

*Improving GeoGebra*

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**SUPPORTING GROUP MATH COGNITION IN VIRTUAL MATH TEAMS WITH SOFTWARE CONVERSATIONAL AGENTS****Gerry Stahl, Carolyn Penstein Rosé, Kate O'Hara, Arthur Powell**

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***Abstract.** This is a research paper on a new tool to support dynamic mathematics in education. The research explores the use of software agents to engage in synchronous interaction with a small group of students working online in the Virtual Math Teams environment. The purpose of the agents is to facilitate discourse by the students that promotes their collaborative learning. In particular, the conversational agents try to encourage academically productive talk, in which students work together in ways that are accountable to each other and to their task. The agents are currently being tested in student groups working on problems in combinatorics. This research will soon be extended to student groups using a multi-user version of GeoGebra.*

**1. THE VISION OF SOFTWARE AGENTS USED TO SUPPORT COLLABORATIVE ONLINE WORK WITH DYNAMIC MATH**

The vision of the Virtual Math Teams (VMT) Project (Stahl, 2009b) is to open up an online opportunity for students to get together in small groups to discuss mathematics. The educational theory behind this is that learning mathematics centrally involves developing skills in mathematical discourse (Sfard, 2008)—see the companion position paper on math as discourse at this conference (Stahl, Rosé & Goggins, 2010). While the Internet allows students from around the world to enter into conversation with each other, turning that abstract possibility into a practical experience with educational benefits requires more than generic online communication media (Stahl, 2006). The VMT environment supplies some of the kinds of tools needed for sharing and discussing mathematical constructions and relationships. It also allows educators to develop well-designed and motivating math topics for exploration and discussion. The VMT environment is currently being extended to incorporate a multi-user version of GeoGebra—see the companion technology paper on this at this conference (Stahl et al., 2010).

In order to enhance the focus of students on math topics and to guide them in productive directions, forms of scaffolding or scripting their discussion are probably important (Kobbe et al., 2007). Of course, well-conceived topic statements can go a long way toward setting a discussion off in a promising direction from the start (Powell et al., 2009). Also, following up on the small-group work with various kinds of feedback afterwards can help to overcome problematic student understandings. For instance, a teacher can

annotate or formally assess the work after an online session, student groups can comment on each other's findings or class discussion following the online group work can check the thinking of individual groups and bring multiple approaches into contact with each other.

In addition to scaffolding before and after the small-group work, it is possible to guide the collaborative process synchronously. It may not be practical to expect a teacher who is supervising several groups to interact effectively with all of them simultaneously. The groups may even be meeting at times when a teacher is not available. In fact, groups of students may decide to discuss math topics with no teacher involved. Our research looks at the possibility of using software conversational agents to guide the student discourse synchronously in some productive way. Software agents have proven to be effective in guiding the mathematical work of individual students. In addition, progress in computer analysis of natural (human) language makes it feasible to design software that can parse typed utterances and respond to them based on their characteristics. This provides the motivation for our investigations of the use of "conversational agents" in the VMT environment (Cui et al., 2009).

This paper will first introduce the VMT Project and the form of mathematical learning that it is designed to foster. Then the paper will discuss the kind of conversational agents that we are developing, particularly the approach of academically productive talk that the agents are trying to promote in collaborative student discussions. The paper will conclude by describing our classroom experiments with topics in combinatorics and in dynamic geometry.

## 2. THE VIRTUAL MATH TEAMS PROJECT

The Virtual Math Teams (VMT) Project has conducted research since 2003 on how to support small teams of students around the world to collaborate in online discussions of stimulating mathematical topics. The project has developed an extensive web-based environment and logged about a thousand sessions of usage. Analysis of usage has resulted in over a hundred academic publications (see <http://GerryStahl.net/vmt/pubs.html>)—the most important of which are collected in *Group Cognition* (Stahl, 2006) and *Studying Virtual Math Teams* (Stahl, 2009b)—and six doctoral dissertations (Çakır, 2009; Litz, 2007; Mühlpfordt, 2008; Sarmiento-Klapper, 2009; Wee, 2010; Zhou, 2009) (see summaries in Çakır, Zemel & Stahl, 2009; Sarmiento & Stahl, 2008).

The VMT environment—available at the Math Forum—currently includes a social-networking portal (<http://vmt.mathforum.org/VMTLobby>), a Java application that integrates synchronous text chat with a shared whiteboard, social awareness indicators, and an asynchronous community wiki. The dynamic math GeoGebra system (<http://www.geogebra.org>) has recently been ported into the VMT environment. The integration of the open-source GeoGebra code enables it to function in a multi-user, synchronous online environment. Integration into the VMT environment supports simultaneous text-chat discussion of dynamic-math diagrams, graphical referencing between chat and diagrams, scrollable history of chat and diagrams, and pasting of diagrams into the associated wiki.

The incorporation of GeoGebra within the VMT environment provides significant mathematical content and functionality to enhance mathematical exploration and communication by virtual math teams. The integration includes the ability to support importing and exporting of GeoGebra dynamic worksheets; this

allows teachers and students to take advantage of available curricular materials; it provides a multi-user version of GeoGebra for the community of teachers and students currently using single-user versions of GeoGebra. Following a period of testing and research, the Math Forum plans to release the new system for worldwide usage, providing a convenient online venue for students to engage in synchronous collaborative learning within a rich environment for mathematical inquiry and knowledge-building interaction.

### **3. A NEW FORM OF MATH LEARNING**

The Math Forum manages a website (<http://mathforum.org>) with over a million pages of resources related to mathematics for middle-school and high-school students, primarily on algebra and geometry, mostly user generated (as a forerunner of the Web 2.0 philosophy). This site is well established; a leading online resource for improving math learning, teaching and communication since 1992, the Math Forum is now visited by several million different visitors a month. A community has grown up around this site, including teachers, mathematicians, researchers, students and parents—using the power of the Web to learn math and improve math education. The site offers a wealth of problems and puzzles, online mentoring, research, team problem solving, collaborations and professional development. Studies of site usage show that students have fun and learn a lot; that educators share ideas and acquire new skills; and that participants become increasingly engaged over time (Renninger & Shumar, 2002).

The Virtual Math Teams Project explores the potential of the Internet to link learners with sources of knowledge around the world, including other learners, information on the Web and stimulating digital or computational resources. It offers opportunities for engrossing mathematical discussions that are rarely found in most schools (Boaler, 2008; Lockhart, 2009). The traditional classroom that relies on one teacher, one textbook and one set of exercises to engage and train a room full of individual students over a long period of time can now be supplemented through small-group experiences of VMT chats, incorporating a variety of adaptable and personalizable interactions (Scher, 2002).

### **4. CONVERSATIONAL AGENTS**

We have integrated the agent technology developed by Carolyn Rose's research group (Cui et al., 2009) into the VMT environment developed by Gerry Stahl's research group (Stahl, 2009a). The conversational agents appear in the VMT interface just like human chat participants (see Figure 1).

There have already been several successful studies of student groups benefitting from the support of automatically triggered conversational agents that enrich the interaction between students (Kumar & Rosé, 2010); many of these studies have employed a version of the Virtual Math Teams environment augmented with this form of dynamic collaborative-learning support (Cui et al., 2009; Kumar & Rosé, 2009). For example, early evaluations measured the extent to which students learned more in conditions when automatic support was offered in the environment in comparison to conditions where it was not (Kumar, Gweon et al., 2007; Kumar, Rosé et al., 2007; Wang et al., 2007). These early studies showed that addition of a support agent into the environment increased pre to post-test learning gains by about one standard deviation, which is a full letter grade. Subsequent studies compared alternative versions of this form of automatic support. These evaluations showed additional increases in effectiveness as we have

refined the design of the support. For example, Chaudhuri et al. (2009) showed that students learned more when the support agents allowed the students to put off discussion with the support agents until they were ready to give it their full attention. Ai et al. (to appear) showed that students learned more when the support agents engaged in social behavior in addition to only offering cognitive support.

**Figure 1.** *The VMT collaborative-math-learning environment. Note along the right-hand side of the interface that a software agent named TutorE is listed as a current user, as the poster of several chat utterances and as the current chat typist.*

## 5. ACADEMICALLY PRODUCTIVE TALK

It is quite easy to program agents to greet students as they enter a VMT chat room and to prompt students to say something when everyone is quiet for an extended period or to prompt a specific student to contribute when that student has been particularly quiet. Another agent strategy might be to suggest mathematical content that is relevant to a current stage of problem solving. This might build on the intelligent tutoring technologies developed for guiding individual math learning. Intelligent tutors maintain a model of one or more standard solutions to a problem and also develop a model of the student understanding or problem-solving strategy, which is then compared step by step with the correct solution. Such an approach may be more problematic where there is a group of students with different understandings and where the goal of the math topic is more to explore than to derive the correct answer. So we are also experimenting with an alternative approach of generic guidance for math discourse and collaboration.

An approach called “academically productive talk” seems promising for scaffolding collaborative math discourse. Academically productive talk strategies have developed in response to observed difficulties

that teachers have in maintaining mathematical rigor and reasoning in their class discussions (Michaels, O'Connor & Resnick, 2008). Academically productive talk has three dimensions: accountability to the community, accountability to math knowledge and accountability to accepted standards of reasoning. The concept of academically productive talk thus highlights the need to combine appropriate classroom discourse, mathematical rigor and student reasoning to achieve powerful mathematics instruction and learning.

The academically productive talk form of classroom interaction is one in which a facilitator (or an agent) poses a question that calls for a relatively elaborated response (in mathematics, both a solution and a reason for the solution) and then presses the group as a whole to develop explanations for the solution. The process includes extended exchanges between teacher and student and among students, and includes a variety of talk moves, such as asking other students to explain what the first respondent has said, challenging students—sometimes via posing of counter examples, or “re-voicing” a student’s contribution (“So let me see if I’ve got your idea right. Are you saying...?”), which makes the student’s idea, reformulated by the teacher, available to the entire group.

## 6. EXPERIMENTS WITH COMBINATORICS

We are currently conducting a series of experiments using a curriculum of problems in combinatorics (Powell, Lai & O’Hara, 2009) specifically designed for the VMT Project by Arthur Powell’s research group. This involves eight problems:

1. The Towers Problem Set
2. The Pascal’s Triangle Problem Set
3. The Pizza Problem Set
4. The Pizza with Halves Problem Set
5. The World Series Problem Set
6. The Taxicab Problem Set
7. The Cuisenaire Rods Problem Set
8. Final Compare-and-Contrast Problem Set

The problems are closely related to each other and to Pascal’s triangle (Powell & Lai, 2009), which is introduced in the second problem. A high school class on finite math taught by Kate O’Hara is working on these problems in small groups. They work on each problem for about two sessions, gradually gaining insight into the structure of typical combinatorics problems.

In these experiments, conversational agents play different roles, as discussed above. In the first place, we have tried a broad range of degrees of intervention. We often use a “wizard-of-oz” approach, in which a human researcher plays the role of the software agent, without the student participants knowing. This makes it easier to try many different approaches, without being too concerned about the practicality of programming them.

In some sessions, the agents play a rather directive role, similar to that of a teacher-centered discourse. Here, the agent recommends steps for students to take, asks questions and provides content-related feedback. The students become quite focused on interacting with the agents—even more than with each

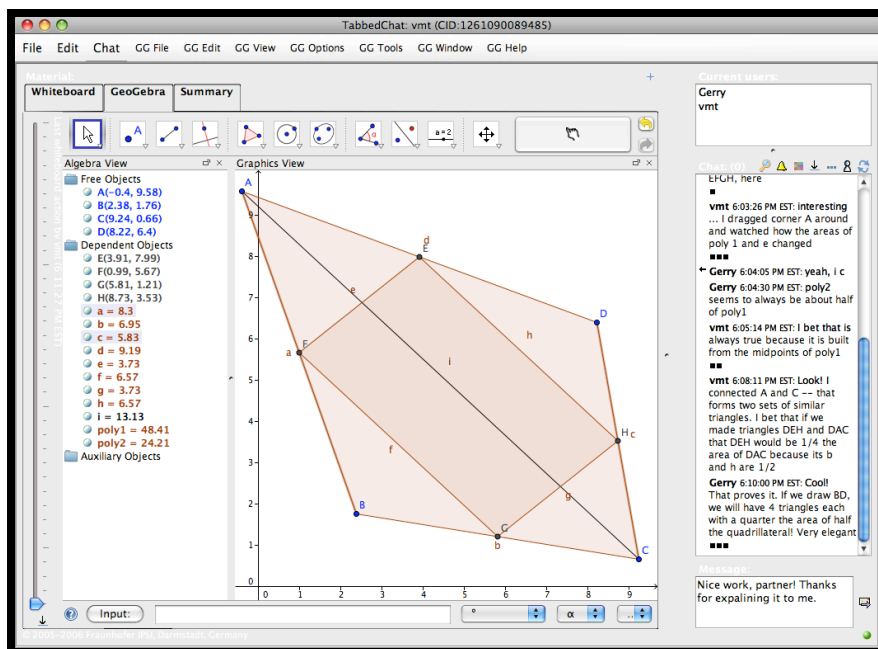
other. They ask the agents to tell them if they have the correct answer and sometimes even ask the agent to give them the answer.

In other sessions, the agent greets the students at the beginning and then informs them that they are accountable for their own discourse and math work, but that the agent is available to answer questions. In such cases, the agent plays the role of an interactive help system without being intrusive. Students can easily access the agent by addressing it in the chat, just as they would address a peer or teacher.

Probably the most interesting role for the agents is as promoters of academically productive talk. Here, they monitor the discourse and occasionally intervene to encourage mutual understanding among the students in a group. Thus, they promote accountability to the collaborative community, to math knowledge and to accepted standards of reasoning in their class.

## 7. EXPERIMENTS WITH GEOGEBRA

When the students finish their sessions on combinatorics, we will move on to dynamic geometry topics involving the use of GeoGebra. Our VMT 2.0 environment, currently in alpha testing in several experimental classrooms, includes a multi-user version of GeoGebra. In Figure 2, a quadrilateral has been constructed and the midpoints of its edges have been connected. The chat participants are discussing the ratio of the area of the interior quadrilateral to that of the original one.



*Figure 2. A GeoGebra construction created and discussed collaboratively in the VMT 2.0 learning environment.*

Of course, the work in Figure 2 is just a scenario, not actual student data. When we conduct experiments with student groups we will be interested in how they integrate work in GeoGebra with the chat discourse. As an additional layer, we will have the interactions in the chat between students and conversational

agents. An added challenge for development of our software agents will be the question of whether the agents need to analyze the work in the GeoGebra tab or whether they can just focus on the chat discourse.

We hope that our experience with these studies will help us to determine the most effective roles for conversational agents in facilitating virtual GeoGebra teams.

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