# Group Cognition Displayed

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**Abstract:** Particularly in contexts of online group collaboration, learning and becoming can be displayed by the participants in ways that render it observable to researchers. In this study, a team of three students displays its growing mastery of dynamic-geometry techniques and discourse. In one session, the team struggles to solve a challenging problem and succeeds as a group. In the next session, it displays its newly acquired practices by immediately applying them in an analogous problem setting. The team displays its shared understanding; the analysis of this interaction data provides a clear example of group cognition and group learning. Further, the team displays its capacity to plan, implement and assess collaborative actions in a temporally unfolding shared situation, illustrating the characteristics of group agency.

## Observing Learning and Becoming at the Group Unit of Analysis

Learning is often conceived as a change in propositional knowledge possessed by an individual student (Thorndike, 1914). Opening up an alternative to this view, Vygotsky argued that students could accomplish epistemic tasks in small groups before they could accomplish the same tasks individually—and that much individual learning actually resulted from the earlier group interactions (Vygotsky, 1930/1978), rather than the group being reducible to its members as already formed individual minds. He conceived the group interactions as mediated by artifacts, such as representational images and communication media. More recently, educational theorists have argued that student processes of becoming mathematicians or scientists, for instance, are largely a matter of mastering the linguistic practices of the field (Lemke, 1993; Sfard, 2008).

Views of learning focused on individual minds require methodologies that test individual changes over time and interpret them in terms of some theory of mental processes that are not directly observable such as mental models, mental representations, cognitive change, cognitive convergence, cognitive conflict, etc. In contrast, a view of learning focused on group interaction can hope to observe processes of group cognition more directly. A reason for this is that in order for several students to work together effectively, they must display to each other what the group is planning, recalling, doing, concluding and accomplishing. These displays take place in the physical world through speech, gesture and action; they are in principle visible to researchers as well as to the participants.

In practical terms, it is difficult for researchers to capture enough of what is taking place in group interactions to be able to reliably understand what is going on as well as the participants do. Capturing face-to-face interaction in an authentic classroom involves many difficulties, including multiple video angles, lighting issues, multiple audio recordings, transcription and coordination of all the data (Suchman & Jordan, 1990). In this paper, we present data that was automatically captured during an online chat involving three students. All of their communication and action that was shared within the group is available to us as analysts in exactly the same format as it appeared for the students, as well as in automatically generated logs. So none of the issues of interpretation and partiality of the data are present here the way they are in face-to-face settings. In particular, all the representational images and language used by the group are available in detail to the researchers. We can use methods of interaction analysis or conversation analysis (Jordan & Henderson, 1995; Schegloff, 1990), adapted to our online math-education setting (Zemel & Çakir, 2009).

For some time, we have proposed the idea of focusing on the small group as the unit of analysis and foregoing any reliance on theories of mental processes in favor of observing the visible interactions (Stahl, 2006). We spent a decade developing an online environment to support collaborative learning of mathematics (Stahl, 2009) and instrumenting the technology to capture group interaction (see Figure 1). Our research and theory now distinguish distinct learning processes at the individual, small-group and community units of analysis. Although we recognize that these processes are inextricably intertwined in reality, we focus methodologically in this paper on the group unit of analysis, which is where individual learning, group becoming and community practices are often most visibly displayed.

During a recent cycle in our design-based research iterations (Stahl, 2013b), we captured sessions of groups of students learning about dynamic geometry. To illustrate the theme of "learning and becoming in practice," we have here selected a sequence of two hour-long sessions in which three girls display their enactment of mathematical practices as a group. (The complete dataset for this group is available at: www.GerryStahl.net/vmt/icls2014data.) In this data, one can observe particularly clearly an extended group interaction in which the group succeeds in accomplishing two related curricular tasks. In the first task, after a lengthy effort, the group finally performs a practice that is important for becoming proficient in dynamic geometry. In the second task, they display that they can perform that practice in the context of a new task; it has become part of their group repertoire. Furthermore, each of the three students participates in the solving of both tasks. In the second task each of the students displays that they have mastered and understood that practice—at least when working within their small group. The data includes both actions by the students in manipulating the shared graphical representations of geometric objects and discussion by the students in the text-chat communication medium. Thus, they display both enhanced geometric construction practices and deepened conceptual reflection on the dependencies at the heart of the construction.

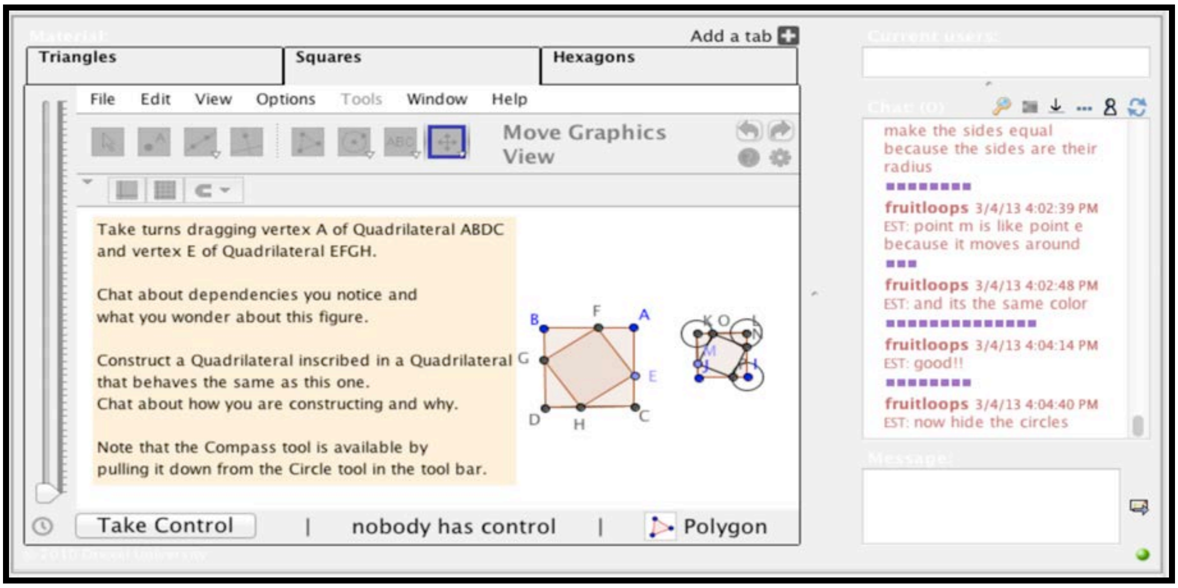

Figure 1: The interface of the collaboration environment.

## The Collaborative-Learning Setting

Our research aims at transforming school mathematics education by supporting collaborative-learning approaches with computational technologies. In our recent work, we have extended our online collaboration environment to support dynamic geometry. Our previous system combined a graphical workspace with a text-chat facility to allow small groups of students to share drawings of mathematical representations while chatting about challenging math problems. Our extension provides a multi-user version of GeoGebra as an option for the shared workspaces. GeoGebra is a popular software implementation of dynamic mathematics, including dynamic geometry (www.GeoGebra.org). We are exploring how to use our collaborative version of dynamic geometry to transform the way that geometry can be taught in schools. That involves guiding teachers and students to work effectively in online collaborative groups and to take advantage of the core functions of dynamic geometry: dynamic dragging, dynamic construction, dynamic dependencies and custom tools.

In Spring 2013, teachers who had taken our professional development course in collaborative mathematics organized "virtual math teams" of students to use our technology and curriculum for eight hour-long sessions after school. The team of three students reported on here chose login names: Fruitloops, Cornflakes and Cheerios. They are middle-school students in 8th grade (about 14 years old). They are currently taking an algebra course and have had very little previous exposure to geometry—only about a week during their previous year's pre-algebra math course.

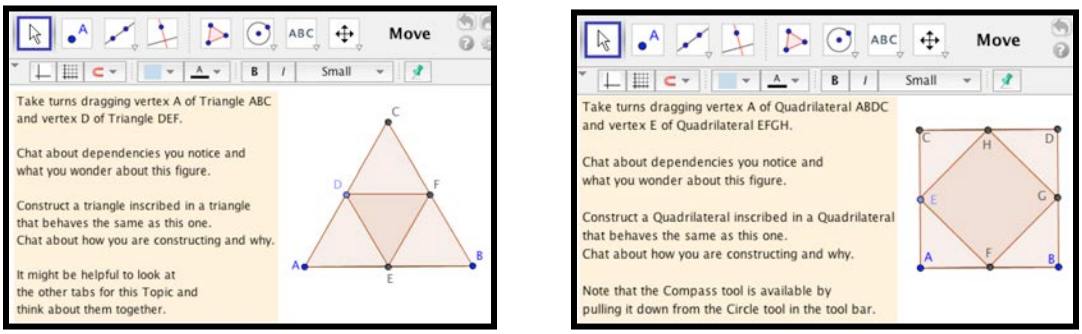

Figure 2: The tabs of Topic 5 and 6: inscribed triangles and squares.

In the following, we review the work of this team on Topics 5 and 6, related problems of inscribed equilateral triangles and inscribed squares. The curricular topics were presented to the team in GeoGebra tabs, as shown in **Figure 2**. The tabs are shared workspaces for teams of students to collaboratively explore and co-

construct dynamic-geometry figures. The online environment includes a text-chat facility to support planning, coordination and reflection. The instructions in the tabs provide the only curricular guidance for the team. This particular set of tabs presents a challenging task that involves both discovery through dragging dynamic-geometric objects and creation through constructing new objects with the same dependencies. We have argued that relations of dependency among geometric objects are central to an understanding of dynamic geometry (Stahl, 2013b). Methods of construction and strategies for proof can be grounded in the design of dependency relationships. Our curriculum is structured to guide student teams toward a gradually deepening understanding, use and articulation of dependencies (Stahl, 2013a).

## Learning to Co-Construct Dependencies

The team starts its fifth online session by beginning to follow the instructions in the opening tab: "Take turns dragging vertex A of Triangle ABC and vertex D of Triangle DEF" (see Log 1).

Log 1: The team explores the triangles.

| Line | Post Time | User | Message |
|---|---|---|---|
| 3 | 15:11:53 | fruitloops | heyyyyyyyyyyyyyy |
| 4 | 15:13:05 | cornflakes | hi |
| 5 | 15:13:30 | cornflakes | i will go first |
| 7 | 15:18:09 | fruitloops | when i move vertex a the whole triangle of abc moves |
| 8 | 15:18:43 | cornflakes | when i moved point c the triangle stayed the same and either increased or decreased in size, butit was equivalent to the original triangle |
| 9 | 15:18:52 | fruitloops | but when i tryed to move vertex d, it couldnt go behond triangle abc |
| 10 | 15:18:54 | cheerios | does the inner triangle change its shape when u move vertex a |
| 11 | 15:19:34 | fruitloops | try moving it... |
| 12 | 15:20:38 | cheerios | nvm it doesnt |

….

| | | | |
|---|---|---|---|
| 24 | 15:26:41 | cornflakes | ecf arent moving |
| 25 | 15:27:00 | fruitloops | point c e and f cant move |
| 26 | 15:27:52 | cornflakes | because they are sconstrained or restricted |
| 27 | 15:27:53 | fruitloops | point d can only make point f and g move but nothing else |
| 28 | 15:28:29 | cornflakes | yea |
| 29 | 15:28:50 | fruitloops | okay want to try to conssrtuct it? |
| 30 | 15:29:01 | cheerios | yup |
| 31 | 15:29:07 | cornflakes | sure |

After greeting each other in the chat room, the students start to follow the instructions by taking turns exploring the constraints of the figure in the GeoGebra tab. The students drag points A and D. They quickly see that the interior triangle DEF is confined to stay inside triangle ABC and that both triangles retain their equilateral shape when dragged. The students note that points C, E and F are "constrained or restricted," so they are not free to be dragged. They also note that dragging point D will move points E and F. This will turn out to be a key dependency, although the students do not yet discuss it as such. They are now ready to begin the construction task. Fruitloops begins the construction with a segment GH and two circles with radius GH centered on points G and H, respectively. Fruitloops gets stuck at line 32 of Log 2 and Cheerios takes over, drawing the triangle connecting point I at the intersection of the circles with points G and H. Fruitloops wants to remove the circles, but seems to understand in line 34 that they cannot delete the circles without destroying the equilateral triangle. Cornflakes hides the circles by changing their properties, without deleting them.

Some of the group's interaction is displayed in the logs, such as those excerpted in this paper. Due to space considerations, the excerpts here have been filtered to only show chat postings. The full spreadsheet logs include other actions in the environment and supplementary metadata. In addition, there is a Replayer, which allows anyone (student, teacher or researcher) to replay and/or step through an online session to see precisely what the students saw. (The Replayer uses the same technology and data as the original session.) The use of the Replayer, in coordination with the chat logs, is essential for an analysis of the group interaction. Using the Replayer, we can see how the three students take turns co-constructing the triangles. We can see and document in as much detail as desired how accomplishments like exploring the given figure and constructing the students' triangle GHI were collaborative achievements of the team, in which the three students each built on each other's contributions and succeeded in tasks that no one of them could have done alone.

Log 2: The team constructs the first triangle.

| | | | |
|---|---|---|---|
| 32 | 15:30:26 | fruitloops | what should i do next? |
| 33 | 15:32:22 | fruitloops | so how do we get rid of the circles then? |
| 34 | 15:32:54 | fruitloops | if we cant delete them, what do we do? |
| 35 | 15:34:37 | fruitloops | so i think triangle igh is like triangle abc |
| 36 | 15:36:30 | fruitloops | now that the first triangle is good, what should we do? |
| 37 | 15:47:48 | fruitloops | d moves but f and e dont |
| 38 | 15:48:04 | fruitloops | so both f and e are dependent on d |
| 39 | 15:48:18 | cheerios | so what does that mean |
| 40 | 15:48:37 | fruitloops | so if we make a line and use the circle thing, maybe we can make it somehow |
| 41 | 15:48:15 | cornflakes | right |
| 42 | 15:49:09 | cheerios | lets try |
| 43 | 15:49:29 | cheerios | and we will jsut figure it out .. by making the line thing |
| 44 | 15:49:14 | fruitloops | how? |
| 45 | 15:50:18 | cheerios | f and e are restricted |
| 46 | 15:51:19 | fruitloops | we can make their d point by just using a point tool on our triangle to make point j |
| 65 | 16:11:35 | fruitloops | so what ere you dong now? |

In line 35 of Log 2, Fruitloops suggests that they have succeeded in replicating the outer triangle. Then in line 38, Fruitloops makes explicit that their previous observation about movement of point D affecting points E and F implies a dependency that may be relevant to their construction task. Cheerios and Cornflakes express interest in this line of argument. They all agree to proceed with trying constructions in order to figure out just what needs to be done. As with designing the exterior triangle, the results of dragging provide an impetus for construction of the interior inscribed triangle, but not a blueprint. The team launches into a trial-and-error process, guided by some vague ideas of things to try.

The students begin their trial with the knowledge that point D is freer than points E and F, which are dependent on D (line 38). Therefore, they decide to start by constructing their equivalent of point D on a side of their exterior triangle (line 46). Note the gap of about 12 minutes from line 46 to the next chat posting. This was a period of intense experimentation by the three students. Unfortunately, they do not chat about what they were doing during this period. We have to look at a more detailed log and step through the Replayer slowly to observe what they were doing. There are actually 170 items in the detailed log and thousands of GeoGebra actions for that period. During most of this activity, the students make very little direct progress on their construction. They construct some lines, circles and points. They engage in considerable dragging: of the original figure, of their new triangle and of their experimental objects.

Finally, Cheerios provides the key analysis of the dependency: AD=BE=CF. The others immediately and simultaneously agree with this analysis. In Log 3, Cheerios goes on to project this dependency onto their construction.

Log 3: The team constructs the inscribed triangle.

| | | | |
|---|---|---|---|
| 66 | 16:18:30 | cheerios | as i was movign d segment da is the same distance as segment be |
| 67 | 16:18:52 | cheerios | and also cf |
| 68 | 16:19:41 | cheerios | our kg is the same as ad |
| 69 | 16:20:06 | cornflakes | agrreeed |
| 70 | 16:20:06 | fruitloops | i agree |
| 71 | 16:21:21 | cheerios | there should be a point on segment gh which is the same distance as kg and also between segment uh |
| 72 | 16:22:00 | cheerios | it should be ih not uh |
| 73 | 16:23:39 | cheerios | so i used the compass tool and measured kg and used point i as the center and created a circle |

Cheerios narrates line 73 while she uses the GeoGebra compass tool to measure the length from the point on the side of the exterior triangle to one of its vertices and to transfer that length to another side from another vertex. This is an important method for establishing dependencies in GeoGebra; it is both conceptually subtle and physically tricky to master. The students had just watched a video of that construction—using the compass tool to copy a length from one line onto another line—in class earlier that day in preparation for this topic, and had previously been introduced to it in Topic 2. Next, Cornflakes takes control of the construction, places a point where the compass intersects the side and then repeats the process with the compass to construct another point on the third side.

Fruitloops takes control and uses the polygon tool to construct a shaded interior triangle connecting the three points on the sides of the exterior triangle. She then conducts the drag test, dragging points on each of the new triangles to confirm that they remained equilateral and inscribed dynamically. Thus, all three group members not only agree with the action plan, but they also all participate in the construction. The team as a collaborative unit thereby accomplishes the solution of the problem in tab A. The team has learned to use the compass tool of GeoGebra to establish dependencies among segment lengths in a constructed figure. The team has also learned to recognize, test and confirm existing dependencies through the presence of compass circles, color-coding of points in GeoGebra and drag tests; it understands that dependencies can still be at work when constraining compass circles are invisible, as long as they have not been deleted. Team members have become more articulate about discussing dependencies.

At that point, the students had been working in the room for over an hour and had to leave quickly. They had hurriedly completed the construction of the inscribed triangles, but had not had a chance to discuss their accomplishment. Furthermore, they had not had any time to work on the other tabs. Three days later, the team reassembles for Session 6 in the same chat room to continue work on Topic 5.

## Establishing Group Practices

Back in the chat room, the team spends some time reflecting on how they constructed the inscribed triangles and several characteristics of the dependencies involved in that construction. Next, they turn to the similar task involving inscribed squares. They take turns dragging the points of the given figure to determine which vertices are free, partially constrained or entirely dependent. Then Fruitloops asks how they can construct a square (Log 4, line 127). They have constructed many triangles in previous sessions, but never a square.

Log 4: The team constructs the first square.

| | | | |
|---|---|---|---|
| 127 | 15:38:45 | fruitloops | how but how do we make the square? |
| 128 | 15:39:11 | fruitloops | like i know how to make the triangle but now the square |
| 129 | 15:39:11 | cheerios | a grid |
| 130 | 15:39:20 | cornflakes | olets start by cinstructing a regular square |
| 131 | 15:39:16 | cheerios | a grid |
| 132 | 15:39:47 | fruitloops | i think we should make perpendicular lines somehow |
| 133 | 15:39:58 | cheerios | use the perpindicular line tool |
| 134 | 15:43:21 | fruitloops | the first line segment would be like ab |
| 135 | 15:43:27 | cornflakes | yes |
| 138 | 15:51:24 | cheerios | how do u know ji is straight |
| 139 | 15:55:40 | fruitloops | i dont know what to do because the points arent the same color |
| 140 | 15:56:38 | fruitloops | now after you make the perpendicular lines try to make the circles\ |
| 141 | 15:57:48 | fruitloops | i think you need to know use the polygon tool and make the square |
| 142 | 15:58:50 | cheerios | i made a line segment which was if than i used the perpendicular line tool and made 2 lines on each side then used the compass tool and clicked on each point and then the center vertex was i and then made a another circle except the center vertex is j and connected all the points |

There is again considerable experimentation taking place in GeoGebra during Log 4. Note from the time stamps that this log spans 20 minutes. The three students take turns trying various approaches using the tools they are familiar with and gradually adding the perpendicular-line tool. They consider the definition of a square as having all right angles, so they first talk about using a grid and then construct perpendiculars. Eventually, Cheerios succeeds in constructing a dynamic square (see Figure 3), and describes the procedure in line 142. A more detailed analysis of the GeoGebra activity shows that although the successful construction was carried out by Cheerios, her process built on the actions and proposals of the team as a whole.

The student construction of the square is quite elegant. It closely mirrors or builds upon the construction of an equilateral triangle, which the students have mastered: it has a base side (segment IJ) and two circles of radius IJ centered on I and J (like Euclid's triangle construction). For the right-angle vertices at the ends of the base, perpendiculars are constructed at I and J. Because segments JK and IL are radii of the same circles as IJ, all three segments are constrained to be equal length (by the same reasoning as for the three sides of Euclid's equilateral triangle). This determines the four corners of a quadrilateral, IJKL, which is dynamically constrained to be a square. In the structure of their eventual construction and in their acceptance of this construction as satisfying the constraints of a dynamic square, the group displays that it has acquired a geometric practice of the mathematical community that dates back at least to (Euclid, 300 BCE/2002). Vygotsky might label this as “internalization” (at the unit of individual development), although here it is an enactment by the team (as a unit of epistemic analysis) and takes place within the small-group situation.

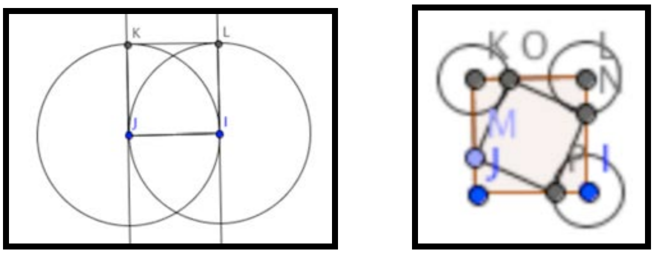


Figure 3: The team constructs a square and then inscribes another square.

As soon as the outer square is constructed, Fruitloops proposes to construct an inscribed inner square by following a procedure analogous to the procedure they used for inscribing the triangle (see Log 5). Note that line 143 by Fruitloops follows 20 seconds after Cheerios’ line 142 (Log 4). While she narrates, the team actually constructs the inscribed square and conducts the drag test on it.

Although the chat log (Log 5) is dominated by Fruitloops, analysis of the dynamic-geometry construction using the Replayer shows that the construction of the inscribed square is a team accomplishment.

Log 5: The team constructs the inscribed square.

| | | | |
|---|---|---|---|
| 143 | 15:59:10 | fruitloops | now we need to use the compass tool lilke we did in the triangles tab |
| 144 | 15:59:57 | fruitloops | because af is equal to ec and dh and bc |
| 145 | 16:01:15 | cheerios | then used to polygon tool and then hid the circles and lines |
| 146 | 16:01:07 | fruitloops | correct |
| 147 | 16:01:36 | fruitloops | and we used the circles to make the sides equal because the sides are their radius |
| 148 | 16:02:39 | fruitloops | point m is like point e because it moves around |
| 149 | 16:02:48 | fruitloops | and its the same color |
| 150 | 16:04:14 | fruitloops | good!! |

In line 143 at 15:59:10, Fruitloops states, “now we need to use the compass tool lilke we did in the triangles tab.” Note the use of the plural subject, “we,” referring to the team and proposing an action plan for the team—based on what the team did in the previous session. Fruitloops is “bridging” back to past team action as relevant to the current situation of the team (Sarmiento & Stahl, 2008). In line 144, she continues to draw the analogy between the line segments in the inscribed squares with those of the inscribed triangles. Cheerios picks up on this plan and creates point M at 16:02:27. Cheerios continues to create point O at 16:03:36, point N at 16:04:02 and point P at 16:04:16. Fruitloops immediately comments approvingly of this construction act (line 150).

While Cheerios does most of the construction of the inscribed squares, everyone on the team takes turns in control of the GeoGebra tools and contributes to the process, displaying in various ways that they are paying attention and supporting the effort. From 16:04:21 to 16:04:43, Cheerios constructs the figure, as shown in **Figure 2**. At 16:04:52, Cornflakes steps in and hides the circles made by the compass tool to define the lengths of the segments along the four sides as equal—just as Cheerios and Fruitloops had discussed in lines 145 and 147. Following this, both Cheerios and Fruitloops perform the drag test to check that their new figure preserves its dependencies of inscribed vertices and equal sides. Cheerios drags point M starting at about 16:05:30 and then Fruitloops drags points I and M starting at about 16:08:00. In line 158 of Log 6, we see Cornflakes responding approvingly to the result of the drag tests and to Fruitloops’ conclusion about the dependencies introduced by the compass tool’s circles. The team expresses its general agreement with their accomplishment, displaying their shared understanding of their group work.

Log 6: The team displays its shared understanding.

| | | | |
|---|---|---|---|
| 155 | 16:09:42 | cheerios | so just plotted a random point on line segment jk and then used the compass tool and clicked on point m and j ( radius) and then clicked k to be the center and then plotted the point where line segment kl intersect with the circle and repeated these steps on the other sides |
| 156 | 16:09:18 | fruitloops | i think points o, n, and p are dark because they weere made using the original circles |
| 157 | 16:08:23 | cornflakes | yess |

| | | | |
|---|---|---|---|
| 158 | 16:09:32 | cornflakes | yea i agreeee |
| 159 | 16:10:27 | cheerios | the distance between m and j is the same between ok and ln and pi |
| 160 | 16:10:02 | fruitloops | yeah i saw and i understand |
| 161 | 16:10:44 | fruitloops | all the radii are the same so the distances from ko,ln, and ip and jm are the same |
| 162 | 16:10:18 | cornflakes | same |
| 163 | 16:10:57 | cornflakes | yup i agrree |
| 164 | 16:11:01 | cheerios | yes |
| 165 | 16:11:03 | fruitloops | should we move on? |
| 166 | 16:11:33 | fruitloops | actually i dont think we have enough time |
| 167 | 16:12:02 | cheerios | yeah so next time |

The chat in Log 6 is confusing because some of the postings overlapped in their typing, so that some lines respond to postings other than the immediately preceding one. We have to use the full log to reconstruct the threading of responses. Cheerios took two and a half minutes (from 16:07:10 to 16:09:42) to type up line 155, carefully documenting her construction steps. In line 159 (16:09:45 to 16:10:27), she continued this description, explaining that the construction created equal line segments. Strikingly, Fruitloops typed an almost identical posting, in line 161 from 16:10:03 to 16:10:45. This displays an impressive degree of alignment. Cornflakes immediately (16:10:52 to 16:10:57) posts line 163, displaying her agreement as well.

Intertwined with the preceding thread are several others. First, Cornflakes' "yess" in line 157 is probably an aligning response to the antecedent drag testing by Cheerios and Fruitloops. Second, Fruitloops responds to line 155 in line160, stating that she saw and understood the construction steps that Cheerios now describes. Cornflakes then joins in by saying "same" in line 162, indicating that she too saw and understood the construction sequence. In addition, Cornflakes agrees in line 158 to Fruitloops' claim about dependencies in line 156 and Cheerios agrees with Fruitloops statement in line 161, which was so similar to Cheerios' own statement in line 159. The need to involve threading relationships and to understand postings as responses to preceding events or as elicitations of future events is indicative of analysis at the group-cognitive unit.

The excerpt in Log 6 displays a high level of agreement among the three participants. Often the actual mathematical problem solving or geometric construction is done jointly by the team, with two or three of the participants taking turns doing the steps. However, even when only one person does the actions, the others are intimately involved in planning the moves, describing them or evaluating them. Each major action is discussed and the team agrees to its correctness before moving on to another task. Generally, each action by an individual is entirely embedded in the group context and situated in the team interaction. Geometry construction acts make sense in terms of team plans in the preceding chat and/or team reflections in the subsequent chat. Individual chat postings make sense as responses to preceding actions or comments. The work of the team can be analyzed in terms of the group interaction, team plans, team activities and team assessments of that work. All of the evidence to support analysis is displayed in the data of the team interaction, with no need to hypothesize non-observable mental processes in the individual students' minds. Of course, we assume that the students have human abilities to understand language, to interact collaboratively and to enact community practices. A full analysis of cognition requires analysis at multiple levels. However, it is possible to analyze team interaction at the group unit of analysis based on evidence displayed by the participants and captured in the session data. The three students in this study repeatedly displayed for each other (and indirectly for us as analysts) that their activity was a team effort. Through their repeated agreements and other group practices, they constituted their activity as such a team effort.

In these excerpts, the team has displayed its mastery of a dynamic-geometry *practice* for constructing inscribed polygons. Where it took much experimentation to figure out how to construct the inscribed triangles, when confronted with the problem of inscribed squares, the team immediately (once the exterior square was constructed) applied the practice they had developed with the inscribed triangles. The three students were in complete agreement about how to do this and were relatively articulate in describing the construction practice as a process of imposing the required dependencies.

## Group Cognition Displayed

The group practices developed by the team—such as using the compass tool to impose dependencies among segment lengths—are closely related to mathematical practices of the mathematics community—in some cases going back 2,500 years to the early geometers. This connection was facilitated by the curriculum, which consisted mainly of the instructions in the GeoGebra tabs of the chat rooms for the sequence of eight online sessions. This curriculum was designed to guide groups of students to experience basic relationships in dynamic geometry and its predecessor versions of Euclidean geometry (Stahl, 2013a).

The analysis of the session logs shows that the team of Cheerios, Fruitloops and Cornflakes displayed their gradual mastery of a variety of practices for online interaction, team collaboration, math discourse, geometric construction and problem solving. All of this is visible at the group unit of analysis as dimensions of group cognition.

Among the practices enacted by the team are those that constitute agency, such as planning, negotiating, choosing, acting and assessing one's actions. In their seminal paper on the nature of agency, Emirbayer and Mische (1998) conceptualize agency as a temporally embedded process of social engagement, informed by the past but also oriented toward the future and toward the present. Accordingly, we can say that the team displays group agency, the capacity to plan, carry out and reflect upon chosen actions (Charles & Shumar, 2009). In the student interactions during the online sessions, the team displays its learning and becoming in terms of practices of group agency.

Of course, one could also investigate the individual learning of these students (e.g., as measured on tests taken outside of the team context) or their shifting community participation (e.g., as displayed when these students report their online work to their teacher and classmates in their physical classroom). Such investigations would require different data and methods. Analyses of individual, group and community cognition could complement each other since the processes at the different units of analysis are essentially intertwined; they are united in reality and only distinguished for the sake of analysis. It is ironic that so much research is focused on the individual and community levels when it is group cognition that is displayed most visibly. We have tried to illustrate in this paper how to analyze displays of group cognition.

## References


Charles, E. S., & Shumar, W. (2009). Student and team agency in VMT. In G. Stahl (Ed.), *Studying virtual math teams.* (ch. 11, pp. 207-224). New York, NY: Springer. Web: http://GerryStahl.net/vmt/book/11.pdf. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_11.

Emirbayer, M., & Mische, A. (1998). What is agency? *American Journal of Sociology. 103*(4), 962-1023.

Euclid. (300 BCE/2002). *Euclid's elements* (T. L. Heath, Trans.). Santa Fe, NM: Green Lion Press.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences. 4*(1), 39-103. Web: http://lrs.ed.uiuc.edu/students/c-merkel/document4.HTM.

Lemke, J. L. (1993). *Talking science: Language, learning and values*. Norwood, NJ: Ablex.

Sarmiento, J., & Stahl, G. (2008). *Extending the joint problem space: Time and sequence as essential features of knowledge building.* Paper presented at the International Conference of the Learning Sciences (ICLS 2008). Utrecht, Netherlands. Web: http://GerryStahl.net/pub/icls2008johann.pdf.

Schegloff, E. A. (1990). On the organization of sequences as a source of 'coherence' in talk-in-interaction. In B. Dorval (Ed.), *Conversational organization and its development.* (pp. 51-77). Norwood, NJ: Ablex.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.

Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Web: http://GerryStahl.net/elibrary/gc.

Stahl, G. (2009). *Studying virtual math teams*. New York, NY: Springer. Web: http://GerryStahl.net/elibrary/svmt. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3.

Stahl, G. (2013a). *Topics in dynamic geometry for virtual math teams*. Web: http://GerryStahl.net/elibrary/topics/topics.pdf.

Stahl, G. (2013b). *Translating Euclid: Creating a human-centered mathematics* (e-book ed.): Morgan & Claypool Publishers. Web: http://gerrystahl.net/elibrary/euclid. Doi: 10.2200/S00492ED1V01Y201303HCI017.

Suchman, L. A., & Jordan, B. (1990). Interactional troubles in face-to-face survey interviews. *Journal of the American Statistical Association. 85*, 232-244.

Thorndike, E. L. (1914). *Educational psychology* (Vol. I-III). New York, NY: Teachers College.

Vygotsky, L. (1930/1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Zemel, A., & Çakir, M. P. (2009). Reading's work in VMT. In G. Stahl (Ed.), *Studying virtual math teams.* (ch. 14, pp. 261-276). New York, NY: Springer. Web: http://GerryStahl.net/vmt/book/14.pdf. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_14.