with a 'Take Control' button, the text 'nobody has control', and a 'Polygon' tool icon.

On the right side of the image, there is a chat window titled 'Current users:' and 'Chat (0)'. The chat contains several messages from a user named 'fruitloops' with timestamps and EST. The messages are:

- make the sides equal because the sides are the radius
- *****
- fruitloops 3/4/13 4:02:39 PM EST: point m is like point e because it moves around
- ***
- fruitloops 3/4/13 4:02:48 PM EST: and its the same color
- *****
- fruitloops 3/4/13 4:04:14 PM EST: good!!
- *****
- fruitloops 3/4/13 4:04:40 PM EST: now hide the circles

Below the chat is a 'Message:' input field.

➤ VMT Topic with triangles, squares, hexagon polygon generalization

Take turns dragging vertex A of Triangle ABC and vertex D of Triangle DEF.

Chat about dependencies you notice and what you wonder about this figure.

Construct a triangle inscribed in a triangle that behaves the same as this one.

Chat about how you are constructing and why.

It might be helpful to look at the other tabs for this Topic and think about them together.

Take turns dragging vertex A of Quadrilateral ABCD and vertex E of Quadrilateral EFGH.

Chat about dependencies you notice and what you wonder about this figure.

Construct a Quadrilateral inscribed in a Quadrilateral that behaves the same as this one.

Chat about how you are constructing and why.

Note that the Compass tool is available by pulling it down from the Circle tool in the tool bar.

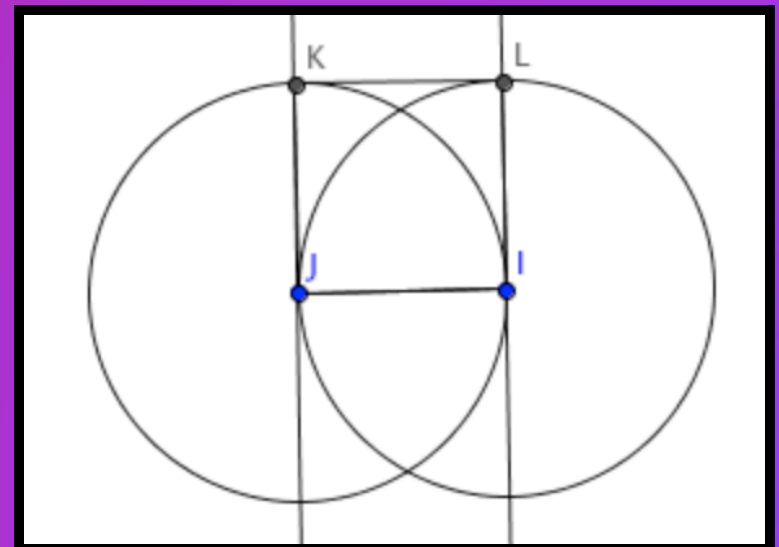
Take turns dragging vertex A of Hexagon ABCDEF and vertex G of Hexagon GHIJKL.

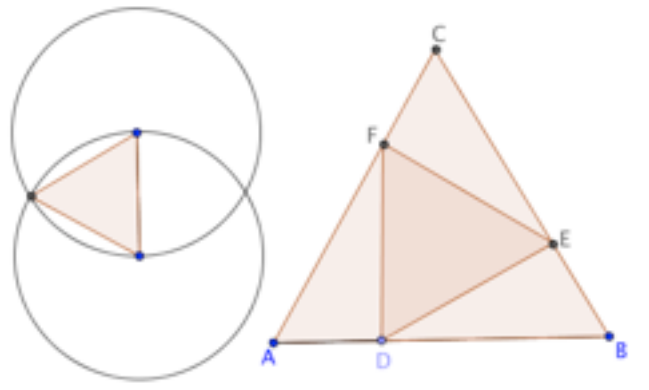
Chat about dependencies you notice and what you wonder about this figure.

Construct a Hexagon inscribed in a Hexagon that behaves the same as this one.

Notice that you can use the Regular Polygon tool. Explore how it works. Chat about how you are constructing and why.

Can you make a conjecture about inscribing regular N-sided polygons? Can you prove (or disprove) your conjecture?



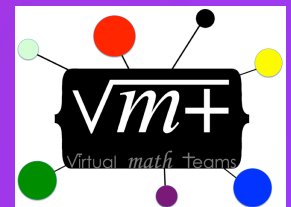
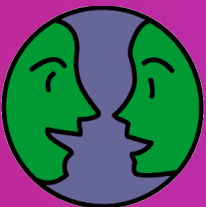


➤ Community: Euclid's 1st proposition (construct equilateral triangle), problem of inscribed triangles, definitions of regular polygons.

➤ Individual: perception of equal lengths, coordinated movements, explorative dragging, memory of similar problem solutions

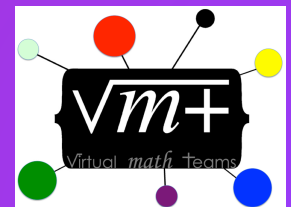
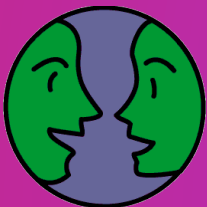
➤ Small group: group practices of taking turns, dragging, coloring, naming, discussing

➤ Group cognition: shared attention, collaborative discourse, joint solution



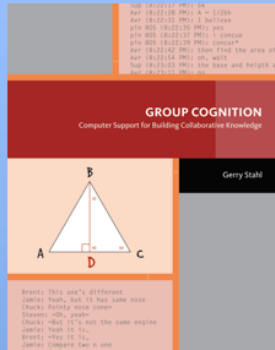
Further Reading

- Öner, D. (2013). Analyzing group coordination when solving geometry problems with dynamic geometry software. *ijCSCL*. 8(1).
- Stahl, G. (2012). Traversing planes of learning. *ijCSCL*. 7(4).
- Stahl, G. (2013). Learning across levels. *ijCSCL*. 8(1).
- Stahl, G. (2013). Transactive discourse in CSCL. *ijCSCL*. 8(2).
- Stahl, G. (2013). *Translating Euclid: Creating a human-centered mathematics*: Morgan & Claypool Publishers. Web: <http://gerrystahl.net/elibrary/euclid>.



The Virtual Math Teams Trilogy

Group Cognition (2006)

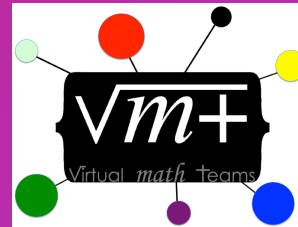


Computer Support for Building Collaborative Knowledge

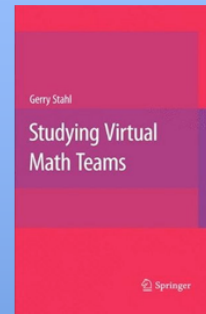
MIT Press, 510 pages
Available for Kindle

The theory of group cognition emerges from several studies of CSCL and CSCW technologies. Analysis of interaction. Theory of CSCL.

www.GerryStahl.net/elibrary/gc



Studying Virtual Math Teams (2009)

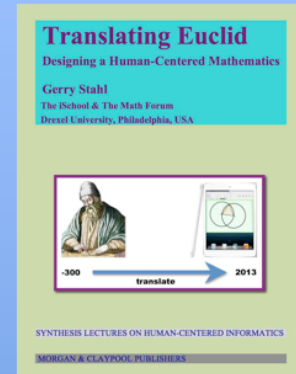


Springer Press, 626 pages
CSCL Book Series, paperback

Studies of the VMT Project technology, pedagogy, analysis, theory by team members and international collaborators

www.GerryStahl.net/elibrary/svmt

Translating Euclid (2013)



Creating a Human-Centered Mathematics

Morgan Claypool Publishers,
325 pages, e-book & paperback

Latest results of this design-based CSCL research from many perspectives.

www.GerryStahl.net/elibrary/euclid