The Display of Learning in Groupwork

Gerry Stahl
The Math Forum and the College of Computing & Informatics
Drexel University
Philadelphia, PA, USA
www.GerryStahl.net
Gerry@GerryStahl.net

ABSTRACT

Methods of evaluating how small groups learn when interacting through groupware systems are not well established. In particular, the most common methods provide comparative measures of outcomes without showing the mechanisms or processes involved. Thus, they do not reveal how specific functionality of the software is or is not effective in supporting productive group work. This paper follows a virtual math team as it engages in dynamic-geometry exploration for an hour in a chat room with a multi-user version of GeoGebra. It describes the display of mathematical reasoning by the team of three seventh-grade students discussing the geometric dependencies of several different quadrilaterals. By analyzing the network of mutual responses displayed in the chat log, it is possible to evaluate the meaning-making process of the team and to derive implications for re-design of the environment for supporting the learning of dynamic-geometry fundamentals.

The need to display learning

Methods of evaluating how small groups learn when interacting through groupware systems are not well established (Stahl, Koschmann & Suthers, 2006). In particular, the most common methods provide comparative measures of outcomes without showing the mechanisms or processes involved. Thus, they do not reveal how specific functionality of the software is or is not effective in supporting productive group work.

A primary concern for designers of groupware should be the extent to which groups using their software are actually supported by the software in the ways intended by the design of the system. Determination of what learning does and does not take place in the environment and the role of specific technical functionality in supporting or failing to support that learning is essential to re-design for subsequent iterations of the development cycle.

For instance, the Virtual Math Teams (VMT) Project has gone through countless design cycles during the past decade, evolving a computer-supported collaborative learning (CSCL) environment for small groups of students to learn mathematics together (Stahl, 2006; 2009; 2013c). Project staff need to know periodically how their prototypes are succeeding. In particular, the designers of the VMT environment have developed software, curricular resources, teacher-professional-development courses and best practices to introduce students to the core skills of dynamic geometry. The VMT Project is a design-based research effort, which means that it undergoes cycles of design, implementation, testing, evaluation and re-design (Design-Based Research Collective, 2003).

The question addressed by this paper is: How well did students in the latest iteration of the VMT Project learn the skills that the environment was intended to support? The point is not to come up with a rating of the success of this groupware, as though the software was in a final state. It is also not to compare how users feel or succeed when using VMT versus not using this support. Rather, the aim is to observe just how teams of students learn targeted skills and how they fail to learn them within the designed environment. These observations should be concrete enough to drive the next cycle of re-design.

The context of developing groupware

The research question of the VMT Project is: "How should one translate the classic-education approach of Euclid's geometry into the contemporary vernacular of social networking, computer visualization and discourse-centered pedagogy?" (Stahl, 2013c, p. 1) The approach is to use a computer-based form of geometry known as dynamic geometry (e.g., Geometer's Sketchpad, Cabri, GeoGebra). Specifically, the project is based on the principle that "the key to understanding dynamic geometry is not the memorization of terminology, procedures, propositions, or proofs; it is *dependencies*" (p. 11). That is, the intention of the VMT Project is to support teams of students to develop their ability to identify dependencies in geometric figures and to use those dependencies in their own construction of similar dynamic-geometry figures.

In the beginning of 2013, the Math Forum sponsored a "WinterFest" in which teams of three or four students participated in a sequence of eight online sessions using the VMT environment. The groups were organized by teachers who had been through a semester-long teacher-professional-development course in collaborative-dynamic-mathematics education, offered by Drexel University and Rutgers-Newark. The VMT environment at that time included the first multi-user dynamic-geometry system, an adaptation to VMT of the open-source GeoGebra system (www.GeoGebra.org). The mathematical topics for the eight sessions were embedded in multiple tabs of VMT chat rooms for the sessions. The topics were developmentally designed to gradually convey an understanding of geometric dependencies.

In order to observe in sufficient detail how a group of students learned over time to work on dynamic geometry and to identify geometric dependencies, the VMT Project staff held weekly "data sessions" (Jordan & Henderson, 1995) in which they looked at the eight sessions of a particular group of three students, who called themselves Cheerios, Cornflakes and Fruitloops. The members of this "cereal team" were 7^{th} graders (about 13 years old) in an after-school activity at a New Jersey public school. They were algebra students who had not yet taken a geometry course.

This team had come to the staff's attention in connection with their performance on Topic 5, which had been worked on by many groups and had become a useful standard for observing groups identifying geometric dependencies (Stahl, 2013c, Ch. 7). The analysis of the cereal team's first session demonstrated how much they had to learn about collaborating, chatting, navigating VMT, using GeoGebra tools, arguing mathematically, manipulating dynamic-geometry objects and constructing figures (Stahl, 2013b). A detailed analysis of their logs for sessions 5 and 6 revealed considerable progress, but still showed some major holes in their understanding of how to design the construction of dynamic-geometry figures to incorporate specific dependencies (Stahl, 2014). As a further step in this longitudinal analysis, the present paper adds the following study of the team's final session.

Analysis to guide design-based research

Although design-based research is a popular approach to the development of groupware, especially in CSCL and Technology-Enhanced Learning, there is little agreement on how to evaluate trials in a way that contributes systematically to re-design (Stahl, Koschmann & Suthers, 2006). The theory of Group Cognition proposed that one could make collaborative learning—or group cognition—visible (Stahl, 2006, Ch. 18), based on the principles of ethnomethodological description (Garfinkel, 1967). This is because meaning making is an intersubjective or small-group process, requiring group members to make their contributions visible to each other, and therefore also to researchers (Stahl, 2006, Ch. 16). As the editor's introduction to (Garfinkel, 2002) explains, "the sounds and movements that comprise social action are meaningful creations that get their meaning from the shared social contexts of expectation within which they are enacted…. Intended meanings, however, can only be shared if they can be successfully displayed before others in expected ways" (p. 57).

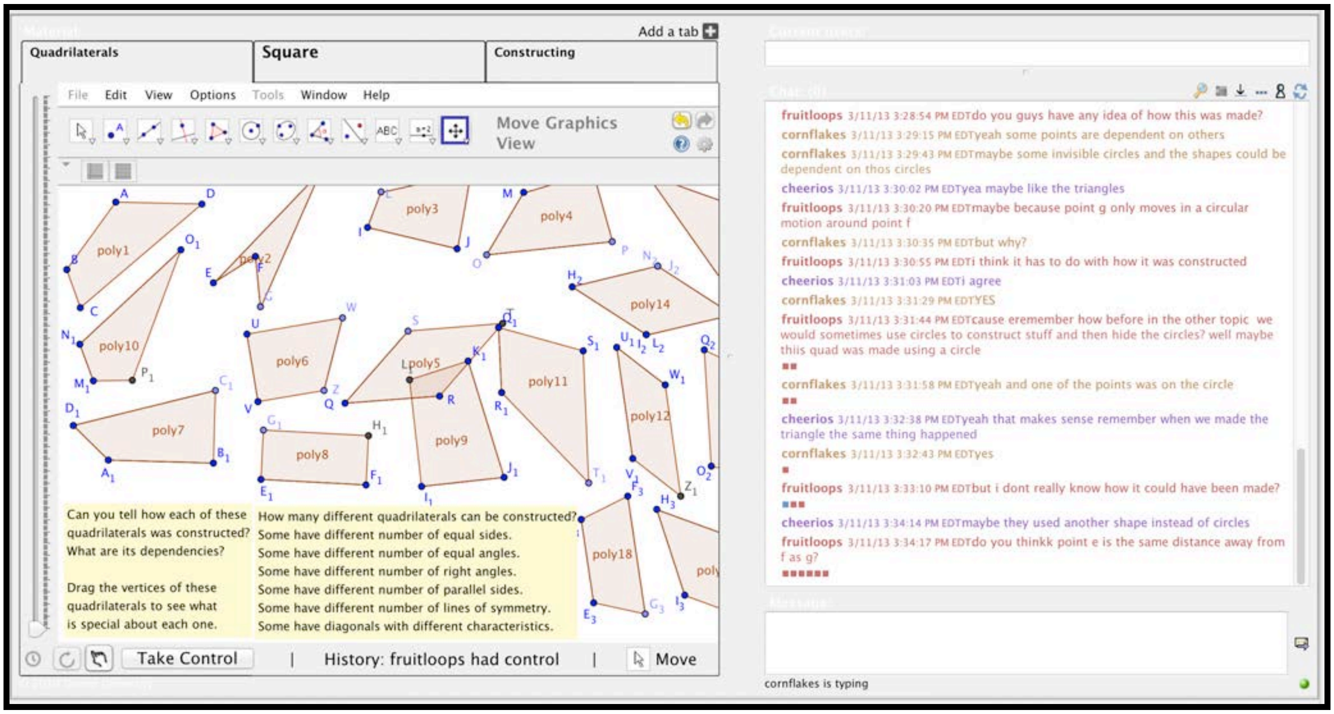

Figure 1. The VMT environment shortly after 3:34:17 pm during the group's last session. The group is dragging points E, F, G and H of Poly2 in the GeoGebra tab named "Quadrilaterals" while it is chatting in the chat tab. Topic instructions are included with the pre-constructed dynamic geometry figures in the GeoGebra tab.

This paper's analysis of the meaning-making process focuses on the sequential response structure of utterances, which build on previous utterances and elicit further possible, anticipated or expected responses (Schegloff, 2007). The analysis re-constructs the web of situated semantic references: "The meaning of the interaction is co-constructed through the building of a web of contributions and consists in the implicit network of references" (Stahl, 2009, p. 523). Most sequential analyses of conversation are limited to brief excerpts; this paper's analysis of the hour-long session—especially considered in the context of the series of eight sessions—goes beyond the analysis of so-called "longer sequences" (Stahl, 2011) toward longitudinal analysis of learning over time.

A chat posting typically responds to the immediately preceding posting. In text chat, people can be typing simultaneously. In the VMT chat system, there is an awareness message indicating who is typing, but one cannot see what is being typed until it is posted as a finished message. Thus, a new posting cannot usually be responding to a previous posting unless that posting was completed before the new one was starting to be typed. In addition, in the VMT environment, a posting may be responding to the dragging of a geometric object or a construction action in a GeoGebra tab. Although not all this information is available in the chat logs as reproduced in this paper, it is all available to researchers (as well as to students and teachers) in spreadsheets and replayer files. The analyses in this paper take the full interaction data into account. (Access to the full data can be obtained by contacting the author.)

The VMT interface is shown in Figure 1. This screenshot during an important episode in the final session corresponds to Log 4 below at about line 79. This image is taken from the VMT Replayer, which allows researchers and others to step through a session. It displays almost exactly what each of the students in the team saw on their computer screens during their live session.

We are particularly interested in seeing how well the group understands how to design dependencies into dynamic-geometry constructions. Thus, the topic instructions begin by asking, "Can you tell how each of these quadrilaterals was constructed? What are its dependencies?" During the one-hour session, the team posted 174 chat messages discussing in order the first seven of the shown four-sided figures, Poly1 through Poly7. We now follow the interaction of the team during this period to see how well the group could identify the dependencies in each of the different quadrilaterals and could surmise how they were constructed.

Poly1: An efficient analysis

The team's discussion of Poly1 is amazingly straightforward and efficient—especially when contrasted to their interactions in their early sessions. The three students enter the room and they each take a brief look at each of the three tabs before beginning to interact (lines 1-12, not shown in the log). Then Fruitloops proposes starting by looking at the first example of a quadrilateral, Poly1, which is labeled ABCD (see line 13 in Log 1).

Log 1. Fruitloops analyzes Poly1.

Line	Post Time	User	Message
13	15:00.2	fruitloops	lets start with quad abcd
14	15:18.5	fruitloops	in the upper lefthand corner
15	15:47.4	cornflakes	ok
16	16:20.1	cheerios	label it by saying its points
17	16:26.5	fruitloops	okay so for poly 1 all the points can move anywhere and i dont think they have resrictions
18	16:42.3	cornflakes	ok
19	17:19.1	fruitloops	so i think this was constructed by just making four points and using a polygon tool
20	17:38.1	fruitloops	you guys can try moving if youd like
21	18:14.2	cornflakes	yeah your right i dont think theres any restrictions
22	18:23.2	cheerios	can i try
23	19:00.7	cheerios	there are no restrictions like you said
24	19:34.6	fruitloops	so do you agree with how i think it was constructed
25	19:38.6	cornflakes	yes
26	19:44.8	cheerios	yes
27	19:53.4	fruitloops	okay good

Fruitloops opens the chat with a post that initiates the discussion of Poly1: "lets start with quad abcd." She directs her teammates' attention to it by referencing its name, vertex labels and position in the displayed GeoGebra tab. She drags each of the vertices and sees that each one moves independently. She drags point A twice and each of the others just once before announcing, "okay so for poly 1 all the points can move anywhere and i dont think they have resrictions."

While waiting for responses by her teammates, she drags the vertices some more and concludes, "so i think this was constructed by just making four points and using a polygon tool." Cornflakes and Cheerios agree to Fruitloops' proposal to start with Poly1. She encourages her partners to try moving the vertices for themselves, and they do so. Then they affirm both her observation about a lack of restrictions on the movement of the vertices and her proposal of how the figure may have been constructed. The team then moves on to the next figure.

They have followed the several steps of the instructions in the tab for Poly1: dragging each vertex, determining dependencies (or lack of them) among the figure's components and suggesting how the figure could have been constructed. Furthermore, they have all taken turns dragging and agreeing to each conclusion.

Poly1 is the simple, base case of a quadrilateral with no special relationships among its sides or angles. Therefore, its construction is a trivial application of the polygon tool of GeoGebra. Led by Fruitloops, the team is incredibly efficient at: focusing on the task of their new topic; exploring the geometric figure's dynamic behavior; concluding about the lack of dependencies; proposing how the figure was constructed; having everyone in the team explore the figure; having everyone agree with the conclusion; having everyone agree with the construction proposal; and then moving on to the next task.

Fruitloops, as an individual, proposed the solution to the task and led the group through it. Because of the simplicity of the task for an individual like Fruitloops, there was no need for group cognition or group agency in this case. Nevertheless, if one compares this chat excerpt with the log of the team's first session, the episode demonstrates that this particular team of three students has learned a lot about collaboration, problem solving, interaction in the VMT environment, the GeoGebra tools and the practices of dynamic geometry.

Poly2: A group semantic network

Now let us move to the team chat about Poly2 (Log 2). Cornflakes volunteers to take the lead with the next polygon. She drags each of the vertices and sees that points E, F and H move freely, but point G does not. She also notes (line 34) that point G is a different color than the other vertices, which indicates a different level of dependency in GeoGebra.

Cheerios asks if this means that point G is "constrained" or "restricted" (line 35 and 36). This becomes a new thread of discussion about the meaning of the terms "constrained" and "restricted" (line 47). Meanwhile, Fruitloops requests control of the GeoGebra tab; she drags point G extensively and then point E as well—for about 14 minutes, from line 40 to line 81. Cheerios asks to have control (line 42), but never really takes over control from Fruitloops and remains focused on discussing the issues of constraint—both the definition of the term and its application to Poly2. This sets up two parallel threads of discussion, which both elaborate on Cornflakes' initial observation.

Log 2. The team explores constraints in Poly2.

28	19:58.3	cornflakes	ill go next?
29	20:13.2	fruitloops	sure
30	20:15.8	cheerios	[fully erased the chat message]
31	20:26.5	cornflakes	ill do polygon efgh
32	20:37.5	cheerios	just say the number its easier
33	21:17.3	cornflakes	okasy polygon 2 has all points moving except point g
34	21:28.8	cornflakes	and point g is also a different color
35	21:32.7	cheerios	[fully erased the chat message]
36	21:40.3	cheerios	do u think it is restricted
37	21:44.7	cheerios	or constrained
38	21:49.4	fruitloops	i feel like poly 1 and poly 2 are almost exactly the same except that poly 2 had one point that is a lighter shade
39	22:04.5	fruitloops	can i try moving it?

40	22:17.1	cornflakes	sure
41	22:25.0	fruitloops	and @ cheerios , i dont know for sure
42	23:18.0	cheerios	ok can i try
43	23:22.7	cornflakes	sure
44	23:23.3	fruitloops	so point g only moves in like a circular motion around point f
45	23:30.8	cornflakes	[fully erased the chat message]
46	23:35.6	cornflakes	@fruitloops yea
47	24:16.7	cheerios	what si the difference between constrained and restricted
48	24:24.3	cheerios	is*
49	24:41.6	cornflakes	constrained is limited function
50	24:46.4	fruitloops	also when you move e, g moves away or closer to f
51	25:08.4	fruitloops	so i think g it definitly constrained
52	25:14.0	cornflakes	yes
53	25:19.8	cornflakes	i think that too
54	25:25.6	cheerios	why though
55	25:59.3	fruitloops	and g moves whenever you move point e and f but it doesnt move when you move h
56	26:20.3	cheerios	okay
57	26:42.4	fruitloops	@ cheerios. i think its constrained because it moves but the function is limited
58	27:04.9	fruitloops	[fully erased the chat message]
59	27:36.8	cheerios	oh i see
60	27:37.5	fruitloops	what is the definition of dependant
61	28:52.4	cheerios	u need the other line or point otherwise it wont work

Fruitloops first explores point G, which Cornflakes had said did not move. (She probably meant that G did not move freely or independently, which is what they were supposed to determine for the vertices). Fruitloops drags point G extensively, noting that point G's movement is confined to a circle around point F (as long as points E, F and H remain fixed): "so point g only moves in like a circular motion around point f" (line 44). She then drags point E and discovers that the position of point G shifts in response to movements of point E, changing the length of segment FG: "also when you move e, g moves away or closer to f" (line 50). Relating her findings to Cheerios' discussion of constraint, Fruitloops concludes that G is definitely constrained (line 51). Specifically, G moves in response to changes in E or F, but not in response to changes in the position of H (line 55). We can see this sequence of exploration and noticings as similar to the analysis of Poly1, in which Fruitloops builds on her own postings to accomplish the task of determining the dependencies in the figure. Cornflakes expresses agreement to Fruitloops' line 44 in her line 46 and to line 51 in lines 52 and 53.

In parallel, Cheerios asks for terminological clarification in line 60: "what si the difference between constrained and restricted." Cornflakes responds that being constrained means having a limited function (line 61). Fruitloops then provides her analysis of point G as an example of a constrained point, because its ability to be dragged is limited by the positions of other points (E and F): "so i think g it definitly constrained" (line 51). Cornflakes agrees with that in lines 52 and 53.

Cheerios questions this example by asking "why though" (line 54). This question may seem ambiguous. However, Fruitloops treats it as asking how her analysis of point G fits the definition of constrained that Cornflakes had offered. Extending her conclusion in line 51 that G is constrained (line 55), Cheerios then adds a remark (line 57) explicitly directed to Cheerios and responsive to her question from line 54. Cheerios expresses satisfaction with Fruitloops' remarks as adequate responses to her question about why point G should be considered constrained. First, she responds, "okay" (line 56) to Fruitloops' summary in line 55 about how point G moves. Then she states, "oh i see" (line 59) to Fruitloops' response to the question in line 57.

Having clarified Cheerios' question about the meaning of "constrained," Fruitloops then reverses their relative positioning as questioner and clarifier and she asks Cheerios "what is the definition of dependant" (line 60). After a pause of a minute, Cheerios replies, "u need the other line or point otherwise it wont work" (line 61). Although presented as an answer to the question, taken by itself the formulation remains rather ambiguous as a self-contained definition. It includes indexical terms ("the other", "it") whose references are missing and it is unclear what she means by not working. However, it does suggest the main idea that the behavior of a given point is somehow determined by some other point or line. The next excerpt (Log 3) will clarify this definition in terms of past experiences of the team as well as the current focus on Poly2.

While Cheerios is typing her response to Fruitloops' question about the definition of "dependent," Fruitloops raises another question, equally based on the topic description in the tab: "do you guys have any idea of how this was made?" (line 62). Note that the instructions given to the students in the original tab were, "Can you tell how each of these quadrilaterals was constructed? What are its dependencies?" The students have enacted these questions by discussing the definition of the terms "constrained" and "dependent," and by asking how Poly2 was made.

Cornflakes responds, bringing together the two threads. First, she affirms and elaborates Cheerios' definition of dependency, stating, "yeah some points are dependent on others" (line 63). Then she responds to Fruitloops' question, using this definition of dependency: "maybe some invisible circles and the shapes could be dependent on thos circles" (line 64). This introduces a discussion by the team that displays their understanding of the role of dependencies in the design of dynamic-geometry figures. This was a key goal of the curriculum developers of the set of activities culminating in this session.

To prepare for the analysis of this chat excerpt, we will formulate a statement that the developers of this topic might have intended for a response about Poly2:

> By dragging the vertices of Poly2, we see that it is a quadrilateral with two adjacent sides equal to each other in length. Side FG is constrained to be equal in length to side EF. That is, the location of point G is dependent upon the locations of points E and F, such that the distance from G to F is always the same as the distance from F to E. Points E, F and H can be located anywhere. We can design a figure like Poly2 using a circle to constrain two sides to be equal radii of the circle (all radii of a given circle are equal by definition of a circle). The figure could have been constructed by first constructing a segment EF. Then construct a circle with center F and radius EF (determining the locus of points where G can be located). Construct point G on the circle, confined to stay on it. This will constrain FG to be equal in length to EF (even if points E or F are moved). Point H can be constructed anywhere. Then use the polygon tool to connect points E, F, G, H and E in that order. Quadrilateral EFGH will display the behavior of Poly2, i.e., that G is constrained to maintain EF=FG and the other points are free.

Now let us see how the team actually discusses the dependencies designed into Poly2 by the curriculum developers in Log 3.

Log 3. The team discusses the possible construction of Poly2.

62	28:54.6	fruitloops	do you guys have any idea of how this was made?
63	29:15.6	cornflakes	yeah some points are dependent on others
64	29:43.4	cornflakes	maybe some invisible circles and the shapes could be dependent on thos circles

65	29:47.6	cheerios	[fully erased the chat message]
66	30:02.4	cheerios	yea maybe like the triangles
67	30:20.3	fruitloops	maybe because point g only moves in a circular motion around point f
68	30:35.3	cornflakes	but why?
69	30:55.5	fruitloops	i think it has to do with how it was constructed
70	31:01.5	cheerios	[fully erased the chat message]
71	31:03.8	cheerios	i agree
72	31:29.1	cornflakes	YES
73	31:44.4	fruitloops	cause eremember how before in the other topic we would sometimes use circles to construct stuff and then hide the circles? well maybe thiis quad was made using a circle
74	31:58.9	cornflakes	yeah and one of the points was on the circle
75	32:38.1	cheerios	yeah that makes sense remember when we made the triangle the same thing happened
76	32:43.0	cornflakes	yes

Consider line 66: Cheerios says, "yea maybe like the triangles." This is a potential pivotal moment in that it brings in a crucial lesson that the team learned in a previous session about constructing dependencies in triangles. However, it is quite different in appearance from the anticipated statement formulated above. It is clearly not a self-contained expression of someone's complete and adequate response to the topic, like Fruitloops' earlier proposal about Poly1. Rather it has the appearance of a semantic fragment, whose meaning is dependent upon its connections to other chat postings.

The first word, "yea," seems to be responding in agreement to a previous statement by another team member. The next word, "maybe," introduces a tentative proposal soliciting a response from others. Finally, "like the triangles" references a previous topic of discussion. Thus, line 66 is dependent for its meaning on its connections to previous postings, to potential future postings and to a topic from another discussion. Line 66 is structured with these various semantic references and the meaning of the posting is a function of its ties to the targets of those references. We will now try to connect line 66 to its references, recognizing that the target postings are also likely to be fragments, dependent for their meaning on yet other postings, ultimately forming a large network of semantic or indexical references.

Line 65 says, "yea maybe like the triangles." The "yea" is registering agreement with line 60, "maybe some invisible circles and the shapes could be dependent on thos circles." Line 66 reaffirms the tentative nature of this joint proposal by repeating line 64's hedge term, "maybe." It thereby further solicits opinions on whether the proposal should be adopted.

Line 66 then adds both detail and evidence in support of the proposal by referencing the lessons that the team experienced in working on "the triangles" in an earlier GeoGebra session. In Session 2, about three weeks earlier, the group had learned how to construct an equilateral triangle by constructing two circles around endpoints A and B of a line segment, both circles with radii of AB. The two circles constrained point C, defined by the intersection of these circles. The fact that the two circles both had the same radius (AB) meant that the sides AC and BC of a triangle ABC (which were also radii of the two circles) would both be equal in length to the base side AB, making triangle ABC always equilateral. So a proposal to take an approach "like the triangles" could involve constructing circles that are later made invisible, but confining new points to those circles to make the figures formed by the new points dependent upon the circles (whether the circles are visible or not) in order to impose equality of specific segment lengths.

The thread from line 66 posted by Cheerios back to line 64 posted by Cornflakes is a response to line 62 posted by Fruitloops: "do you guys have any idea of how this was made?" Line 62 is a call to address the main questions of the session's topic: "Can you tell how each of these quadrilaterals was constructed? What are its dependencies?" When applied to Poly2, it asks how quadrilateral EFGH was constructed, taking into account its dependencies, which the team has just finished exploring.

So the meaning of line 66 is that it proposes an answer to the topic question as expressed in line 62, building on and confirming the tentative partial response in line 64. *The meaning does not inhere in line 66 on its own* or on that posting as an expression of Cheerios' mental state, but as a semantic network uniting at least the three postings by the three team members, and therefore only making sense at the group level of the interaction among multiple postings by multiple team members.

The meaning-making network of postings continues with line 67 by Fruitloops: "maybe because point g only moves in a circular motion around point f." Again, this posting begins with "maybe," establishing a parallel structure with lines 64 and 66, unifying the postings of all three team members. The posting goes on to provide specifics about how the proposed invisible circle could be working, similarly to how it worked for the equilateral triangles. It names point G as the point that moves on the circle and point F as the point at the center of the circle. This is based on Fruitloops' extensive dragging of point G.

The next chat post, by Cornflakes, questions why G would move around F: "but why?" (line 68). Fruitloops responds in detail in lines 69 and 73, tying the observed behavior to the conjecture by Cornflakes and Cheerios in lines 64 and 66 about how Poly2 may have been constructed with a circle. This is based on the team's earlier experience constructing point C of an equilateral triangle on circles and then hiding the circles but having C remain at a distance AB from points A and B. She types: "i think it has to do with how it was constructed" and "cause eremember how before in the other topic we would sometimes use circles to construct stuff and then hide the circles? well maybe thiis quad was made using a circle."

Lines 71 and 72 from Cheerios and Cornflakes agree emphatically with lines 69 and 73. Line 74 (Cornflakes) then elaborates: "yeah and one of the points was on the circle." This clarifies that not only was it necessary to construct a circle, but then it was necessary to construct one of the points on that circle.

Line 75 sums up this whole discussion relating to the experience from Topic 2: "yeah that makes sense remember when we made the triangle the same thing happened." Cornflakes agrees in line 76 with Cheerios' conclusion. Line 75 is a quite explicit and strikingly literal affirmation of successful sense making: "that makes sense ...." It appeals to the team to "remember" the previous experience as directly relevant to their current issue.

The team has effectively bridged from their current task of understanding how Poly2 was constructed back to their past lesson about how to construct an equilateral triangle. The team has—through an effort of remembering that involved all three team members working together—recalled relevant aspects of the past shared experience and situated those aspects in the current situation(Sarmiento & Stahl, 2008). They have made sense of their current problem with the help of their past experience. This excerpt of the chat has displayed for the team and for us evidence of what might be considered group learning or even transfer—and has illustrated certain methods the team used to recall the past experience and tie it to the current joint problem context.

Log 4. The team becomes confused about Poly2.

77	33:10.4	fruitloops	but i dont really know how it could have been made?
78	34:14.5	cheerios	maybe they used another shape instead of circles
79	34:17.4	fruitloops	do you thinkk point e is the same distance away from f as g?
80	35:03.6	cornflakes	we coulda had a shape on a triangle or square made it invisible but in reality the other shape is still there therefore making one of tth e points that was on the shape dependent on that shape

81	35:31.9	cheerios	i think it is the same tool maybe they used the compass tool cuz they have the same distance
82	36:13.9	fruitloops	and h is just completely unrestriced
83	36:26.4	cheerios	[fully erased the chat message]
84	36:30.8	cornflakes	yeah it probably wasnt built on anything
85	36:31.7	cheerios	agreed
86	36:37.6	fruitloops	agreed
87	36:55.4	fruitloops	so h was probably the first point construceted in building the shape
88	37:02.3	cheerios	[fully erased the chat message]
89	37:05.9	cheerios	yeah
90	37:09.8	cheerios	[fully erased the chat message]
91	37:13.4	cheerios	[fully erased the chat message]

After posting line 73, Fruitloops resumes her exploration of Poly2. She drags point G, perhaps confirming her posting back in line 67 that "point g only moves in a circular motion around point f," but not stating anything about her observations. Rather, she posts line 77, "but i dont really know how it could have been made?" (See Log 4.) This posting destroys the coherence of the team effort. It puts into question the progress the group made without providing any specifics about what the problem might be, let along indicating a path for group inquiry.

Cheerios and Cornflakes try to respond to the problem, but their responses do not seem to reflect attention to the GeoGebra dragging of point G that Fruitloops has been doing. Cheerios suggests "maybe they used another shape instead of circles" (line 78). This ignores the apparent circular motion of G around F. Simultaneously, Cornflakes reiterates how the dependency of a point on a line remains even when the line is hidden: "we coulda had a shape on a triangle or square made it invisible but in reality the other shape is still there therefore making one of tth e points that was on the shape dependent on that shape" (line 80). (Note that here Cornflakes references both their previous work on the square in Topic 5 as well as the triangle in Topic 2.)

Fruitloops ignores the postings of the others and asks, "do you thinkk point e is the same distance away from f as g?" (line 79). She then actively drags the points of Poly2 more to explore this conjecture. While she is doing this, Cheerios reverses her previous suggestion and argues for the circle rather than some other shape: "i think it is the same tool maybe they used the compass tool cuz they have the same distance" (line 81). She indicates that "maybe" the construction used the compass tool rather than the circle tool, because the compass also creates a circle and is used for making line segments stay the same length as each other in their past experiences.

Fruitloops continues to ignore the others and produces another proposal: "and h is just completely unrestriced" (line 82). The initial "and" syntactically ties her new posting to her previous one as a continuation, in effect ignoring the postings of the others. Cornflakes and Cheerios quickly agree with the new proposal. Cornflakes draws a consequence of the unrestricted behavior of point H for its construction: "yeah it probably wasnt built on anything" (line 84). Cheerios simply agrees with Fruitloops, posting "agreed" (line 85). Fruitloops posts the identical "agreed" in response to Cornflakes' consequence. The team now seems to be aligned once more.

Fruitloops next proposes a further consequence: "so h was probably the first point construceted in building the shape" (line 87). Cheerios quickly affirms this. However, during the next 6 seconds, there may have been some second thoughts about this. The awareness messages in the chat system indicate that Cheerios started to type another message twice and then deleted it, while Fruitloops also started to type a message that was never posted. Finally, Cornflakes questioned the latest proposal with "whatb do you mean?" (line 92, see Log 5). Cheerios then asked if Fruitloops meant "the first point plotted?" (line 95). Cornflakes built on Cheerios' post about the order of constructing points and clarified her non-specific previous question in lines 96 and 97: "doesnt it go in alphabetical order?" and "efg and then h."

Log 5. The team is unsure of how Poly2 was constructed.

92	37:14.9	cornflakes	whatb do you mean?
93	37:20.7	fruitloops	[fully erased the chat message]
94	37:25.0	cheerios	[fully erased the chat message]
95	37:33.2	cheerios	the first point plotted?
96	37:51.7	cornflakes	doesnt it go in alphabetical order?
97	38:06.7	cornflakes	efg and then h
98	38:10.5	cheerios	[fully erased the chat message]
99	38:16.7	fruitloops	well if h can move anywhere it was probably made first cause if you just put a random point anywhere it is the same dark blue color as h and it can move anywhere
100	38:45.0	cornflakes	but e and f are the same color>>>>>>>>>>
101	39:02.3	fruitloops	but e and f are constrained..
102	39:14.6	fruitloops	i dont know for sure maybe youre right
103	39:57.6	cheerios	im not very sure either
104	40:02.4	cornflakes	meneither

Line 99 from Fruitloops presents an argument for why point H was probably the first point constructed in building Poly2: "well if h can move anywhere it was probably made first cause if you just put a random point anywhere it is the same dark blue color as h and it can move anywhere." The team knows from their previous construction activities that if one simply constructs a point by itself, it appears in the same dark blue color as point H and one can drag it freely, the way that point H can be dragged. (In GeoGebra points are colored differently if they are free, constrained or dependent. The students have learned to use this as a clue for determining how a figure may have been constructed.)

Cornflakes points out in line 101, "but e and f are the same color>>>>>>>>>>" In other words, E or F could have been constructed before H because they are the same color as H, indicating that they are also free points. Furthermore, they come earlier alphabetically. Fruitloops responds (line 101) that they are not free like H, but can be seen through the dragging that she previously did to be constrained: "but e and f are constrained.." Presumably, since the behavior of points E and F is constrained, they must have been constructed after some other points, like H. However, Fruitloops admits that she is not convinced that she is right and Cornflakes is wrong: "i dont know for sure maybe youre right" (line 102).

Cornflakes then takes control in GeoGebra and drags the vertices of Poly2 extensively for a half a minute. At the end of that, Cheerios concludes, "im not very sure either" (line 103). Cornflakes agrees: "meneither" (line 104).

This concludes the team's work on Poly2. It seemed that they had figured out the dependencies—that point G maintained a fixed distance from point F and that sides EF and FG were equal, while point H was free. They also seemed close to concluding that Poly2 could be constructed by confining point G to a circle around point F. If they had started to explore such a construction, they would probably have discovered that the circle should have a radius of EF and that would ensure that EF=FG. Unfortunately, the team restricted its explorations to dragging vertices. Of course, this is what the instructions told them to do. They had looked ahead to the instructions for the other tabs and seen that trying to construct the quadrilaterals was reserved for the third tab, which they did not have time to work on.

The work on this quadrilateral contrasts strongly with that on Poly1. The chat interaction is rich, complex and intertwined. Meaning is created across postings by all three students. Meaning making also incorporates references to the GeoGebra actions, the instructions in the tab, the definitions of key terms, techniques of dynamic geometry and even lessons learned weeks ago. Discussions of the definitions of the terms “restricted,” “constrained” and “dependent” are interwoven with observations about relationships between geometric objects. Despite considerable productive work, the team ends in doubt about its conclusions. Poly2 seems to be a case that is particularly hard to analyze by just dragging; if the team had engaged in construction to explore their ideas about how Poly2 was built, they might have been more successful and confident in their findings.

Poly3: A confused attempt

Having agreed that they were not sure how Poly2 was constructed, the team moved on to Poly3, with Cheerios volunteering to be in control of the initial dragging this time (see line 105 in Log 6). The others agreed (lines 108 and 111).

Log 6. Confusion about Poly3.

105	40:19.3	cheerios	can i do the next polygon
106	40:21.9	fruitloops	should we move on or??
107	40:26.8	cheerios	polygon 3?
108	40:29.9	fruitloops	sure
109	40:32.7	cheerios	[fully erased the chat message]
110	40:34.4	cheerios	alright
111	40:49.6	cornflakes	cheerios your turn
112	40:50.2	cheerios	l is constraned
113	41:14.4	fruitloops	how is l constrained?
114	41:20.4	cheerios	k j l are not restrcited they can move freely
115	41:22.1	cornflakes	yeaH??
116	41:50.0	cheerios	sorry my bad i isnt constrained
117	42:05.8	fruitloops	is I constrained
118	42:08.4	fruitloops	?
119	42:12.8	cheerios	it is I that is constrained
120	42:42.3	cheerios	there is at least one right angle
121	42:42.5	cornflakes	can i get control for a sec?
122	42:49.9	cheerios	sure
123	43:13.0	cornflakes	im not sure
124	43:26.9	fruitloops	i dont really get what you are saying cheerios
125	44:04.5	cheerios	what dont u get
126	44:20.0	cheerios	i dont understand what u mean
127	45:12.8	cheerios	[fully erased the chat message]

128	45:23.7	fruitloops	[fully erased the chat message]
129	45:27.5	fruitloops	nevermind'
130	45:43.4	cheerios	okay lol

Cheerios drags point L vigorously and sees that it moves the other vertices, so she says, "l is constraned" (line 112). She may have selected L to explore first because it is colored light blue, like constrained points. Fruitloops has presumably been watching all the movement of the vertices of Poly3 and asks for more detail about how Cheerios thinks that L is constrained, "how is l constrained?" Cornflakes reinforces this with "yeaH??"

However, Cheerios—who has continued to drag all the vertices of Poly3 as far as possible in the tab—meanwhile changes her analysis repeatedly: "k j l are not restrcited they can move freely" (line114); "sorry my bad i isnt constrained" (line 116); "it is l that is constrained" (line 119); "there is at least one right angle" (line 120).

Fruitloops asks to be given control of GeoGebra and she drags each of the vertices in many directions. Fruitloops seeks clarification from Cheerios, but it is not forthcoming. After some mutual questioning, they both seem unable to pursue the discussion, erasing their attempts to respond. They mutually agree to move on to the next quadrilateral.

The movements of Poly3 in response to the dragging of a vertex seem quite complex and confusing. Especially if one pulls a vertex a long distance, the whole quadrilateral becomes distorted in strange ways. The problem is that the dependency designed into Poly3 involves sides, not individual vertices. The dependency is that the length of side IJ is equal to the length of side KL (a pair of equal opposite sides). Because any change to the length of IJ will cause side KL to change—while the quadrilateral as a whole has to remain linked up, most attempts to drag any given vertex will cause movements of most of the other vertices. It is a lot harder to see what is going on here than in previous cases. No individual point seems either completely independent or completely dependent on another individual point. It is probably necessary to declare a conjecture (like IJ=KL) and then see if it holds up under dragging. Conjectures about individual points do not help. Cheerios' conjecture that "there is at least one right angle" (line 120) also did not pan out.

Poly4: Vertices swinging around circles

It is again Fruitloops' turn to drag as the team moves to Poly4 (see Log 7). After two minutes of dragging, she determines that "so pont o and p are constrained" (line 132) and more specifically that "point p moves around point n in a circular pattern and o does the same for m" (line 135).

Log 7. Constraints in Poly 4.

131	47:27.3	fruitloops	okay ill do poly 4 now
132	49:36.9	fruitloops	so pont o and p are constrained
133	50:07.7	cheerios	agreed
134	50:15.1	cornflakes	right they are also diff colors
135	50:16.7	fruitloops	point p moves around point n in a circular pattern and o does the same for m
136	50:29.4	cheerios	can i try
137	50:34.3	cornflakes	maybe they were constructed ona circle?
138	50:56.7	fruitloops	maybe
139	51:13.2	cheerios	om and pn are like the radiuses
140	52:16.0	cornflakes	right

141	52:27.6	cheerios	maybe the compass tool?
142	52:40.6	fruitloops	yeah and also when you move point m it changes the distance poitn n is from p and when you move point n it changes the distance between m and o
143	53:11.6	cornflakes	yeah
144	53:32.9	cheerios	yup

Poly4 is apparently easier to analyze. The team can see that points P and O (which are colored as dependent points) swing around points N and M like endpoints of radii of circles. Furthermore, the two radii are connected, so that when you change the length of one that changes the length of the other. The team agrees that this could have been constructed using the compass tool. They then move on.

The team does not remark that when O swings around M, it passes directly over N, indicating that the length of side MO equals the length of side MN. Similarly, NP=MN, so that sides MO, NP and MN are all constrained to be equal by confining O and P to circles of radius MN. The team never addresses the third question in the instructions, to see what is special about each figure—Poly4 has three equal sides.

Poly5: It's restricted dude

Cornflakes tries to drag point T in Poly5 (see Log 8) and finds she cannot move it directly. Cornflakes applies the term "point t is restricted" (line 148). Fruitloops affirms, citing that point T is colored black, which indicates that it is fully dependent for its position on other objects.

Log 8. A restricted point in Poly 5.

145	53:49.9	cornflakes	oky im going to do polygon 5 now
146	54:33.8	fruitloops	okay
147	54:34.3	cornflakes	[fully erased the chat message]
148	54:52.3	cornflakes	point t is restricted
149	55:13.9	fruitloops	agreed because off the color
150	55:33.5	fruitloops	so t only moves when you move the other points
151	55:46.7	cheerios	yea thats one way to prove that is constrained
152	56:03.4	fruitloops	[fully erased the chat message]
153	56:09.6	fruitloops	i thought it was restricted
154	56:09.9	cornflakes	and when you move point r all the pointsmove around point q
155	56:29.9	cornflakes	yeah its restricted dude
156	56:48.0	cheerios	sorry that is what i mean
157	57:02.3	fruitloops	okayyy dudeee
158	01:05.3	cheerios	[fully erased the chat message]

Fruitloops adds, "so t only moves when you move the other points" (line150). Cheerios agrees: "yea thats one way to prove that is constrained" (line 151). Fruitloops questions the use of the term "constrained," saying "i thought it was restricted," which Cornflakes supports: "yeah its restricted dude" (line 155). Cheerios agrees with them that the correct term is "restricted." Point T is not merely partially constrained, for instance to move in a circle maintaining a fixed distance to another point and being constrained to a circular path, but is fully restricted to a specific position relative to other objects.

The team continues to drag Poly5 for several minutes. They drag it into a state where all four vertices are roughly on top of each other. They are not able to drag the vertices apart, but only succeed in dragging labels of the points. So they give up on Poly5 and move on under time pressure.

Poly6: A rectangle?

The team moves on to Poly6 (see Log 9). Cheerios drags point Z back and forth a little, ending with Poly6 in a rectangular shape. Cheerios concludes, "z is constrained and it is a square and has 2 sets of parallel lines and has 4 right angles" (line 161). Cornflakes and Fruitloops agree. This is a strange conclusion since the shape does not look completely square. However, it is possible that the students have not learned the distinction between square and rectangle because they have not had a formal course in geometry yet. Actually, Cheerios gives a very nice formal definition of rectangle in terms of what the tab lists as special possible characteristics: having 2 sets of parallel sides and 4 right angles. Still, it is strange that the team accepts this description for Poly6 since it was not rectangular in its original position.

Log 9. Parallel lines and right angles in Poly6.

159	01:13.3	fruitloops	lets move on to poly 2
160	01:16.7	fruitloops	6*
161	02:13.6	cheerios	z is constrained and it is a square and has 2 sets of parallel lines and has 4 right angles
162	03:38.2	cornflakes	i agrere
163	03:57.3	fruitloops	i agree*******
164	03:58.9	cheerios	w is constrained also
165	04:05.0	cornflakes	*agree

Cheerios next drags point W and concludes, "w is constrained also" (line 164). The rest of the team moves on.

Poly7: A final attempt

Cornflakes starts to move several of the quadrilaterals out of the way and Fruitloops then moves Poly7 into the cleared space in the tab. After four minutes of silence, she announces "so c1 is deff cpnstricted" (line 166) (see Log 10). Cornflakes agrees (line 167) and, when prompted, Cheerios does as well (line 174).

Log 10. A constricted point in Poly7.

166	07:54.9	fruitloops	so c1 is deff cpnstricted
167	08:12.9	cornflakes	yes
168	08:16.1	cornflakes	agreed upon
169	08:19.9	fruitloops	definitly constricted
170	08:55.6	fruitloops	definitely*
171	09:05.7	fruitloops	[fully erased the chat message]
172	09:06.9	fruitloops	[fully erased the chat message]
173	09:14.2	fruitloops	cheerios do you agrere?
174	09:14.8	cheerios	yea i agrree
175	09:30.5	cheerios	agree
176	10:33.7	fruitloops	sorry
177	10:38.7	cornflakes	soorry
178	11:25.1	fruitloops	that was by accident
179	11:36.3	cheerios	its okay

180	12:05.2	fruitloops	when you mkove a1 c1 also moves
181	12:09.5	cornflakes	yeah
182	12:14.7	cheerios	yeah
183	13:25.7	cornflakes	toodles
184	13:37.2	fruitloops	goodbye fellow peers
185	13:43.5	cheerios	toodles
186	13:55.7	cheerios	nice working with you

Fruitloops continues to test Poly7, mainly by dragging point A_1. She concludes simply that "when you mkove a1 c1 also moves" (line 180). The rest of the log is taken up with repairing typos, apologizing for accidentally sending blank chat messages and saying goodbye at the end of the final session. That ends the cereal team's involvement in WinterFest 2013.

Implications for design

The team had varying success in the exploration of seven quadrilaterals, each with different constraints or dependencies. This variety revealed a significant range in the group's capabilities, from an impressive facility in analyzing dynamic constraints and expressing conjectures about the hidden construction mechanisms to a contrasting inability to see what is going on in other, similar figures:

- Poly1: Fruitloops dragged the figure, noted its lack of dependencies and proposed that it was constructed with a simple use of the polygon too. The other students took turns dragging the vertices and agreed with Fruitloops. The task was accomplished by Fruitloops as an individual and she led the group to a consensus. The collaboration was simple and efficient. The team demonstrated mastery of completing VMT tasks, particularly when their interaction here is compared with that of the early sessions. The learning environment seems to have been successful.
- Poly2: The team worked intensively together on this figure. They brought in many resources, including reflections on constraints and lessons from past sessions. In the end, they were unsure of their findings; it might have helped if they had engaged in exploratory construction.
- Poly3: The relationships in Poly3 were apparently hard to see by just dragging vertices. It might have helped if the team had proposed conjectures, had discussed relationships among sides rather than just between points.
- Poly4: Fruitloops analyzes the dependencies of the vertices. Cornflakes and Cheerios propose how the quadrilateral was constructed.
- Poly5: The team finds a point that is not just constrained to follow a path, but is fully restricted and can only be moved indirectly by dragging another point. The students clarify their understanding of the terms "restricted," "constrained" and "dependent."
- Poly6: Here, Cheerios finds that sides are constrained to be parallel. Therefore, they see that relationships can be among sides as well as vertices.
- Poly7: Fruitloops finds a constricted point, but time runs out for the team.

The analysis in this paper, revealing both the remarkable gains in mathematical understanding and the fragility of this understanding, suggests the following implications for the re-design of the VMT environment, especially the curricular resources:

- The team does not have time to explore all the quadrilaterals or to do any work involving active construction of the figures. This is unfortunate. In previous sessions, the students have also had too little time to do some of the important constructions. Although they did construct an equilateral triangle and a square, they did not construct an isosceles triangle, which might have given them a clearer understanding of the use of circles and the compass tool for imposing the dependency of one segment length on another. Therefore, it would be better to narrow the breadth of topic coverage and focus on a few topics that intensively involve construction of dependencies.

- The team seems to be close to a grasp of constraints and dependencies in GeoGebra, but their understand is quite fragile and they often have to simply give up on certain figures. It could be quite productive to extend their introduction to dynamic geometry by a couple more sessions. The group has become very collaborative and efficient, so by the later sessions they can really focus on the geometric understanding. A couple more sessions with time-on-task might allow them to become much surer—both as a team and as individuals—of how to explore and construct dynamic-geometry figures with dependencies.
- The team moves from task to task without specific opportunities to reflect on their accomplishments, to compare the results of multiple tasks, to receive hints or helpful feedback about cases where they were stuck or to coalesce their findings in some form of persistent inscriptions. The team could be encouraged and scaffolded to formulate summaries of their findings, noticings and conjectures. They could receive teacher feedback between sessions and have time to revisit the previous topics armed with such feedback. Teams could report on their findings in whole-class discussions after all the teams are finished with a given topic.
- Some cases require lengthy investigations and discussions, while others can be completed very quickly. Different teams bring different levels of mathematical experience and expertise to the curriculum. There should be a way for teams to pace their progress through the topics flexibly. That way, novice teams could spend more time enacting the basic practices in ways that are meaningful to them and teams that are more expert (such as teams of math teachers) could move through the same set of topics faster and reflect on them at a higher level of mathematical sophistication.
- Although there was only a small set of instructions for the topic worked on in this team's final session, in general there are a number of instructions when a team works through several tabs of activity. It is always productive to revise the wording of the instructions based on the observed use of those instructions. Here, it might have been more productive to have the team try to construct their own version of each quadrilateral right after they explored that figure. In that way, their observations would be fresh and could be immediately extended through the effort of re-construction.

These implications largely motivated the latest round of development in the VMT Project. Based on the analysis of the interactions and achievements of the cereal team and other student groups in WinterFest 2013, the VMT environment was extensively re-designed for WinterFest 2014:

- Of course, the collaboration software was further developed to eliminate known bugs and to introduce new features. However, the major change was to curricular resources. The teacher-professional-development course was focused more on construction of geometric dependencies, giving teams of teachers considerably more hands-on experience with the kinds of tasks that their students would face. The WinterFest curriculum was extended from 8 sessions to 10 sessions, but the number of tabs for each session was reduced to about half as many and the tasks were simplified so that student teams could be expected to complete all the work within one-hour-long sessions.
- The WinterFest 2014 curricular resources are restricted to dragging and construction of basic dynamic-geometry objects and exploration of the characteristics of triangles. The use of the compass tool for defining dependencies is presented in detail and the construction of isosceles and equilateral

triangles are explored extensively. YouTube videos are included, illustrating clearly the role of the "drag test" and the use of the compass tool. In addition, students are involved in programming their own construction tools, so that they understand more intimately how dependencies are constructed in dynamic-geometry systems.

- Students are given workbooks, which motivate the topics in the tabs, provide some background narrative and provide spaces for students to record their observations and questions (Stahl, 2013a). The wording of the instructions for each of the topics has been edited for clarity. The text now emphasizes the consideration of geometric dependencies. Students are encouraged to preview upcoming topics and to continue work that their team did not complete. Teams are encouraged to return to complete or reconsider work on previous topics.
- Because a variety of arrangements are organized for student groups to participate in WinterFest—such as after-school math clubs, in class lessons, at-home networking—the teachers are given considerable latitude in how they facilitate the groups. However, the teachers have been involved in reflecting on their own group's work during the professional-development course and they are required to summarize the work of their student groups. They receive credit for their involvement in WinterFest and are prompted to set pedagogical goals for their students' involvement and to compare these goals with perceived achievements. Teachers often gather WinterFest participants together between sessions for feedback, discussion and reflection.

Conclusions

This paper has reviewed the work of a particular student team during their last of eight hour-long online sessions of dynamic geometry in the VMT Project's WinterFest 2013. It has analyzed the sequential responses of the team members to each other as the team tries to determine the dependencies in a series of seven quadrilaterals constructed in dynamic geometry.

The team worked well together, efficiently moving from task to task and collaborating effectively. They took turns leading the explorations of the dynamic-geometry figures, proposing analyses of the dependencies in the figures and deciding when to move on. The team members consistently made sure that they all agreed on team conclusions.

The team had varying success in their work on the different figures. With some figures, they were able to make quite complete analyses and come up with reasonable descriptions of how the figures could have been constructed. With other figures, they had much less success.

The review of the team's efforts at the detailed granularity of the responses of utterances to each other suggested a number of implications for re-design. The VMT curriculum for WinterFest 2014 incorporates most of these suggestions. Over a hundred students are starting to participate in WinterFest 2014 with the revised topics as this paper is submitted.

REFERENCES

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher. 32*(1), 5-8.

Garfinkel, H. (1967). *Studies in ethnomethodology*. Englewood Cliffs, NJ: Prentice-Hall.

Garfinkel, H. (2002). *Ethnomethodology's program: Working out durkeim's aphorism*. Lanham, Md.: Rowman & Littlefield Publishers. xiii, 299 p. pages.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences. 4*(1), 39-103. Web: http://lrs.ed.uiuc.edu/students/c-merkel/document4.HTM.

Sarmiento, J., & Stahl, G. (2008). Group creativity in inter-action: Referencing, remembering and bridging. *International Journal of Human-Computer Interaction (IJHCI).* 492–504. Web: http://GerryStahl.net/pub/ijhci2007.pdf.
Schegloff, E. A. (2007). *Sequence organization in interaction: A primer in conversation analysis*. Cambridge, UK: Cambridge University Press.
Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Web: http://GerryStahl.net/elibrary/gc. Doi: http://mitpress.mit.edu/books/group-cognition.
Stahl, G. (2009). *Studying virtual math teams*. New York, NY: Springer. Web: http://GerryStahl.net/elibrary/svmt. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3.
Stahl, G. (2011). *The structure of collaborative problem solving in a virtual math team.* Paper presented at the iConference 2011. Seattle, WA. Web: http://GerryStahl.net/pub/iconf2011.pdf.
Stahl, G. (2013a). *Explore dynamic geometry together*. Web: http://GerryStahl.net/elibrary/topics/explore.pdf.
Stahl, G. (2013b). Seminar: Analyzing virtual math teams enacting geometric practices. Presented at the LinCS Seminars, University of Gothenburg, Sweden. Web: http://GerryStahl.net/pub/analyzing.pdf.
Stahl, G. (2013c). *Translating Euclid: Designing a human-centered mathematics* (paperback & ebook ed.). San Rafael, CA: Morgan & Claypool Publishers. 221 pages. Web: http://gerrystahl.net/elibrary/euclid. Doi: http://dx.doi.org/10.2200/S00492ED1V01Y201303HCI017.
Stahl, G. (2014). *The display of learning in groupwork.* Paper presented at the ACM Conference on Supporting Groupwork (GROUP 2014). Sanibel Island, FL. Web: http://GerryStahl.net/pub/icls2014.pdf.
Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), *Cambridge handbook of the learning sciences.* (pp. 409-426). Cambridge, UK: Cambridge University Press. Web: http://GerryStahl.net/elibrary/global.