

A contribution to the ICLS 2010 symposium: “*Social construction of mathematical meaning through collaboration and argumentation*” by Shirley Atzmon, Rina Hershkowitz (organizer), Chris Rasmussen, Baruch Schwarz (organizer and chair), Gerry Stahl, Megan Wawro, Michelle Zandieh, Mitchell Nathan (discussant)

Computer Mediation of Collaborative Mathematical Exploration

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Two central principles of contemporary learning science theory (Stahl, Koschmann & Suthers, 2006) are:

- Student learning involves *construction* of meaning by the learners (constructivism).
- Learning often takes place originally *inter-subjectively* (socio-cultural theory).

For learning in the discipline of mathematics, these principles imply:

- Student *exploration* of mathematical issues should play a significant role in math education.
- Collaborative approaches should be incorporated, in which *small groups* of students build mathematical understanding together.

The field of computer-supported collaborative learning (CSCL) proposes that these aims could be furthered through the use of networked computers. Online collaboration environments can mediate activities designed to promote math learning in multiple ways. They can provide a workspace, representations of math objects and tools for manipulating the representations, to provide stimulating experiences of math exploration, discovery and meaning making. They can also provide communication media to support productive collaboration and the sharing of knowledge by individuals, small groups and larger communities, such as classrooms and communities of people interested in math.

The Virtual Math Teams (VMT) Project (Stahl, 2009d) is developing an online environment for small groups of students to explore mathematics together. As a result of a design-based research effort at Drexel University since 2003, our software system has evolved to integrate synchronous text chat, shared whiteboard, asynchronous portal and community wiki. Analysis of student interactions in this environment have documented processes of group cognition (Stahl, 2006), which move between graphical, narrative and symbolic modes of collaborative mathematical meaning making (Çakır, Zemel & Stahl, 2009).

We have found that mathematics can be accomplished collaboratively, even by small groups of novice math students helping each other, building sequentially on each other’s moves and exploring together, even across sessions (Sarmiento & Stahl, 2008). Virtual math teams can engage in graphical constructions, verbal accounts and symbolic derivation, using conventional methods of interaction adapted to online media, such as methods of social acknowledgment, proposal offerings, deictic references, questioning and agreement. Issues of presence, orientation and embodiment—so important in face-to-face mathematical discourse—remain critical in virtual media, although in transformed ways.

Our approach to computer-supported collaborative mathematics learning is based on the theory that math cognition is at base a matter of math exploration (Livingston, 1999; Lockhart, 2009) and math discourse (Sfard, 2008; Stahl, 2008). Of course, math discourse involves drawings and symbolic expressions as well as terms, propositions and arguments in natural language. According to (Netz, 1999), the origin of mathematical deduction in ancient Greece was a function of the development of labeled diagrams and a specialized math dialect used for asynchronous collaboration. Consider the flowering of Western mathematics through the exchange of text and diagrams on clay tablets and papyrus documents among Euclid’s colleagues around the Mediterranean; now imagine analogous digital versions of this within online global communities equipped with specialized computer media and tools.

Based on theories of embodied cognition, distributed cognition, discursive cognition and mediated cognition, we are currently extending the VMT virtual learning environment to support dynamic mathematics. We are integrating the first multi-user dynamic geometry system into VMT. This is a port of the single-user GeoGebra system, which provides coordinated symbolic, graphical and spreadsheet representations of geometric constructions. This will allow online teams of math students to propose, explore and solve problems from algebra, geometry, matrices, trigonometry, conics and elementary calculus. The VMT system is instrumented to capture the entire interaction of these groups for replay and analysis by students, teachers and researchers—allowing the students themselves, their teachers and educational researchers to observe and reflect on actual collaborative math processes.

The design of the VMT software supports scripting of educational activities integrating individual, small-group and community meaning making (Stahl, 2009a). The synchronous textual chat and graphical workspaces are associated with asynchronous wiki pages and web browsers. Curriculum units (Powell, Lai & O’Hara, 2009)—

which can be tuned by teachers to classroom circumstances—guide students to explore sequences of open-ended mathematical topics, to pose their own questions, to reflect on multiple problem-solving paths, to bring in mathematical terminology and principles, to summarize their argumentation and to post accounts of their work for other student groups.

As learning-science researchers, we are interested in the details of how math learners make meaning within various learning environments. The VMT system, with its logging and replay facilities, makes visible and persistent the details of the interactions by means of which small online groups co-construct their understandings of mathematical problems and situations. In this symposium presentation, we will take a more detailed and comprehensive look at the interaction data that we began to analyze in CSCL 2007, CSCL 2009 and ICCE 2009 (Koschmann, Stahl & Zemel, 2009; Stahl, 2007; 2009b; Stahl, Zemel & Koschmann, 2009). This is data from Team B during VMT Spring Fest 2006. Three students spent four hours online together discussing, exploring and reflecting upon a number of related pattern problems. We approach this data with a form of interaction analysis based on conversation analysis (Stahl, 2009e). Our new look at the data will extend sequential analysis to consider longer sequences than adjacency pairs—including group-cognitive moves and larger-scale thematic work. It will also look at a more detailed scale to identify different kinds of references—indexical, temporal, semantic and deictic, for instance—that link a given utterance to multiple past postings, drawings, events, resources and future potentialities.

Although chat postings often appear to be chaotic and inscrutable (Stahl, 2009c), it is possible to reconstruct their implicit response structure and to recreate the indexical relations that underlie their ubiquitous deictic references. We can then build up an explicit analysis of the semantic and linguistic structure that was implicit in the student collaboration. (This corresponds to the tacit understanding that participants maintain of the on-going interaction.) From the detailed relationships among lexical elements of the chat postings, we can identify pairs of initiating proposals and responses to them. These can be seen as communicative methods by which the students achieve group-cognitive moves that advance their larger discussion themes. In this way, among others, we analyze the mediation (within computer-based media) of collaborative mathematical exploration. By observing at this unprecedented level of detail the methods by which students actually coordinate their mathematical work and their shared understandings, we can see how collaborative mathematics can take place in a setting like VMT. We observe and analyze how: students co-construct, communicate, negotiate and make shared sense of math representations (graphical, narrative and symbolic); decide as a group what to do next; inquire about things they do not understand; agree on their joint findings. An understanding of student methods for doing such things deepens our sense of how to design technology to support collaborative math exploration and how to guide, script or scaffold it pedagogically. It also advances our understanding of collaboration as sequential interaction, math exploration as discourse and learning as a social process.

References

- Çakır, M. P., Zemel, A., & Stahl, G. (2009). The joint organization of interaction within a multimodal CSCL medium. *International Journal of Computer-Supported Collaborative Learning*, 4(2), 115-149. Web: http://GerryStahl.net/pub/ijCSCL_4_2_1.pdf Doi: <http://dx.doi.org/10.1007/s11412-009-9061-0>
- Koschmann, T., Stahl, G., & Zemel, A. (2009). “you can divide the thing into two parts”: *Analyzing referential, mathematical and technological practice in the VMT environment*. Paper presented at the international conference on Computer Support for Collaborative Learning (CSCL 2009), Rhodes, Greece. Web: <http://GerryStahl.net/pub/cscl2009tim.pdf>
- Livingston, E. (1999). Cultures of proving. *Social Studies of Science*, 29(6), 867-888
- Lockhart, P. (2009). *A mathematician's lament: How school cheats us out of our most fascinating and imaginative art forms*. New York, NY: Belevue Literary Press.
- Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

- Powell, A. B., Lai, F. F., & O'Hara, K. (2009). *Supplemental curriculum unit for online, collaborative problem solving in VMT*. Web: <http://GerryStahl.net/vmt/combinatorics.pdf>
- Sarmiento, J., & Stahl, G. (2008). *Extending the joint problem space: Time and sequence as essential features of knowledge building*. Paper presented at the International Conference of the Learning Sciences (ICLS 2008), Utrecht, Netherlands. Web: <http://GerryStahl.net/pub/icls2008johann.pdf>
- Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.
- Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. 510 + viii pages. Web: <http://GerryStahl.net/mit/>
- Stahl, G. (2007). Meaning making in CSCL: Conditions and preconditions for cognitive processes by groups. *international conference on Computer-Supported Collaborative Learning (CSCL '07)*. Web: <http://GerryStahl.net/pub/cscl07.pdf>
- Stahl, G. (2008). Book review: Exploring thinking as communicating in CSCL. *International Journal of Computer-Supported Collaborative Learning*, 3(3), 361-368. Web: <http://GerryStahl.net/pub/Sfardreview.pdf> Doi: <http://dx.doi.org/10.1007/s11412-008-9046-4>
- Stahl, G. (2009a). Designing a mix of synchronous and asynchronous media for VMT. In G. Stahl (Ed.), *Studying virtual math teams* (ch. 16, pp. 295-310). New York, NY: Springer. Web: <http://GerryStahl.net/vmt/book/16.pdf> Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_16
- Stahl, G. (2009b). *Keynote: How I view learning and thinking in CSCL groups*. Paper presented at the International Conference on Computers and Education (ICCE 2009), Hong Kong, China. Web: <http://GerryStahl.net/pub/iccekeynote2009.pdf>
- Stahl, G. (2009c). Meaning making in VMT. In G. Stahl (Ed.), *Studying virtual math teams* (ch. 26, pp. 505-527). New York, NY: Springer. Web: <http://GerryStahl.net/vmt/book/26.pdf> Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_26
- Stahl, G. (2009d). *Studying virtual math teams*. New York, NY: Springer. 626 +xxi pages. Web: <http://GerryStahl.net/vmt/book> Doi: <http://dx.doi.org/10.1007/978-1-4419-0228-3>
- Stahl, G. (2009e). Toward a science of group cognition. In G. Stahl (Ed.), *Studying virtual math teams* (ch. 28, pp. 555-579). New York, NY: Springer. Web: <http://GerryStahl.net/vmt/book/28.pdf> Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_28
- Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), *Cambridge handbook of the learning sciences* (pp. 409-426). Cambridge, UK: Cambridge University Press. Web: http://GerryStahl.net/cscl/CSCL_English.pdf in English, http://GerryStahl.net/cscl/CSCL_Chinese_simplified.pdf in simplified Chinese, http://GerryStahl.net/cscl/CSCL_Chinese_traditional.pdf in traditional Chinese, http://GerryStahl.net/cscl/CSCL_Spanish.pdf in Spanish, http://GerryStahl.net/cscl/CSCL_Portuguese.pdf in Portuguese, http://GerryStahl.net/cscl/CSCL_German.pdf in German, http://GerryStahl.net/cscl/CSCL_Romanian.pdf in Romanian, http://GerryStahl.net/cscl/CSCL_Japanese.pdf in Japanese
- Stahl, G., Zemel, A., & Koschmann, T. (2009). *Repairing indexicality in virtual math teams*. Paper presented at the International Conference on Computers and Education (ICCE 2009), Hong Kong, China. Web: <http://GerryStahl.net/pub/icce2009.pdf>