Group cognition in online chat: How virtual math teams construct their collaborative experience

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How do groups construct their shared experience of collaborating online? While answers to many questions in human-computer interaction have been formulated largely in terms of individual psychology, questions of collaborative experience require consideration of the group as the unit of analysis. Naturally, groups include individuals as contributors and interpreters of content, but the group interactions have structures and elements of their own that call for different analytic approaches.

In the Virtual Math Teams project, we are studying how middle school students do mathematics collaboratively in online chat environments. We are particularly interested in the *methods* that they develop to conduct their interactions in such an environment. Taken together, these methods define a culture, a shared set of ways to make sense together. The methods are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to the participants. These methods help define the nature of the *collaborative experience* for the small groups that develop and adopt them.

We have adapted the scientific methodology of conversation analysis to the micro-analysis of online, text-based, mathematical discourse. In this paper, we share some of our preliminary findings about how small groups make sense collaboratively in the settings we study (see Acknowledgments). For instance, we distinguish between expository and exploratory modes of narrative, show how individual and group knowledge is intertwined, analyze a proposal-response pair that is typical in math chats and look at referencing patterns that determine chat threading.

Through the use of the kinds of methods analyzed in this paper, small groups construct their collaborative experience. The chat takes on a flow of interrelated ideas for the group, analogous to an individual's stream of consciousness. The referential structure of this flow provides a basis for the group's experience of intersubjectivity, common ground and a shared world. We call this experience *group cognition*—a form of distributed cognition that involves advanced levels of cognition like mathematical problem solving.

As designers of educational chat environments, we are particularly interested in how small groups of students construct their interactions in chat media that have different technical features. How do the students learn about the meanings that designers embedded in the environment and how do they negotiate the methods that they adopt to turn technological possibilities into practical means for mediating their interactions? Ultimately, how can we design with students the technologies, pedagogies and communities that will result in desirable collaborative experiences for them?

Participant methods for "doing" collaboration

In order to understand the experience of people and groups collaborating online in our Virtual Math Teams service at the Math Forum, we look in detail at the interactions as captured in the computer log. In particular, we are studying groups of three to six middle- or high-school students discussing mathematics in chat rooms. The logs that we collect allow us to see what the participants see to a good approximation.

We conceptualize the patterns of interaction that we observe as *methods*. This is a concept that we take from ethnomethodology (Garfinkel, 1967; Heritage, 1984; Livingston, 1987). Ethnomethodology is a phenomenological approach to sociology that tries to describe the methods that members of a culture use to accomplish what they do, such as how they carry on conversations (Sacks, Schegloff, & Jefferson, 1974) or how they do mathematics (Livingston, 1986). In particular, the branch of ethnomethodology known as conversation analysis (Psathas, 1995; Sacks, 1992; ten Have, 1999) has developed an extensive and detailed scientific literature about the methods that people deploy in everyday informal conversation and how to analyze what is going on in examples of verbal interaction.

Methods are seen as the ways that people produce social order and make sense of their shared world. For instance, conversation analysis has shown that there are well-defined procedures that people use to take turns at talk. There are ways that people use to determine when they can speak and how they can signal that others may take a turn at conversation (Sacks *et al.*, 1974).

We adopt the general approach of conversation analysis, but we must make many adaptations to it given the significant differences between our chat logs and informal conversation. Our data consists of chat logs of student messages about mathematics. The messages are typed, not spoken, so they lack intonation, verbal stress, accent, rhythm, personality. The participants are not face-to-face, so their bodily posture, gaze, facial expression and physical engagement are missing. Only completed messages are posted; the halting process of producing the messages is not observable by message recipients (Garcia & Jacobs, 1998, 1999). The messages are displayed in a particular software environment and the messages are designed by their posters to be read and responded to in that environment (Livingston, 1995; Zemel, 2005). The textual messages are persistent and may be read or ignored at will, and may be re-read later—although they scroll off-screen after several other postings appear. Several participants may be typing messages at the same time, and the order of posting these messages may be unpredictable by the participants (Cakir *et al.*, 2005). Consequently, messages do not necessarily appear immediately following the messages to which they may be responding. In addition to these features of chat, our logs are concerned with mathematics and are created within educational institutional contexts—such as the Math Forum website and sometimes school-related activities or motivations. Thus, the chats may involve building mathematical knowledge, not just socializing and conversing about opinions or everyday affairs.

These differences between our chats and normal conversation mean that the rules of turn-taking, etc. have all been transformed. What remains, however, is that people still develop methods for creating and sustaining social order and shared meaning making. Chat participants are skilled at creating and adapting sophisticated methods that accomplish their tasks in these unique environments. It is the analyst's job to recognize and describe these methods, which are generally taken for granted by the participants.

Among the student chat methods of interest to us are the interactional means that the students use:

- To adapt to institutional settings
- To socialize; to have fun; to flirt
- To get to know each other better

- To establish interpersonal relations or roles
- To form themselves into groups
- To define a problem to work on
- To start working on a problem
- To agree on how to proceed
- To bring in math resources
- To agree on solutions
- To stop problem solving

Research methodology for studying collaboration

In the chat context, participants exchange textual postings. This is the sole visible basis for interaction, communication, mutual understanding and collaborative knowledge building within a generic chat environment. We are developing a chat environment that supplements this with some social awareness features and with a shared whiteboard for drawing geometric figures, but for the moment let us consider a generic chat room. In addition to the content of the typed postings, their order, sequentiality and timing typically play a significant role in how the postings are understood. The participants log in with a chat “handle” that is associated with their postings; the wording of this handle may imply something about the person so named. The postings by a given participant are linked together as his (or hers?) via the handle. Furthermore, we assume that the participants come to the chat room with specific expectations and motivations—in our case, because it is part of the Math Forum site and may be recommended by a teacher, parent or friend. Thus, there is an open-ended set of factors that may enter the chat from its socio-cultural context. There is also more-or-less shared language (e.g., English and basic math terminology) and culture (e.g., contemporary teen subculture and classroom math practices) that can play a role in the chats.

To study what takes place among students in chat rooms, we hold *data sessions* (Jordan & Henderson, 1995). These are meetings in which a number of researchers take a careful look at chat logs and discuss what appears to be taking place. Focus is directed toward brief extracts that seem to present interactions that are of analytic interest to the research group. The chat log reveals to the researchers most of what was visible to the student participants. The researchers can take into account the institutional context in which the chat took place when it is made relevant within the chat. As members of the broader society to which the students also belong, the researchers share to a large extent a competent understanding of the culture and language of the chat. Thus, they are capable of making sense of the chat because they see the same things that the participants saw and can understand them in similar ways. Moreover, by repeatedly studying the persistent log of the chat and by bringing their analytic skills to it, researchers who have made themselves familiar with this genre can make explicit many aspects of the interaction that were taken for granted by participants in the flow of the moment. By working collaboratively, the researchers can minimize the likelihood of idiosyncratic analyses.

Ethnomethodology provides a further theoretical justification for the ability of researchers to produce rigorous analyses of recorded interactions. This has to do with the notion of *accountability* (Garfinkel, 1967; Livingston, 1987). When people interact, they typically construct social order (such as conducting a fun chat or developing a math solution) and may produce social objects (like textual postings). These objects are accountable in the sense that they are designed to reveal their own significance. A brief text, for instance, is written to be read in a certain way; its choice of wording, syntax, references and placement in the larger chat are selected to show the reader how to read it

(Livingston, 1995). The account that a chat posting gives of itself for the other students in the chat can also be taken advantage of by the researchers. The researchers in a data session discuss the log in order to agree on the accounts of the postings, individually and in their interactive unity.

The Virtual Math Teams service at the Math Forum is being developed by the VMT research project (Stahl *et al.*, 2005). We are building on the established Problem-of-the-Week service at the Math Forum digital library (http://mathforum.org) by systematically opening this service up to small groups of students, rather than primarily to individual students. We have taken a design-based research approach (Design-Based Research Collective, 2003) to co-evolve the software, pedagogy, mathematics and service through an iterative process of trial, analysis and design modification. The software started with generic, commercial and educational chat systems and now involves development of a research prototype. The pedagogy started with principles of mathematics education and computer-supported collaborative learning and is now incorporating efforts to build a user community engaged in discussing math and facilitating collaborative practices. The math problems started out using the same Problems-of-the-Week offered to individuals and are now providing opportunities for groups to explore open-ended mathematical worlds as well as to work on issues that the participants generate themselves. The service started as occasional offerings and is now gearing up for continuous availability supported by as-needed monitoring and feedback.

As the trials progress, we analyze the resultant logs in ways described in this paper and use our results to inform our redesign of the software, pedagogy, mathematics and service. Thereby, ethnomethodologically-informed video analysis or interaction analysis provides the analytic component of design research, a component that is not often specified in discussions of design-based research (Koschmann, Stahl, & Zemel, 2006). The usage of our insights into how students interact in chat is at odds with the usual practices of ethnomethodology and conversation analysis, which claim not to impose researcher or designer interests on their data. While we try to understand what the student participants are up to in their own terms and how they are making sense of the activity structure that we provide for them, we are doing this in order to motivate design decisions. Our goal is not just to understand the student meaning-making processes, but to use that understanding to modify the VMT service to allow groups to engage in more effective math discourse.

Expository and exploratory discourse

Although our ethnomethodological chat analysis methodology modeled on conversation analysis has so far yielded the most insight into our data, we are pursuing a variety of approaches including coding (Strijbos & Stahl, 2005), statistical (Zemel, Xhafa, & Cakir, 2005) and ethnographic (Sarmiento, Cakir, & Stahl, 2006) investigations. These independent approaches can shed important light on the data and inform each other. Ethnographic analyses of the socio-cultural context, such as the classroom experiences of individual chat participants or their other activities in the Math Forum community help to clarify the personal motivations and the math resources that students bring into the chat (Renninger & Shumar, 1998).

In our project, a statistical analysis led to an interesting conversation analytic result. A statistical comparison of codes between chats in which students had time to work on math problems individually prior to the chats (condition A) and those where they first saw the problem in the collaborative chat context (condition B) led to a puzzling anomaly (Zemel, Xhafa, & Stahl, 2005). While most of the chats in both conditions were clustered together, one chat from each condition clustered more with the chats from the other condition. A conversation analysis of the two anomalous chats led to a distinction between *expository narrative* and *exploratory inquiry*. In conversation analytic terms, this is largely a difference in turn-taking methods. In exposition, one person makes a bid to "tell a story" about how they solved a problem. The other group members offer the expositor an extended turn at talking (or posting). The expositor dominates the discourse, providing a sequential account across several unusually long turns. The other group members listen (read) attentively, provide brief encouraging exclamations, pose questions and provide an audience. In a math problem-solving session, there may be multiple expositions concerning subsequent parts of the problem solution, possibly by different people. In exploratory inquiry, the turns are more equally shared as the group collectively investigates the problem and constructs a solution path. The steps in exploration may each involve several participants, with one person proposing a move and others agreeing, making the move or challenging it. The distinction of exposition versus exploration parallels that between cooperation (people dividing up tasks to reach a common goal) and collaboration (people working together on each task) (Dillenbourg, 1999).

The statistical quandary was resolved by noticing that the anomalous chat from condition A consisted largely of collaborative exploration despite the fact that the students may have had a chance to produce their own solutions in advance. In the anomalous chat from condition B, the students took time in the chat to first work out at least partial solutions on their own before contributing to the chat; they then provided expositions on what they found. These examples demonstrate that external conditions do not mechanically determine the methods that people use to interact. In fact, it is common for students in a chat to alternate between cooperative expository and collaborative exploratory sequences of interaction.

The group of individuals

The difference between cooperative exposition and collaborative exploration in math problem solving chats is related to the difference between individual solution and group solution. A given math chat log can be ambiguous as to whether it should be analyzed as a set of contributions from individual thinkers or whether it should be analyzed as a group accomplishment. Often, it is helpful to view it both ways and to see an intertwining of these two perspectives at work (Stahl, 2005d).

We tried an experiment with college students where we had them solve standard math problems individually and then solve the same problems in chat groups. In the group that we tracked, the group not only correctly solved all the problems that were solved by any one member of their group individually, but also solved some that no one did by themselves. Here is one that was solved by the group:

> Three years ago, men made up two out of every three internet users in America. Today the ratio of male to female users is about 1 to 1. In that time the number of American females using the internet has grown by 30,000,000, while the number of males who use the internet has grown by 100%. By how much has the total internet-user population increased in America in the past three years?
>
> (A) 50,000,000 (B) 60,000,000 (C) 80,000,000 (D) 100,000,000 (E) 200,000,000

When we first looked at the log, it appeared that one student (Mic) who seemed particularly weak in math was clowning around a lot and that another (Cosi) managed to solve the problem herself despite this distraction in the chat room. We synthesized her contributions to the chat, putting them into the format of a coherent paragraph:

> I think it's more than 60,000,000. It can't be exactly 60,000,000 because the men and women cannot increase equally and even out from an unequal starting point to a 1-to-1 ratio. . . . Oh, no wait, I mean it's less than 60,000,000. It must be 50,000,000. Yeah, I'm pretty sure that is what it is, because the women population had to grow more than the men in order to equal out—so the men must have grown less than 30,000,000. So the total must be less than 60,000,000 and the only answer like that is 50,000,000. <Cosi>

In thinking about why Cosi could solve this problem in the group context but not alone, we noticed that she was not simply solving the problem as one would in isolation (e.g., setting up algebraic equations), but was interacting with the group effort. In particular, Dan, Mic and Hal had set up a certain way of thinking about the problem and of exploring possible solutions. Cosi was reflecting on the group approach and repairing problems in its logic. The numbers, words and considerations that she used were supplied by the context of on-going interactive activities and shared meanings.

If we combine the proposals from Mic, Dan, Hal and Cosi, they read like the cognitive process of an individual problem solver:

> How can I figure out the increase in users without knowing the total number of internet users? <Mic> It seems to all come from the 30,000,000 figure. <Dan> 30,000,000 is the number of increase in American females. Since the ratio of male to female is 1 to 1, <Mic> the total of male and female combined would be 60,000,000. <Hal> No, I think it must be more than 60,000,000 because the male and female user populations can't get higher at equal rates and still even out to a 1 to 1 ratio after starting uneven. No, I made a mistake, the total must be less than 60,000,000. It could be 50,000,000, which is the only multiple choice option less than 60,000,000. <Cosi> Very smart. <Dan>

Clearly, Cosi made some contributions to the group that were key to the group solution. They were acknowledged as such. Cosi was termed "very smart" — although this could equally well be said of the group as a whole. While no individual in the group could see how to solve the problem, everyone contributed to exploring it in a way that rather efficiently led to a solution. In fact, Mic's clowning around can be seen to be an extremely effective facilitation of the group process. By joking and laughing a lot, the group relieved some of the pressure to solve a problem that was beyond any individual's reach and to open a social space in which ideas could be put forward without fear of being harshly judged.

Mathematical problem solving is a paradigm case of human cognition. It is common to say of someone who can solve math problems that he or she is smart. In fact, we see that taking place in the chat. Here, the group has solved the problem by constructing an argument much like what an individual might construct. So we can attribute group cognition or intelligence to the group. Attributing the solution to the group rather than to the sum of the individuals in the group can be motivated by seeing that the construction of mathematical meaning in the solution process was done across individuals. That is, meaning was created by means of interactions among individual contributions (postings) to the chat — such as through what are called *adjacency pairs* in conversation analysis — more than by individual postings construed as expressing a series of personal mental representations.

Math proposal adjacency pairs

In an early chat of the VMT project using AOL-IM, a popular chat environment, we observed a repeated pattern of interaction that we have since found to be common in math chats (Stahl, 2005c). Here is an excerpt from that chat (line numbers added; handles anonymized):

17.	Avr (8:23:27 PM):	i think we have to figure out the height by ourselves
18.	Avr (8:23:29 PM):	if possible
19.	pin (8:24:05 PM):	i know how
20.	pin (8:24:09 PM):	draw the altitude'
21.	Avr (8:24:09 PM):	how?
22.	Avr (8:24:15 PM):	right
23.	Sup (8:24:19 PM):	proportions?
24.	Avr (8:24:19 PM):	this is frustrating
25.	Avr (8:24:22 PM):	I don't have enough paper

In this log we see several examples of a three-step pattern:

a. A proposal bid is made by Avr in lines 17 and 18 for the group to work on: “I think we have to ….”
b. The bid is taken up by someone else (Pin in line 19) on behalf of the group: “I know how”
c. There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step, such as Pin’s new proposal bid in line 20.

The third step initiates a repeat of the three-step process:

a. A proposal bid is made by Pin in line 20 for the group to work on: “Draw the altitude”
b. An acceptance is made by someone else (Avr in line 22) on behalf of the group: “Right!”
c. There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step, such as Sup’s new proposal bid in line 23.

But here the pattern breaks down. It is unclear to us as analysts what Sup’s proposal bid, “Proportions?” is proposing. Nor is it responded to by the other group members as a proposal. Avr’s lines 24 and 25 ignore it and seem to be reporting on Avr’s efforts to work on the previous proposal to draw the altitude. Breakdown situations are often worth analyzing carefully, for they can expose in the breach practices that otherwise go unnoticed, taken for granted in their smooth execution.

Our analysis of Pin’s “failed proposal” helps to specify—by way of counter-example—the conditions that promote successful proposals in math chats: (a) a clear semantic and syntactic structure, (b) careful timing within the sequence of postings, (c) a firm interruption of any other flow of discussion, (d) the elicitation of a response, (e) the specification of work to be done and (f) a history of helpful contributions. In addition, there are other interaction characteristics and mathematical requirements. For instance, the level of mathematical background knowledge assumed in a proposal must be compatible with the expertise of the participants and the computational methods must correspond with their training.

We call the three-step pattern described above a *math proposal adjacency pair* (Stahl, 2005a). It seems to be a common interaction pattern in collaborative problem solving of mathematics in our chats. As we see in other chats, however, not all student groups adopt this method. We call this a form of “adjacency pair” in keeping with conversation analysis terminology (Duranti, 1998; Schegloff, 1991), even though in chat logs the two parts of the pair may not appear adjacent due to the complexities of chat postings: e.g., line 22 responds to line 20, with line 21 intervening as a delayed response to line 19.

References and threading

The more we study chat logs, the more we see how interwoven the postings are with each other and with the holistic Gestalt of the interactional context that they form. There are many ways in which a posting can reference elements of its context. The importance of indexicality to creating shared meaning was stressed by Garfinkel (1967). Vygotsky also noted the central role of pointing for mediating intersubjectivity in his analysis of the genesis of the infant-and-mother’s pointing gesture (1930/1978, p. 56). Our analysis of face-to-face collaboration emphasized that spoken utterances in collaborative settings tend to be elliptical, indexical and projective ways of referencing previous utterances, the conversational context and anticipated responses (Stahl, 2006, chapter 12).

We have recently adopted ConcertChat, a chat environment that not only includes a shared whiteboard, but has functionality for referencing areas of the whiteboard from chat postings and for referencing previous postings (see figure 1). The shared whiteboard is necessary for supporting most geometry problems. (This will save Avr the frustration of running out of paper, and also let Pin and Sup see what she is drawing and add to it or reference it.) Sharing drawings is not enough; students must be able to reference specific objects or areas in the drawing. (Sup could have pointed to elements

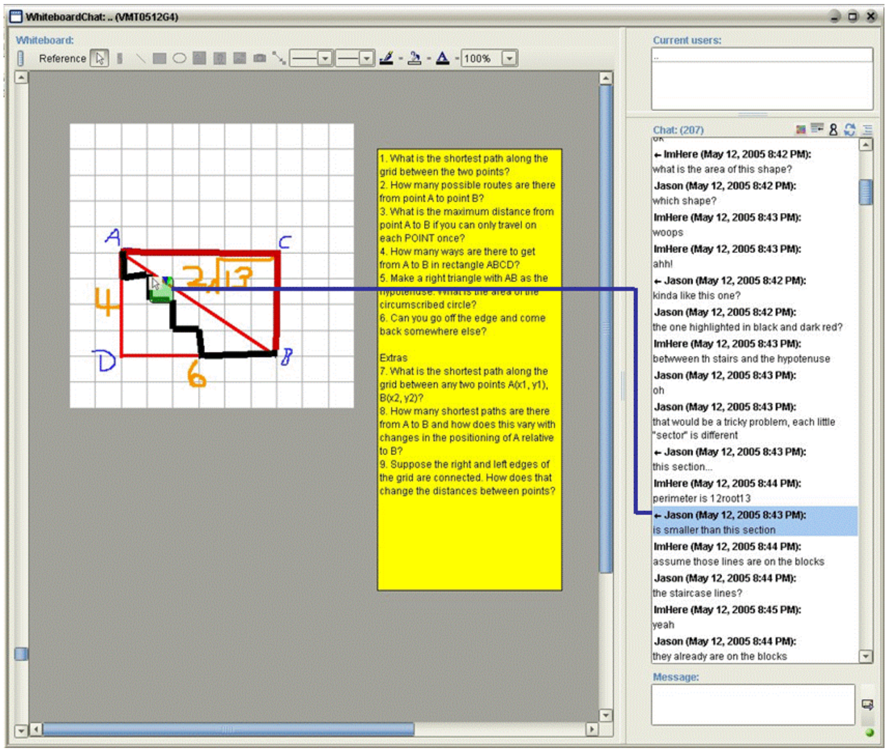

Figure 1. Screen view of ConcertChat with referencing. Line 12 of the chat is selected.

of the triangles that he felt to be significantly proportional.) The whiteboard also provides opportunities to post text where it will not scroll away. (Sup could have put his failed proposal in a text box in the whiteboard, where he or the others could come back to it later.) The graphical references (see the blue line from a selected posting to an area of the drawing) can also be used to reference one or more previous postings from a new posting, in order to make the threads of responses clearer in the midst of "chat confusion" (Pimentel, Fuks, & Lucena, 2005).

In one of our first chats using ConcertChat, the students engaged in a particularly complex interaction of referencing a figure in the whiteboard whose mathematics they wanted to explore (Stahl, Wessner *et al.*, 2006). Here is the chat log from figure 1 (graphical references to the whiteboard are indicated by "[REF TO WB]" in the log.):

1	ImH:	what is the area of this shape? [REF TO WB]
2	Jas:	which shape?
3	ImH:	woops
4	Imh:	ahh!
5	Jas:	kinda like this one? [REF TO WB]
6	Jas:	the one highlighted in black and dark red?
7	ImH:	between th stairs and the hypotenuse
8	Jas:	oh
9	Jas:	that would be a tricky problem, each little "sector" is different
10	Jas:	this section [REF TO WB]
11	ImH:	perimeter is 12root3
12	Jas:	is smaller than this section [REF TO WB]
13	ImH:	assume those lines are on the blocks
14	Jas:	the staircase lines?
15	ImH:	yea
16	Jas:	they already are on the blocks

Line 1 of the chat textually references an abstract characteristic of a complex form in the whiteboard: "*the area* of *this* shape." The software function to support this reference failed, presumably because the student, ImH, was not experienced in using it and did not cause the graphical reference line to point to anything in the drawing. Line 5 provides a demo of how to use the referencing tool. Using the tool's line, a definite textual reference ("*the* one") and the use of line color and thickness in the drawing, lines 5 and 6 propose an area to act as the topic of the chat. Line 7 makes explicit in text the definition of a sub-area of the proposed area. Line 8 accepts the new definition and line 9 starts to work on the problem concerning this area. Line 9 references the problem as "that" and notes that it is tricky because the area defined does not consist of standard forms whose area would be easy to compute and add up. It refers to the non-uniform sub-areas as little "sectors". Line 10 then uses the referencing tool to highlight (roughly) one of these little sectors or "sections". Line 12 continues line 10, but is interrupted in the chat log by line 11, a failed proposal bid by ImH. The chat excerpt continues to reference particular line segments using deictic pronouns and articles as well as a growing vocabulary of mathematical objects of concern: sectors, sections, lines, blocks.

Progress is made slowly in the collaborative exploration of mathematical relationships, but having a shared drawing helps considerably. The students use multiple textual and graphical means to reach a shared understanding of mathematical objects that they find interesting but hard to define. In this excerpt, we start to get a sense of the complex ways in which brief textual postings weave dense webs of relationships among each other and with other elements of the collaborative context.

Constructing the collaborative experience

Our goal in the VMT Project is to provide a service to students that will allow them to have a rewarding experience collaborating with their peers in online discussions of mathematics. We can never know exactly what kind of subjective experience they had, let alone predict how they will experience life under conditions that we design for them. Our primary access to information related to their group experiences comes from our chat logs. The logs capture most of what student members see of their group on their computer screens. We can even replay the logs so that we see how they unfolded sequentially in time. Of course, we are not engaged in the interaction the way the participants were and recorded experiences never quite live up to the live version because the engagement is missing. We do test out the environments ourselves and enjoy the experience, but we experience math and collaboration differently than do middle-school students. We also interview students and their teachers, but teenagers rarely reveal much of their life to adults.

So we try to understand how collaborative experiences are structured as interpersonal interactions. The focus is not on the individuals as subjective minds, but on the human, social group as constituted by the interactions that take place in the group.

Replies, up-take, pairs and triplets

Figure 2 provides a diagram of the responses of postings in the chat discussed above involving Avr, Pin and Sup (Stahl, 2005b). The numbered posts from each participant are placed in chronological

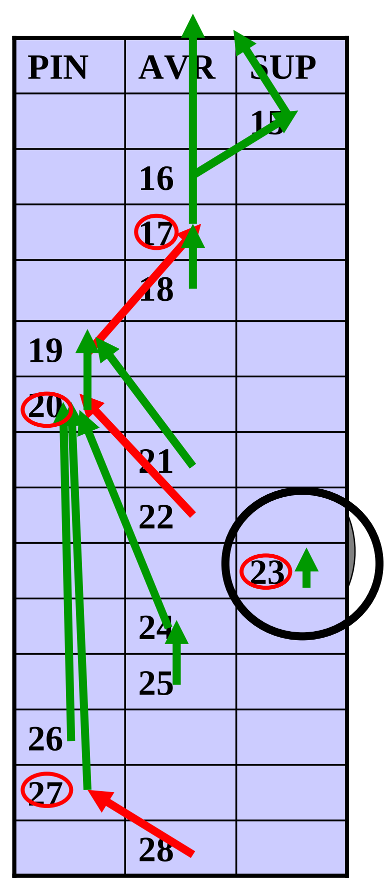

Figure 2. Threading of adjacency pairs and other uptake.

order in a column for that participant. Math proposal adjacency pairs are indicated with red arrows and other kinds of responses are indicated with green arrows. Note that Sup's failed proposal bid (line 23) is isolated. Most of the chat has coherence, flow or motion due to the fact that most postings are responses to previous messages. This high level of responses is due to the fact that many postings elicit responses, the way that a greeting invariably calls forth another greeting in response, or a question typically produces an answer. In a healthy conversation, most contributions by one participant are taken up by others. Conversationalists work hard to fit their offerings into the timing and evolving focus of the on-going interaction. In chat, the timing, rules and practices are different, but the importance of up-take remains.

The fact that the group process and the cross-ties between people are central to collaborative experiences does not contradict the continuing importance of the individuals. The representation of figure 2 uses columns to indicate the connections and implicit continuity within the sequence of contributions made by an individual. We may project psychological characteristics onto the unity of an individual's postings, attributing this unity to personal interests, personality, style, role, etc. Such attributions may change as the chat unfolds. The point is that the individual coherence of each participant's contributions adds an important dimension of implicit connections among the postings.

Adjacency pairs like math proposals, greetings and questionings provide important ties that cut across the connections of individual continuities. They form the smallest units of meaning precisely by binding together postings by different people. A proposal bid that is not taken up is not a meaningful proposal, but at best a failed attempt at a proposal. A one-sided greeting that is not recognized by the other is not an effective greeting. An interrogative expression that does not call for a responce is no real questioning of another. These are all interactional moves whose meaning consists in a give-and-take between two or more people. When we hear something that we recognize as a proposal, a greeting or a question, we feel required to attempt an appropriate response. We may ignore the proposal, snub the greeter or refuse to answer the question, but then our silence is taken as a response of ignoring, snubbing or refusing—and not simply a lack of response or up-take.

In fact, the way that a response is taken is also part of the interaction itself. In discussing the building of "common ground," Clark argues that shared understanding by A and B of A's utterance involves not only B believing that he understands A, but also A believing that B understands (Clark & Brennan, 1991). This requires an interaction spanning at least multiple utterances. Thus, for instance, the most prevalent interaction in classroom discourse is when a teacher poses a question, a student provides an answer demonstrating understanding and then the teacher acknowledges the student response as such an understanding (Lemke, 1990). Here, the elemental cell of interactional meaning making is a sequence of contributions by at least two different people. It is clear that the meaning is constructed through the interaction of multiple people, and is not a simple expression of pre-existing mental representations in any one individual's head.

Longer sequences

Although much attention has been given to adjacency pairs in conversation analysis and although such pairs can be thought of as the cells of meaning making in collaborative interaction, they form only one of many levels of analysis. For instance, there are longer sequences, episodes and topics in dialogs and chats that provide layers of structure and sense (Linell, 2001; Zemel, Xhafa, & Cakir, 2005). An hour-long chat is not a homogeneous interchange. A typical math chat might start with a period of introductions, greetings, socializing. Then there could be some problem-solving work. This might be periodically interrupted by joking, playing around, or silliness. People may come and go, requiring catching up and reorganizing. Each of these episodes has boundaries during which the group members must negotiate whether to stop what they were doing and start something else. These transitions may themselves be longer sequences of interaction, especially in large groups. We have barely begun to explore these different layers.

The chat excerpt from ConcertChat above was from the second hour-long session in a series of four chats with the same groups. The sessions referred back to previous sessions and prepared for future ones. We hope to foster a community of Math Forum users who come back repeatedly to math chats, potentially with their friends. Their chats will reference other chats and other online experiences, building connections at the community level. This adds more layers of interconnections.

Constructing proofs

In our chats, students work on math problems and themes. In solving problems and exploring math worlds or phenomena, the groups construct sequences of mathematical reasoning that come close to proofs. Proofs in mathematics have an interesting and subtle structure. One must distinguish: the problem situation; the exploratory search for the solution; the effort to reduce a haphazard solution path to an elegant, formalized proof; the statement of the proof; and the lived experience of following the proof (Livingston, 1986, 1987). Each of these has its own structures and practices. Each necessarily references the others. To engage in mathematics is to become ensnarled in the intricate connections among them. To the extent that these aspects of doing math have been distinguished and theorized, it has been done as though there is simply an individual mathematician at work. There has been virtually no research into how these could be accomplished and experienced collaboratively—despite the fact that talking about math has for some time been seen as a priority in math education (NCTM, 1989).

The stream of group consciousness

Psychologists like Williams James and novelists like Jack Kerouac have described narratives that we tell ourselves silently about what we are doing or observing as our stream of consciousness. This "inner voice" rattles on even as we sleep, making connections that Sigmund Freud found significant (if somewhat shocking in his day). In what sense might online chats—with their meanderings, flaming, associative referencing, unpredictable meaning making and unexpected images—deserve equal status as streams of (group) consciousness? Group cognition can be self-conscious.

Our sense of time and the rhythms of life are largely reliant upon the narratives we tell ourselves (Trausan-Matu, Stahl, & Sarmiento, in preparation). We know that we have already lived through a certain part of the day or of our life because we place the present in the nexus of its ties to our memories of the past or our hopes for the future. In similar ways, the web of references in a chat that connects postings to prior postings to which they respond and to future postings that they elicit defines a temporality of the chat. This is a lived sense of time that is shared by the group in the chat. Like our individual internal clocks, the group temporality must be attuned to the larger world outside—the world of family life that calls the students away from the chat for dinner or the world of school that interrupts a chat with class changes or homework pressures. The temporality that is constructed as a dimension of the collaborative experience is constrained by nature of the social situation and technological environment.

Group cognition in chat

The fact that meaning is created at the group unit of analysis rather than by particular individuals suggests the notion of group cognition (Stahl, 2006). The traditional view of cognition, particularly in Western philosophy since Descartes, is that meaning, ideas and thoughts are created in individual minds. Recent theories in cognitive science formulate this in terms of mental representations in the heads of individuals—an approach that has been critiqued by more recent theories of situated and distributed cognition. The mental contents of individual cognition—in the traditional view—can subsequently be expressed in language and communicated through the external world, to be then interpreted in the minds of other individuals. Meaning, in this view, exists only in individual minds, and cognition is always personal.

Whether or not one accepts some version of the cognitivist view in general, it seems that in situations of collaboration notions of shared meaning and group cognition are useful and important. Here, “shared meaning” has a deeper significance than what seems to underlie Clark’s analysis of common ground, where sharing is reduced to coordination among the individual mental contents of several minds (Clark & Brennan, 1991). Shared meaning is constructed across pairs or triplets of postings by more than one participant. It is not that an answer to a question implies that the answerer has in mind the same thing as the questioner, but that the answer and the question by themselves are fragmentary; they have meaning only as part of the question-answer interaction. The unit of meaning is the interaction itself, and this is a group phenomenon not an individual one. Moreover, with adequate capture of collaborative interactions, it is possible to see the construction of meaning in the traces of interaction; it is not necessary to hypothesize about hidden mental operations or contents.

Of course, in some sense it is easier to visualize individuals than groups as cognitive agents. As Vygotsky’s analysis of the infant’s gesture shows, we are used to identifying other individuals as meaning-expressing agents. Given our perceptual orientation to a primarily visual world (Merleau-Ponty, 1945/2002), it is more natural for us to assign agency to physical objects like human bodies than to more abstract entities like online groups. What, we may well wonder, *is* a collaborative group?

Groups constitute themselves. We can see how they do this in the chat logs. At one level the Math Forum service brings several students together and locates them in a chat room together. It may supply a math problem for them to work on and it may provide a facilitator who introduces them to the environment. At this point, they are a potential group with a provisionally defined membership. The facilitator might say something like, “Welcome to our first session of Virtual Math Teams! I am the facilitator for your session. . . . As a group, decide which question you would like to work on.” (This is part of the facilitator script from the session involving ImH and Jas excerpted above.) Here we can see that the facilitator has defined the group (“as a *group* … *you*”) and distinguished her own role as outside the group (“*I* am the facilitator … *your* session”). The potential group projected by the facilitator need not necessarily materialize. Individual students many come to the setting, look around, decide it is lame, and leave as individuals. However, this rarely happens. Sometimes an individual will leave without ever interacting, but as long as there are enough students there, a group will emerge.

Students come to the chat environment with certain motivations, expectations and experiences. These are generally sufficient to get the group started. One can see the group form itself. This is often reflected in the shift from singular to plural pronouns: “Let’s get started. Let *us* do some math.” We saw this in Avr’s proposal: “*I* think *we* have to figure out the height by *our*selves.” The proposal bid comes from the individual, but the projected work is for the group. Through her use of “we,” Avr constitutes the group. Through her proposal bid, she constitutes the group as a recipient of the bid and elicits a response from them. Someone other than Avr must respond to the bid on behalf of the group. When Pin says, “I know how: draw the altitude,” he is accepting Avr’s proposal as a task for the group to work on and in so doing he makes a proposal about how the group should go about approaching this task (by making a geometric construction). In this interchange, the group (a) is projected as an agent in the math work (Lerner, 1993), and (b) is actually the agent of meaning making because the meaning of Avr’s proposal is defined by the interaction within the group.

If the group experience is a positive one for the participants, they may want to return. Some chats end with people making plans to get together again. In some experiments, the same groups attended multiple sessions. We would like to see a community of users form, with teams re-forming repeatedly and with old-timers helping new groups to form and learn how to collaborate effectively.

The recognition that collaborative groups constitute themselves interactionally and that their sense making takes place at the group unit of analysis has fundamental methodological implications for the study of collaboration. The field of computer-supported collaborative learning (CSCL) was founded a decade ago to pursue the analysis of group meaning making (Stahl, Koschmann, & Suthers, 2006). We view the research described here as a contribution to the CSCL tradition.

How groups construct their experience

We are designers. Our goal is to design an exciting mathematical group experience for students. We want to design an online collaborative service, with strong pedagogical direction and effective computer support. We approach this by trying to understand how groups of students construct their experience in such settings. Because we are designing a computer-supported experience that has never before existed and because we want our design to be based on detailed study of how students actually created their collaborative experience in the environment we are designing, we follow a highly iterative try-analyze-redesign cycle.

When students enter our website, they are confronted by a densely designed environment. The lobby to our chat rooms is configured to help students find their way to a room that will meet their needs. In the room, there is a bewildering array of software functionality for posting and displaying chat notes, drawing geometric forms and annotating them, keeping track of who is doing what and configuring the space to suit oneself. There may be a statement of a math problem to solve or an imaginary world to explore mathematically. The service, problems and software are all designed to enhance the user's experience. But how can a student who is new to all this understand the meanings of the many features and affordances that have been built into the environment?

Groups of students spontaneously develop methods for exploring and responding to their environments. They try things out and discuss what happens. A new group may doodle on the whiteboard and then joke about the results. They bring with them knowledge of paint and draw programs and skills from video games, SMS and IM. The individuals may have considerable experience with single-user apps, but react when someone else erases their drawing; they must learn to integrate coordination and communication into their actions. The math problems they find in the chat rooms may be quite different than the drill-and-practice problems they are used to in traditional math textbooks. It may take the group a while to get started in productive problem solving, so the group has to find ways to keep the group together and interacting in the meantime. There may be various forms of socializing, interspersed with attempts to approach the math. As unaccustomed as the math may be, the students always have some knowledge and experience that they can bring to bear. They may apply numerical computations to given values; try to define unknowns and set up equations; graph relationships; put successive cases in a table; use trigonometric relationships or geometric figures; draw graphical representations or add lines to an existing drawing. Mainly, they put proposals out in the chat stream and respond to them. Sometimes the flow of ideas wanders without strong mathematical reflection. Other times, one individual can contribute substantial progress and engage in expository narrative to share her contribution with the group.

Groupware is never used the way its designers anticipated. The designers of ConcertChat thought that its referencing tools would immediately clarify references to elements of drawings and transform chat confusion into logical threaded chat. But our studies of the actual use of these designed functions tell a quite different and more interesting story. The shared whiteboard with graphical references from the chat may allow more complex issues to be discussed, but they do not make pointing problem free. We saw above how much work ImH and Jas engaged in to clarify for each other what they wanted to focus on. In the excerpt and in the longer chat, they used a variety of textual, drawing and referencing methods. In the process, they learned how to use these methods and they taught each other their use. In a matter of a fraction of a minute, they were able to reach a shared understanding of a topic to work on mathematically. In that brief time, they used dozens of indexical methods, some that would prove more useful than others for the future.

Chat is a highly constrained medium. Participants feel various pressures to get their individual points of view out there. In a system like ConcertChat, there is a lot to keep track of: new postings, changes to the whiteboard, signs that people are joining, leaving, typing, drawing. Small details in how something is written, drawn or referenced may have manifold implications through references to present, past or future circumstances. Students learn to track these details; apply them creatively; acknowledge to the group that they have been recognized; check, critique and repair them. Each group responds to the environment in its own way, giving group meaning to the features of the collaborative world and thereby putting their unique stamp on their group experience.

In the process, they create a group experience that they share. This experience is held together with myriad sorts of references and ties among the chat postings and drawings. Often, what is not said is as significant as what is. Individual postings are fragmentary, wildly ambiguous, and frequently confusing. In lively chats, much of what happens remains confusing for most participants. Clarity comes only through explicit reflections, up-takes, appreciations, or probing. The interactions among postings, at many levels, coheres into a stream of group consciousness, a flow of collaboration, a shared lived temporality and, with luck, an experience of mathematical group cognition.

As we have seen in this paper, when students enter into one of our chats they enter into a complex social world. They typically quickly constitute a working group and begin to engage in activities that configure a group experience. This experience is conditioned by a social, cultural, technological and pedagogical environment that has been designed for them. Within this environment, they adopt, adapt and create methods of social practice for interacting together with the other students who they find in the chat environment. Over time, they explore their situation together, create shared meaning, decide what they will do and how they will behave, engage in some form of mathematical discourse, socialize and eventually decide to end their session. Then our job begins: to analyze what has happened and how the environment we are designing conditions the collaborative experiences that groups construct there.

Acknowledgments

This paper is an attempt to reflect the thrust of research being conducted in a multi-disciplinary project, informed by studies conducted using several methodologies and theories derived from various traditions. The argument and presentation of the paper are those of the authors. Many details and formulations in the paper would be put differently by other members of the research group; the paper does not necessarily represent their individual or professional views accurately. For more detailed discussions, see the cited papers.

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