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Repairing Indexicality in Virtual Math Teams

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Abstract: Meaning making in the online collaboration settings of CSCL takes special forms depending on the affordances of the software. Here we analyze how virtual math teams in a synchronous environment combining text chat and shared whiteboard repair problems of chat confusion. We observe the central role of indexicality in establishing common ground and facilitating group cognition.

1. Repairing Chat Confusion in Virtual Math Teams

The problem of "chat confusion" has been much discussed in analyses of computer-mediated communication (Herring, 1999). It is commonly attributed to the fact that the system of turn taking, which structures face-to-face conversation, does not operate in online text chat (Fuks, Pimentel, & Pereira de Lucena, 2006; Garcia & Jacobs, 1998, 1999; O'Neill & Martin, 2003). We have argued that the turn-taking structure of conversation is replaced by a threading structure of responses in chat (Çakir, Xhafa, Zhou, & Stahl, 2005; Zemel & Çakir, 2007). For this reason, we recommend that an analysis of text-chat interaction should typically start with a clarification of the threading structure of the responses of postings to each other (Stahl, 2009, Ch. 20, 26, 28). We took this approach to a particularly interesting but confusing chat excerpt in (Stahl, 2007a) and concluded that there was still ambiguity about what the participants were saying.

In this paper, we extend that analysis. We look at the source of confusion at a deeper level: as being a matter of issues of *indexicality*. For instance, when one student refers to "**the second formula**" another student misunderstands which formula is being indexed as the second one. The students are working in a virtual environment in which their text chat postings reference mathematical formulae and diagrams in a shared whiteboard. The team works hard to repair misunderstandings concerning indexicality. It is by working out a shared system of indexing that they are able to effectively use the deictic referencing that is taken to such an extreme in text chat, with its characteristically brief, elliptical use of pronouns, articles and numbers in place of noun phrases and clauses. This intersubjective indexical field (Hanks, 1992) can be seen as the basis for establishing common ground (Clark & Brennan, 1991) and a joint problem space (Teasley & Roschelle, 1993).

In mathematics, symbols like **x** or **n** are used to index things like the unknown value being sought or the current stage in an increasing pattern. In the interaction that we study in this paper, there is also a problem in understanding the indexicality of the symbol **n** in the formulae under discussion. This problem is of particular concern for the participants and—in contrast to the confusion about the indexicality of "**second**" in "**the second formula**"—this problem is never resolved. In fact, we will see that this confusion may be related to a subtle problem of the value of **n** in the formula, leading to an error in the

student work, which is never brought to light or corrected. This may be a result of the novice status of the students as mathematicians and the fact that they have not adopted the full set of mathematical practices that might have avoided such a problem (Livingston, 1999; Sfard, 2008; Stahl, 2008), such as defining their terms explicitly and labeling indexed objects with persistently visible letters.

We will investigate problems of indexicality and their repair using data from the Virtual Math Teams project at Drexel University. This CSCL research project has previously been presented at ICCE (Stahl, 2005; Stahl, Wee, & Looi, 2007) and at CSCL (Stahl, 2007a, 2007b). The background for it is discussed in (Stahl, 2006) and many results are gathered in (Stahl, 2009). The specific excerpt is taken from the beginning of the last of four hour-long sessions. An initial analysis of the excerpt to determine its threading was undertaken in (Stahl, 2007a; revised version in Stahl, 2009, Ch. 26), This analysis was taken up in (Medina, Suthers, & Vatrapu, 2009), which traced back through the sessions to document the establishment of several group math problem-solving practices that were at work in the excerpt. In the following section, we go back to the beginning of the fourth session (at 19:00:00) and review the interaction up to and including the previously studied excerpt (from 19:29:46 to 19:33:11).

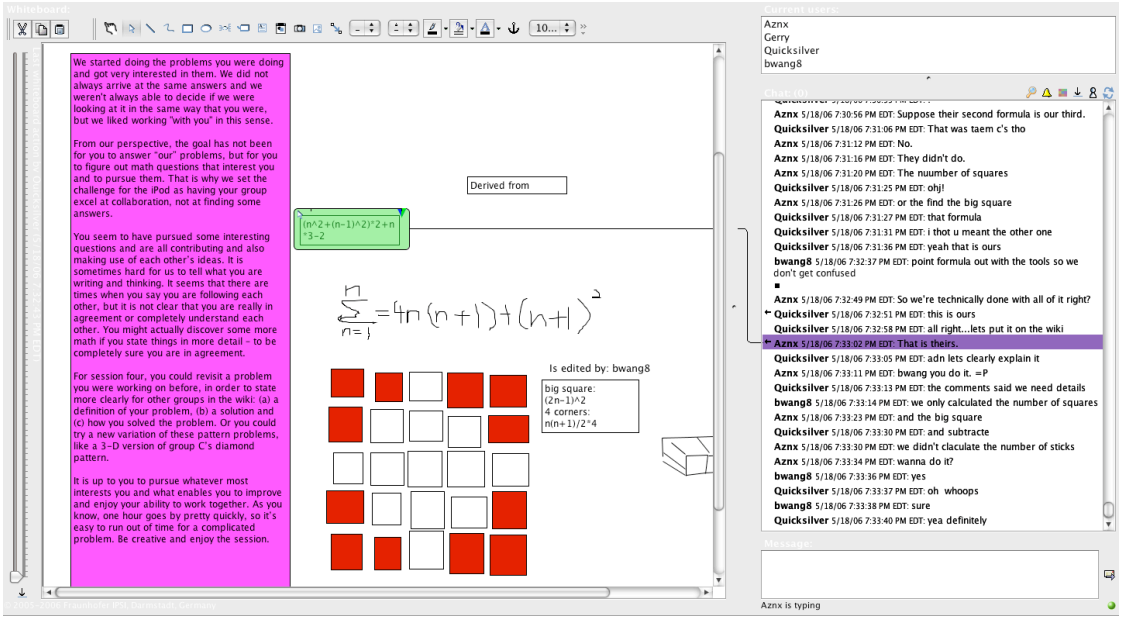

Figure 1. Aznx is pointing to Team C's formula

2. Analysis of the Work the Students Do in the Chat and Whiteboard

In this section, we proceed systematically through the log of student work to trace the chat references to various formulae in the shared whiteboard. In doing so, we can observe how their concern with the formulae arises, how confusion in indexing specific formulae unfolds and how the team repairs the confusion so that they can continue with their work.

[19:00:00 – 19:14:28] The students return to the chat room for their fourth and final session. They orient to a textbox of feedback on the whiteboard from the VMT mentors. The feedback raises for them the issue of whether their discussion in the previous session was clearly expressed—both for them and for others: "**it is not clear that you are really in agreement or completely understand each other.**" It suggests that they review the derivation of their math findings for posting to the wiki: "**For session four, you could revisit a problem you were working on before, in order to state more clearly for other**

groups in the wiki: (a) a definition of your problem, (b) a solution and (c) how you solved the problem."

1279	19:15:57	Quicksilver	so where were we?
1280	19:16:00	bwang8	so right now we know that we must calculate the number of squares on each level by making a big square and minus the 4 extra corners
1281	19:16:03	Aznx	I'd say, we work on the pyramid problem, solve it thoroughly, and then state the solution as they suggested in the feedback. Then, if we have enough time, which probably will ,we'll sytart on the pyramid problem.
1282	19:16:21	Quicksilver	u said two pyramid problems?
1283	19:16:27	Quicksilver	read ur thing again
1284	19:16:27	Aznx	OOps
1285	19:16:34	Aznx	I meant in the first part
1286	19:16:37	Aznx	the diamond problem
1287	19:16:41	Aznx	not the pyramid
1288	19:16:41	bwang8	lol
1289	19:16:45	Quicksilver	so do diamond?
1290	19:16:49	Aznx	so we first work on the diamond solutions
1291	19:16:51	Aznx	yeah
1292	19:16:57	Aznx	we pretty much solved it didnt we?
1293	19:17:09	bwang8	yeah
1294	19:17:11	Aznx	Well 50% of it I should say.
1295	19:17:15	Quicksilver	lets just recap the process
1296	19:17:27	Quicksilver	from the point of view who had never seen this problem
1297	19:17:32	bwang8	we know how to calculate the big square in a level
1298	19:17:44	Quicksilver	ok hold on
1299	19:17:50	bwang8	as in this
1300	19:17:56	bwang8	whole thing
1301	19:17:57	Quicksilver	our objective is to find the amount of squares and sticks in each level righrt?
1302	19:18:03	bwang8	yeo
1303	19:18:04	bwang8	yep
1304	19:18:08	Aznx	Yeah, intending that it is n.
1305	19:18:10	Quicksilver	that was stpe a
1306	19:18:15	Quicksilver	from the comments
1307	19:18:18	Aznx	no, step one
1308	19:18:21	Quicksilver	we defined the problem
1309	19:18:26	Aznx	oh
1310	19:18:27	Aznx	yes
1311	19:18:40	Quicksilver	lets put that in the wiki now\
1312	19:18:45	Aznx	So we dfined the problem.
1313	19:18:50	Aznx	Hold on.
1314	19:18:56	Aznx	Let's finish the ewntire thing up first.
1315	19:19:04	Aznx	We can always look back if we mess up.
1316	19:19:07	Quicksilver	ok
1317	19:19:24	bwang8	the formula is correct, right?
1318	19:19:24	Aznx	So now we should focus on integrating the solutioni and how we found it.
1319	19:19:42	Quicksilver	yup
1320	19:19:44	bwang8	this one
1321	19:19:47	bwang8	ok
1322	19:19:47	Aznx	Yeah.
1323	19:19:55	Aznx	We can always double check, and it's darn right.
1324	19:20:05	Aznx	So we solve it by really looking at a bigger picture.
1325	19:20:15	Quicksilver	or bigger square in this case
1326	19:20:20	Aznx	In this case, the "square" itself.
1327	19:20:23	Aznx	Yeah.
1328	19:20:34	bwang8	i think the 4 corner is growing like this
1329	19:20:43	bwang8	0,1,3,6,10
1330	19:20:48	bwang8	what is the pattern
1331	19:20:56	Aznx	Triagnular numbers.
1332	19:20:58	Quicksilver	triangular numbers!
1333	19:21:00	bwang8	yep
1334	19:21:03	Aznx	We had already figured that out.
1335	19:21:10	bwang8	we can use the equation from session 1
1336	19:21:11	Quicksilver	yes
1337	19:21:20	Aznx	Yup.
1338	19:21:36	bwang8	n(n+1)/2
1339	19:21:56	bwang8	4*n(n+1)/2= the four corners
1340	19:21:57	Quicksilver	this right?
1341	19:22:03	bwang8	yes
1342	19:22:06	Aznx	Yeah
1343	19:22:28	bwang8	(2n-1)^2-2n(n-1)
1344	19:22:48	bwang8	this is the equation for each level
1345	19:22:52	Aznx	So how do we know what to mulitply/change the formula by?
1346	19:23:04	Quicksilver	we can use the brute force method
1347	19:23:15	Quicksilver	burt im sure there's a better wayu
1348	19:23:19	bwang8	wait what do you mean
1349	19:23:19	Aznx	Suppose we didn't know the formula.
1350	19:23:36	Quicksilver	hmm..
1351	19:23:39	Aznx	Not n(n+1)/2
1352	19:23:47	Quicksilver	so we don't know that?
1353	19:23:50	bwang8	can you explain this
1354	19:23:57	Aznx	look
1355	19:24:02	Quicksilver	he means as the levels increase
1356	19:24:06	Aznx	first there's n(n+1)/2 right?
1357	19:24:09	Quicksilver	what is the pattern
1358	19:24:12	Aznx	So now we nkow
1359	19:24:19	Aznx	that the number of squares in the pattern
1360	19:24:24	Aznx	is related to this formula
1361	19:24:32	Aznx	becuase the numbers are triangular numbers
1362	19:24:43	Aznx	So from there, what do we know what to do?
1363	19:25:13	bwang8	n(n+1)/2*4
1364	19:25:28	Quicksilver	because of four corners
1365	19:25:30	Quicksilver	right?
1366	19:25:36	bwang8	that is the number of squares in four corners
1367	19:25:40	Quicksilver	ok
1368	19:25:43	Aznx	But that's not what it ends up to be.
1369	19:25:56	Aznx	If you double check with our already-given formula

1370	19:26:00	Quicksilver	why?
1371	19:26:07	Aznx	It's this
1372	19:26:12	Quicksilver	oh yeah
1373	19:26:14	Quicksilver	it doesn't work
1374	19:26:16	Aznx	The first one
1375	19:26:29	bwang8	no
1376	19:26:39	bwang8	it is the second one that calculate the square
1377	19:27:11	Quicksilver	are you talking about this?
1378	19:27:21	Aznx	Then what's the first one for?
1379	19:27:27	Quicksilver	the sticks\
1380	19:27:33	Aznx	Oh!
1381	19:27:40	Aznx	Then the formula makes sense.
1382	19:27:45	Quicksilver	but pretend we don't know those yet
1383	19:27:47	Aznx	Yeah, I got it.
1384	19:27:51	bwang8	lol
1385	19:28:01	Aznx	I got confused with all the formulas lol.
1386	19:28:16	Quicksilver	i suppose so
1387	19:28:22	Aznx	So is that all?
1388	19:28:37	Quicksilver	what is the actual solution then? those equations?
1389	19:28:43	Aznx	Yeah.
1390	19:28:59	Quicksilver	but when we put in the wiki how we did it....what will we write
1391	19:29:20	Aznx	Um.
1392	19:29:42	Aznx	I don't know how to exactly word it.
1393	19:29:46	Quicksilver	(a) was define the problem, (b) was the solution which we got...
1394	19:29:48	bwang8	we calculated the # of square if the diamond makes a perfect square
1395	19:29:48	Aznx	We can define the problem.
1396	19:29:55	Aznx	We got the solutions.
1397	19:30:12	Quicksilver	yes
1398	19:30:16	Quicksilver	the added corners
1399	19:30:18	Aznx	But I'm not sure how to explain how we got to the solutions, although it makes prefect sense to me.
1400	19:30:19	Quicksilver	to make a square
1401	19:30:24	Aznx	I'm just not sure how to explain it.
1402	19:30:25	Quicksilver	and we found those were triangular numbers
1403	19:30:32	Aznx	Well, I can explain the second formula.
1404	19:30:35	Quicksilver	lets go step by step
1405	19:30:37	Quicksilver	NO!
1406	19:30:42	Quicksilver	we don't know hte second formula
1407	19:30:45	Aznx	It was done through the method of finsing the pattern of triangular #s.
1408	19:30:50	Aznx	Yes we do.
1409	19:30:55	Quicksilver	?
1410	19:30:56	Aznx	Suppose their second formula is our third.
1411	19:31:06	Quicksilver	That was taem c's tho
1412	19:31:12	Aznx	No.
1413	19:31:16	Aznx	They didn't do.
1414	19:31:20	Aznx	The nuumber of squares
1415	19:31:25	Quicksilver	ohj!
1416	19:31:26	Aznx	or the find the big square
1417	19:31:27	Quicksilver	that formula
1418	19:31:31	Quicksilver	i thot u meant the other one
1419	19:31:36	Quicksilver	yeah that is ours
1420	19:32:37	bwang8	point formula out with the tools so we don't get confused
1421	19:32:49	Aznx	So we're technically done with all of it right?
1422	19:32:51	Quicksilver	this is ours
1423	19:32:58	Quicksilver	all right...lets put it on the wiki
1424	19:33:02	Aznx	That is theirs.
1425	19:33:05	Quicksilver	adn lets clearly explain it
1426	19:33:11	Aznx	bwang you do it. =P
1427	19:33:13	Quicksilver	the comments said we need details
1428	19:33:14	bwang8	we only calculated the number of squares
1429	19:33:23	Aznx	and the big square
1430	19:33:30	Quicksilver	and subtracte
1431	19:33:30	Aznx	we didn't claculate the number of sticks
1432	19:33:34	Aznx	wanna do it?
1433	19:33:36	bwang8	yes
1434	19:33:37	Quicksilver	oh whoops
1435	19:33:38	bwang8	sure
1436	19:33:40	Quicksilver	yea definitely

[19:14:38 – 19:17:15] The students discuss what topic to pursue during this session. They decide to continue to work on the diamond problem from their third session and to "**solve it thoroughly, and then state the solution as they suggested in the feedback**."

[19:17:15 – 19:20:23] They proceed to recap their previous findings. They want to post their findings to the wiki, but decide to conduct a thorough review in chat first to get their story straight. At 19:18:31 Bwang posts a textbox: "**big square: (2n-1)^2**" to start the review of their derivation. He indexes it in chat and with a graphical reference at 19:19:44, asking for agreement on the formula's correctness. All members associate Bwang's symbolic formula with the word "**square**".

[19:20:24 – 19:21:56] Bwang proposes that the number of blocks in the corners (the red squares in the whiteboard diagram of the red and white big square) grow like this: 0, 1, 3, 6, 10. The others identify this pattern with "**triangular numbers**," and Bwang affirms their responses in an instructor-like fashion. Bwang then provides a formula for the

number of squares in the four corners, based on the (Gaussian sum) formula from previous sessions, which he had already posted: "**4*n(n+1)/2= the four corners**."

[19:21:39 – 19:22:06] While Bwang does that, Quicksilver drags a textbox from the top right margin of the whiteboard into a prominent position: "**Derived from N(n+1)/2**" and Aznx similarly drags another box, with two formulae: **"(n^2+(n-1)^2)*2+n*3-2 n^2+(n-1)^2**." Quicksilver asks if his box is correct and the others agree. No one—including Aznx—comments in the chat on Aznx' move in the whiteboard.

[19:22:28 – 19:22:51] Bwang posts the expression in chat: **"(2n-1)^2-2n(n-1)**" and says, "**this is the equation for each level**." This is visibly a combination of his two previous formulae, for the number of blocks in the big square minus the number of blocks in the four corners.

[19:22:52 – 19:23:19] Aznx responds to this expression with the question, "**So how do we know what to multiply/change the formula by?**" He then twice starts to type another posting, but erases it without posting. Bwang tries emphatically to ask Aznx what he meant by this. At 19:23:19, Bwang wrote, "**wait what do you mean**" and at 19:23:50 he asked, "**can you explain this**" and pointed back to Aznx' posting. Bwang's appeal that all discussion "**wait**" until Aznx explains his question and Bwang's use of the graphical reference to point back to the question a minute later indicate the high level of Bwang's concern about not understanding Aznx' strange question. As Bwang had said when he posted the expression, it is the "**equation for each level**"—where the variable "**n**" indicates the level and is the basis for change in the formula. Aznx' question raises the possibility that he does not understand the role of the variable "**n**" in equations like these. Aznx had previously expressed some uncertainty about the role of "**n**": at 19:18:08 he had responded to Quicksilver's statement, "**our objective is to find the amount of squares and sticks in each level righrt?**" with "**Yeah, intending that it is n**." When Quicksilver continued by saying, "**that was step a**," Aznx objected at 19:18:18, "**no, step one**." He later understood that Quicksilver was referring to step (a) of the feedback, but this could show that Aznx took the formula with "**n**" to be only for the first step, n=1, rather than for all values of n.

[19:23:19 – 19:25:40] Aznx next asks, "**Suppose we didn't know the formula. . . . Not n(n+1)/2**". The group discusses this formula and clarifies that it is the formula for the number of squares in each of the four corners. It is not clear where Aznx is going with this, but Quicksilver and Bwang try to clarify things for him.

[19:25:43 – 19:26:16] Aznx now says: "**But that's not what it ends up to be. . . . If you double check with our already-given formula. . . . It's this. . . . The first one**". He points to the textbox that he had dragged out at 19:22:01 with the content, **"(n^2+(n-1)^2)*2+n*3-2 n^2+(n-1)^2**." Bwang (19:26:39) and Quicksilver (19:27:27) clarify for Aznx that the first formula in his textbox is for the number of sticks, not the number of blocks. These formulae were not derived by Team B, but were copied from Team C's work on the wiki and remained on the side of the whiteboard from previous sessions until Aznx dragged the textbox into the center. Aznx concludes, "**I got confused with all the formulas lol**."

[19:28:22 – 19:30:25] The team then discusses posting the solution to the wiki and decides to review their derivation in the chat first. This brings us to the analysis in (Stahl, 2009, Chapter 26) and the confusion about "**the second formula**."

[19:30:32 – 19:30:56] Aznx says, "**Well, I can explain the second formula**." To this Quicksilver responds emphatically, "**NO! . . . We don't know hte second formula**". Aznx then responds, "**Yes we do. . . . Suppose their second formula is our third**." The group has repeatedly gone over their derivation of the formula for the number of blocks in the diamond pattern as the number of blocks in the big square minus the number of blocks in the four stair-step corners. So Aznx claims he can now explain this. However, he indexes the formula he is referring to in a way that is not clear to the other group members. He calls it "**the second formula**." Subsequently, he refers to "**their second formula**" and "**our**

third". So now there is a system of indexicals distinguishing first, second and third formulas in sets of ours and theirs.

[19:31:06 – 19:31:36] Quicksilver says, "**That was taem c's tho**." Here, "**that**" is presumably referencing the subject of Aznx' previous statement, "**their second formula**." The "**tho**" indicates that the second formula is not a proper subject for Team B to report in the wiki because it is not theirs, but Team C's (at least originally, as indicated by "**was**"). Aznx explains that he can not be referring to a formula from Team C because, "**No. . . . They didn't do. . . . The number of squares. . . . or the find the big square**." Quicksilver then sees that Aznx must be referring to their own formula based on the number of squares in the big square.

[19:32:37 – 19:33:02] After a minute during which nothing was posted in the chat, Bwang suggested that they "**point formula out with the tools so we don't get confused**." Quicksilver then points with the graphical referencing tool to the textbox that he had dragged out, saying in the chat, "**this is ours**." Aznx points with the graphical referencing tool to the textbox that he had dragged out, saying in the chat, "**That is theirs**" (see Figure 1). This clarifies the categorization of the three formulae: formula one and formula two in Aznx textbox are Team C's formula for the number of sticks. Formula three in Quicksilver's textbox is Team B's own formula for the number of blocks.

[19:32:58 – 19:33:40] Having resolved the referential confusion, the group can now proceed with their work. The resolution made explicit that the group had only solved the problem for the number of blocks, not the number of sticks. So they decide to tackle the problem of the number of sticks.

3. Discussion of Indexicality

In the context of this VMT chat about math, the group of students has to coordinate the joint understanding of a complex system of tightly related graphical, symbolic and linguistic resources (e.g., the white diamond in a red square image in the whiteboard; the math formulae in the whiteboard and chat; the terms like "**big square**," "**corner**," "**triangular numbers**," "**diamond**"). The meaning-making context in which these resources are embedded stretches over multiple sessions (days), much of which is no longer visible in the currently displayed computer interface. To engage in their collaborative task, the students must be able to reference/index the resources in a mutually understood way. They need to recall, explain and reason with these resources in shared ways. For novices in mathematics and in online collaborative problem solving, the three students are confronted with an extremely complex set of resources, existing in multiple media, multiple times (previous sessions, prior actions, projected future activities) and multiple interaction spaces (chat, whiteboard areas, wiki pages, possibly private workspaces). The open-ended math problem may be more challenging than they are used to and they are being held to high standards of expressing their ideas clearly for each other (some of whom they have never met in person) and for various ill-defined audiences (other groups, VMT mentors).

Trained mathematicians take advantage of domain practices that were originally developed by the early Greek geometers (Latour, 2005; Netz, 1999). The rubric of a formal proof involves maintaining an ordered sequence of logical derivation steps that is persistently visible. Major representations, expressions and findings are often numbered, named or labeled to provide for unambiguous and easy referencing. Terms used in the proof are defined explicitly. The vocabulary used in a proof is limited and controlled. Students such as those in Team B have not been socialized into these practices and use the unmediated linguistic resources of ordinary language, causing referential ambiguities, interpersonal misunderstandings and indexical confusions.

In this episode, we see at least two indexical confusions: (a) what is indexed by "**the second formula**" in Aznx's post at 19:30:32 and (b) what is "**n**" in Team B's formula. (a) The first confusion is resolved with the use of VMT's explicit graphical referencing tool. It is attributed by Aznx to his confusion with "**all the formulas**" and by Quicksilver to a confusion between the group's equations and Team C's equations. Much ambiguity remains in this discussion, but the group is able to proceed productively to new work. (b) The second confusion results in a mathematical error that the group never recognizes, despite the fact that Bwang got it right at 19:22:28. Aznx seems to be confused about the role of "**n**" in the formula for number of blocks—see Bwang's concern regarding 19:22:52 at 19:23:19 and 19:23:50. This could be related at a deeper level to Aznx' confusion about variables in formulae generally. On the other hand, Aznx' confusion may have just had to do with referring to the wrong formulae—e.g., to Team C's when his group was discussing their own formulae.

For both the participants and the analysts, understanding what is taking place in a VMT session involves understanding the mathematical relationships that are being discussed—much of which is included in background knowledge that is not made explicit in the postings, but is implicit in the work done by the postings. A case in point involves the variable "**n**" in Team B's formula for the number of blocks in a diagonal pattern. If we take the pattern as starting with one block for n=1, then the big enclosing square contains (2n-1)^2 blocks, as the team noted. However, when n=1, there are no blocks in the corners. So the Gaussian sum is not for 1+2+…+n, but rather for 0+1+…+(n-1), as Bwang actually indicated at 19:20:43 when he said, "**I think the 4 corner is growing like this. . . . 0, 1, 3, 6, 10.**" Accordingly, the sum is (n-1)n/2 rather than n(n+1)/2. Bwang seems to have used this correct formula at 19:22:28 when he wrote, "**(2n-1)^2-2n(n-1)**". However, when he added it to his textbox at 19:26:15 he wrote "**big square: (2n-1)^2, 4 corners: n(n+1)/2*4**". It was never explicitly noted that **n** started at 1 for the big square and at 0 for the corners. This difference in algebraic indexing was never shared and was lost in the discussion, resulting in a mathematically erroneous formula, unbeknownst to the team. Again, rigorous mathematical practices would have avoided this problem. Even checking the formula of simple cases would have raised questions that could have led to discovering the problem.

We have seen in this session how the group learns to conduct effective collaborative math work by indexing more clearly their references to resources. By reviewing the derivation of their prior findings, they make progress in tying together their complex system of resources in a mutually understood way.

Here we can see that the establishment of "common ground" in a situation like this is much different than Clark's (1991) concept of exchanging expressions of mental representations to assure their isomorphism or identity. Rather, what is needed is the co-construction of a joint indexical field (Hanks, 1992). Similarly, what could be construed as a conversational "repair"—namely clarifying what Aznx meant by "**the second formula**"—centrally involves determining which symbolic expression is being indexed.

The analysis also sheds light on Sfard's (2008) notion of multiple realizations of a math object. It is not just that the math object "diamond pattern" consists of a tree of realizations such as the drawings, symbolic formulae and narratives related to this pattern. Rather, these realizations only "make sense" within the context of a much larger indexical field, including other patterns, formulae and concepts. For instance, the formula that is the students' solution indexes the n^{th} stage of the pattern, the enclosing square, the excluded corners, the graphical illustrations, the phrase "**diamond pattern**," the original problem statement, and so on. In a phenomenological sense, the whole world is "given" (i.e., indexed implicitly) in the meaning of a single math object. Within the VMT context, it is clear that this whole world is an intersubjective one and the indexical field is necessarily a

co-constructed and jointly reproduced one. The group production and maintenance of a shared indexical network is central to collaborative meaning making and group cognition.

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