

Multi-User GeoGebra for Virtual Math Teams

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ABSTRACT. *The Math Forum is an online resource center for pre-algebra, algebra, geometry and pre-calculus. Its Virtual Math Teams (VMT) service provides an integrated web-based environment for small teams to discuss mathematics. The VMT collaboration environment now includes the dynamic mathematics application, GeoGebra. It offers a multi-user version of GeoGebra, which can be used in concert with VMT's chat, web browsers, curricula and wiki repository.*

1. The Virtual Math Teams Project

The Virtual Math Teams (VMT) Project grew out of the Problem-of-the-Week (PoW) service at the Math Forum. The Math Forum is a well-established online resource for improving math learning, teaching and communication (Renninger & Shumar, 2002). Operating since 1992, the Math Forum is now visited by several million different visitors a month. Its PoW service provides challenging problems of K-12 students on a weekly basis. These problems are primarily oriented to individual student work, and exemplary student solutions are posted to the <http://mathforum.org> site. The original idea of the VMT Project was to provide similar stimulating problems for small groups of students to work on collaboratively over the Internet (Stahl, 2006; 2009).

In our design-based research at the VMT Project, we started by hosting student chats in a variety of commercially available environments, including AOL Instant Messenger, Babylon, WebCT and Blackboard. Based on these early investigations, we concluded that we needed to include a shared whiteboard for drawing geometric figures and for persistently displaying notes. We also found a need to minimize “chat confusion” by supporting explicit referencing of response threads. We decided to adopt and adapt ConcertChat, a research chat environment with special referencing tools (Mühlpfordt & Wessner, 2009). By collaborating with the software developers at Fraunhofer IPSI in Germany, our educational researchers have been able to successively try out versions of the environment with groups

of students and to gradually modify the environment in response to what we find by analyzing the chat logs.

2. Referencing support for collaboration

The ConcertChat environment integrates text chat with a shared whiteboard. A unique feature of ConcertChat is its support for graphical referencing (see Figure 1 below for an example). It allows for three forms of referencing from the text chat:

- A chat message can point to one or more earlier textual postings with a bold connecting line. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the selected chat posting above.
- While someone types a new chat message, they can select and point to a rectangular area in the whiteboard. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the area of the whiteboard.
- While someone types a new chat message, they can select and point to a graphical object in the whiteboard. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the area of the whiteboard.

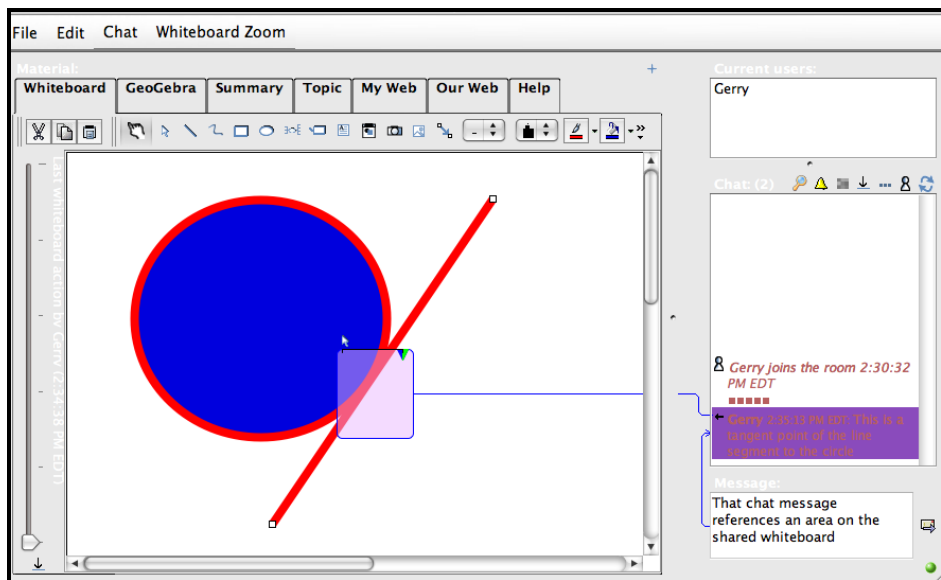


Figure 1. A VMT chat room. Note the multiple workspace tabs on the left and the chat area on the right. The selected chat message is referencing an area on the whiteboard and the new message being typed references that previous message. The small rectangles above the selected chat message provide awareness within the chat area that a series of actions have taken place within one of the workspaces. The rectangles are color coded to match the color of the chat messages of the user who made the actions.

Referencing is critical to supporting online collaboration. In face-to-face situations, like a groups standing around a physical whiteboard, we tend to take for granted that people point to and gesture at items in the whiteboard. People also take turns drawing on the whiteboard by exchanging possession of the marker—and it is visible to everyone who is doing the drawing. In an online context, other forms of referencing and awareness are needed. The Concert-Chat referencing tool can be used to avoid or clarify confusions in text-chat discussions. The action indicators (shown in Figure 1) provide another form of awareness to someone focused on the chat that other participants are active in the whiteboard. In addition, notices are displayed announcing who is typing in the chat, editing text boxes in a whiteboard or creating new objects in a drawing. The box above the chat maintains a list of who is currently logged in the room.

3. The VMT lobby, chat room and wiki

This referencing is just one form of integration of media in the VMT environment. The overall technological integration of the VMT Lobby (or portal), chat room/shared whiteboard, and wiki should be understood theoretically as a pedagogical integration of learning at the individual, small-group and community levels: (a) The VMT Lobby provides a portal for the *individual user* to browse the people and topics of the community and to select a room for group work. (b) The chat rooms are basically meeting and work places for the *small groups* as they engage in synchronous collaborative learning. (c) The wiki, on the other hand, primarily provides an asynchronous *community space* in which the work of all groups is coordinated, commented upon and perhaps summarized.

(a) The *VMT Lobby* provides a social networking portal for students to log into the system. It includes tools for defining and viewing personal profiles. In general, students in a VMT group have no knowledge about each other except for what is revealed in the chat interaction; with the functionality available in the VMT Lobby, they can define their own profiles and view profiles of each other, as well as send messages to individuals or groups in their communities (projects). Communities are defined for various VMT constituencies, such as participants in a given online contest or in a given course. There is also support for defining buddies, listing favorite chat rooms, etc. In addition, there is an interface for searching and browsing available chat rooms, usually listed for a given community. This provides access to chat rooms on different topics. Students may be told by their teachers to find certain rooms, may be invited by buddies, may search for rooms on interesting topics or may create new rooms and invite peers to join them.

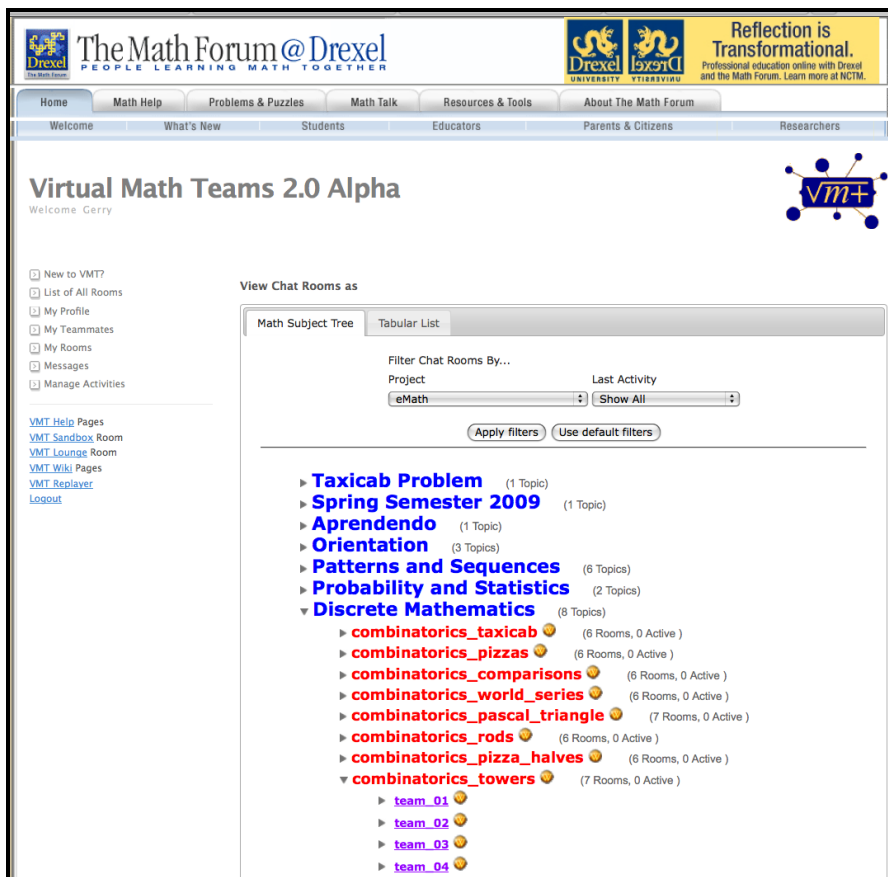


Figure 2. The VMT Lobby. In this view, a list of chat rooms is displayed for Project eMath. The math subjects for which there are rooms in this project are listed; under each subject are math topics (here, a set of related combinatorics topics) that have rooms defined; under each topic are a list of rooms. Clicking on a room name opens the associated chat room.

(b) A typical *VMT chat room* consists of the text chat interface on the right and a shared whiteboard on the left (see Figure 1 above). The history of the whiteboard state can be scrolled through, much like that of the chat, but unlike the chat it usually retains inscriptions in the visible board as long as they are relevant. VMT chat rooms have a tabbed interface, with multiple workspaces—and users can add additional spaces as needed. One kind of workspace is the shared *Workspace*, supporting graphics and text boxes. Another is a similar shared whiteboard, intended for preparing a *Summary* of the group’s work for posting to a special wiki page associated with this chat room. A third tab may display the *Topic* for the room, stored on a wiki page by an instructor. A *Wiki* tab displays a page of the VMT wiki; a special page is created for each room, linked to other pages on the Topic, math Subject or Community. A *Browser* tab provides a simple multi-user web browser that can support the graphical referencing tool from the chat and a history scrollbar. A final tab can display wiki pages containing the VMT *Help* manual and associated information. As described below, we have recently added an optional *GeoGebra* tab. This provides a

complex, but integrated set of spaces for a group to work and communicate together. A group working on a math topic can bring in resources from the different tabs and everyone can see what the others are viewing and working on.

(c) The *VMT wiki* can act as a digital library repository for summaries of work posted by teams. If there is a course that involves multiple chats by several teams, a wiki home page can be constructed for the course. The home page would then point to pages describing the course and each assignment. Group assignments are all posted to linked wiki pages. The course wiki includes index pages that bring together the student assignments in various combinations and allow the instructor to post feedback that is visible to all. The student groups can also rate and provide feedback to each other's previous reports.

4. Integration of tools in the environment

The VMT environment has come a long way from the simple AOL Instant Messaging system to the current lobby/chat/tabbed-spaces/wiki multiple-interaction space. In part, this increased complexity parallels the shift from simple math exercises to open-ended explorations of math worlds, from one-shot meetings to multiple-session Fests, from problem-solving tasks to knowledge-building efforts. Along with the considerable gain in functionality come substantial increases in complexity and the potential for confusion. This has been countered by trying to extend and supplement the integration approaches of ConcertChat. The graphical referencing and the history scrollbars have been extended to the multiple tabs. New social awareness notices have been added to track which tab each group member is viewing or referencing.

The VMT collaboration environment has been tuned to the needs of high-school math students. There are specifically math-oriented functions—like a partial implementation of MathML for displaying equations (see <http://vmt.mathforum.org/VMTLobby/VMTHelp/mathequations.html>) and the whiteboard's stock of Euclidean shapes. In addition, there are tools for integrating the multiple workspaces—like the graphical referencing from chat, the creation of wiki pages corresponding to each chat room and the posting of summary text to the proper wiki page.

Integration across modules has been important. Logins and passwords have been unified across the Lobby, chat rooms and wiki, so that logging into one automatically logs into the others. People registered in one module show up in the profiles and messaging system, by their selected community. When a new chat room is created, it is categorized by a community (e.g., a school), subject (e.g., combinatorics), a topic (e.g., Week 3's assignment) and a group (e.g., Team D). A new wiki page is generated for posting the summary from this room. The MediaWiki functionality of categories automatically associates this new page with aggregation pages for the community, subject, topic and group.

5. Porting GeoGebra to VMT

Our most recent enhancement to the VMT environment was to port the single-user GeoGebra application into VMT as a multi-user component of the tabbed chat room. This allows groups of users to co-develop and co-explore a GeoGebra geometric construction. They can chat about the drawing and reference parts of it from their chat postings. There is

a history slider, so users can scroll back and forth, watching the changes take place in the drawing for convenient review and reflection.

The version of GeoGebra in VMT is fully multi-user. VMT integrates GeoGebra as a tab of the environment (see Figure 3). GeoGebra is a particularly appropriate dynamic math application for this project because its source code is freely available as open source, there is a development community to support on-going development, the lead developer and the founder are consulting with us, the application supports a wide range of math from Euclidean construction to calculus and 3-D, GeoGebra has won international prizes, and it has been translated into about 50 languages.

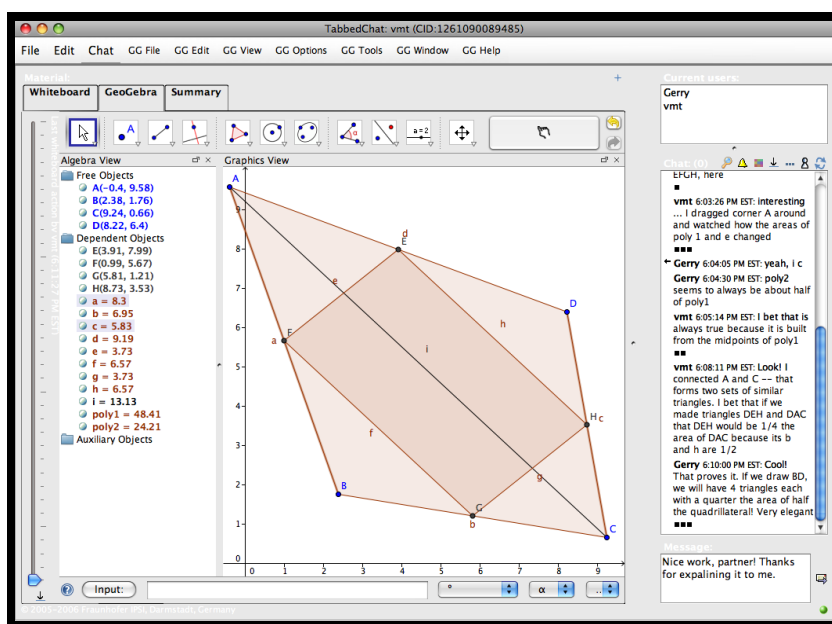


Figure 3. A GeoGebra construction created and discussed collaboratively in the VMT 2.0 learning environment.

Like all other dynamic math applications, GeoGebra previously existed only as a single-user application. While users could send their static constructions to each other, display screen images, or awkwardly include a view of the GeoGebra application within other environments (Blackboard, Moodle, Elluminate, etc.), only one person could dynamically manipulate the construction. Our port converted GeoGebra into a client-server architecture, allowing multiple distributed users to manipulate constructions simultaneously and to all observe everyone's actions in real time. Every action in the GeoGebra tab is immediately broadcast by the server to all collaborating clients.

In addition, incorporation of GeoGebra in the VMT environment framework allows users to engage in text chat while manipulating the construction. Importantly, users can graphically point from a chat posting to an area of the construction that they want to index—an important support for math discourse that is unique to VMT (or its now-defunct basis, ConcertChat). They can also scroll back and forth through the history of the

GeoGebra construction, animating its evolution—a powerful way to explore many mathematical relationships. In addition, a complete record of the collaborative construction is available to the participants, their teachers and project researchers, allowing them to analyze and reflect upon the complete interaction, including the construction actions synchronized with the chat.

The VMT version of GeoGebra is compatible with the standard version. Thus, constructions can be imported and exported seamlessly between the two versions. This facilitates use of legacy GeoGebra curriculum within the collaborative VMT environment. Images of GeoGebra co-constructions can be created and pasted by users into the VMT wiki or into Word documents. Logs of the corresponding chats can also be saved as spreadsheet files and pasted into documents.

The integration of GeoGebra significantly enhances the mathematical domain-orientation of the VMT system. On the other hand, for the GeoGebra community, it makes available for the first time truly multi-user dynamic geometry support within a rich collaborative environment. With the flexible system of tabbed components, a curriculum designer, instructor or even a student can define topics for rooms with just GeoGebra and chat or with a more complicated mix of additional browsers and support components.

For researchers of math learning, the enhanced environment provides a flexible laboratory for hosting virtual math teams engaged in GeoGebra-based tasks. The entire interactions of these teams will be logged in detail. Not only can the logs be generated in a variety of convenient formats, but also the team interactions can actually be replayed from the logs like digital videos for careful study (see Figure 4). With these tools, researchers can explore the group cognition of small teams accomplishing creative problem solving involving geometric constructions that are shared, visible and dynamic.

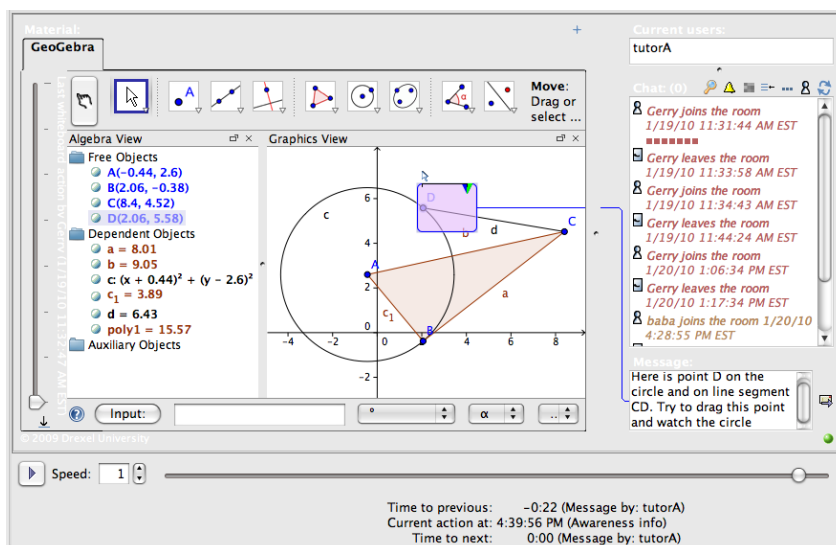


Figure 4. The VMT Replayer. The slider at the bottom can be set to scroll forward in real time or fast forward, as well as being dragged for browsing or stepped forward and backward action by action for detailed observation of coordinated construction and chat. Note that the new chat message being typed is graphically referencing a point in the GeoGebra construction for others to see.

6. Designing for multi-user issues

Making GeoGebra multi-user has involved many technical, underlying changes to the software and has necessitated a number of trade-offs and design decisions. In terms of the software technology, we treated the GeoGebra application as a client and embedded it in a Concert-Chat tab. Every action performed in the tab is immediately broadcast across the Internet to the VMT server. The server logs the action in its database and then broadcasts the action to the client of every user who is logged into the same room, including the originating client. In this way, each action performed by someone in a given VMT room is displayed identically for everyone who is working together. Minimizing Internet traffic is a major concern, especially with potentially large GeoGebra interdependent objects, and we had to make changes to Concert-Chat and GeoGebra implementations to keep traffic under control.

A major issue with multi-user systems is what to do when two users try to do conflicting things at the same time. We have recently implemented a locking mechanism, so that when two clients are creating objects at the same time or are manipulating the same object simultaneously, the changes are not broadcast until the end of the operations. This causes even some delay in sharing what people are doing; however, we believe it is necessary to avoid serious confusion. Imagine if several clients were moving point A in opposite directions at the same time. If the system broadcast changes every tenth of a second, point A would be jumping back and forth wildly, making it hard for either user to move it sensibly. Where would point A end? We have decided to have point A end where the last user to release it leaves it. If two clients are simultaneously creating an initial triangle ABC, then without locking we would get multiple points with the same names. Our locking mechanism avoids these problems by noting the conflict and assigning different names to the points, but at some cost to mutual awareness.

In the near future, we plan to try to implement two mechanisms to counteract the problem of delayed mutual awareness: (1) labeling actions and (2) simulating dragging. (1) We would like to display awareness notices in the drawing area stating who is creating, editing or moving a graphical object. This would indicate when multiple users are simultaneously at work, and perhaps some of the users would then wait to see what the others have done. (2) If point A is dragged to a new position, ending up, say, 5 units to the right, rather than having point A suddenly jump to the new position in everyone's client, we would simulate the dragging motion by interpolating 10 steps at tenth-of-a-second intervals. Then point A would appear to move to its new position through a smooth and straight motion. This would not be true to the actual dragging motion, but would give more of a feel for a dragging manipulation, which we believe to be important to the GeoGebra manipulation experience.

Of course, other trade-offs are possible, depending upon the technical architecture. We are trying certain approaches and testing them out. We hope to soon have students trying our system. Gradually, we will learn of additional problems and evolve some solutions. The experience will never be the same as having a group of geometers standing around a physical whiteboard—although in some ways it will be better because there will be a permanent record of all interactions, which can be replayed for reflection and analysis. We hope that the integration of GeoGebra with text chat will help to overcome problems that arise from imperfect mutual awareness by allowing people to discuss in text what they are doing in constructions.

Mathematics is often thought of as a solitary experience. However, our findings in the Virtual Math Teams Project show that it can be an exciting, engaging, motivating and rewarding experience when conducted collaboratively. To promote this effectively online, one must provide a carefully crafted set of tools. We believe that GeoGebra can play an important role as a central tool in the VMT environment and we look forward to working with the GeoGebra development and user community to tune our environment to meet the needs of math education globally.

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Math Forum @ Drexel University

Virtual Math Teams

Search VMT... **GO**



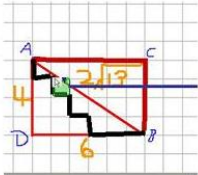
The **Virtual Math Teams** (VMT) Project @ the Math Forum offers students the opportunity to work together and enjoy doing math online. VMT is also an educational research project that investigates how to make collaborative math problem solving a fun learning experience for all students. The VMT project is supported by grants from the NSDL, IERI and SoL programs of the US [National Science Foundation](#).

Math Forum @ Drexel University

Virtual Math Teams

Search VMT... **GO**

ImH: what is the area of this shape?
 Jas: which shape?
 Jas: kinda like this one?
 Jas: the one highlighted in black and dark red?
 ImH: between the stairs and the hypotenuse
 Jas: oh
 Jas: that would be a tricky problem, each little sector is different
 ImH: perimeter is $12\sqrt{3}$
 Jas: really?



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