Social construction of mathematical meaning through collaboration and argumentation

Shirley Atzmon, the Hebrew University <shirlyazmon@walla.com>, Rina Hershkowitz (organizer), the Weizmann Institute <rina.hershkovitz@weizmann.ac.il> Chris Rasmussen, San Diego State University <chrisraz@sciences.sdsu.edu>, Baruch Schwarz (organizer and chair), the Hebrew University <msschwar@mscc.huji.ac.il>, Gerry Stahl, Drexel University <gs47@drexel.edu>, Megan Wawro, San Diego State University <meganski110@hotmail.com>, Michelle Zandieh, Arizona State University zandieh@asu.edu,
Mitchell Nathan (discussant), University of Wisconsin-Madison <mnathan@wisc.edu>

Symposium overview

The mathematics education research community has considerably evolved during the two last decades towards and within a socio-cultural approach (e.g., Cobb & Bauersfeld, 1995). This approach has first led to fine-grained descriptions in which they could identify specific entities that are central to learning processes in classroom contexts. For example, they identified *socio-mathematical norms* (Yackel & Cobb, 1996) and important practices such as *collective argumentation* (Krummheuer, 1995) or *collective reflection* (Cobb & Yackel, 1996) that are central for understanding learning in classroom contexts. These theoretical tools have helped in describing learning processes in specific social settings (Hershkowitz & Schwarz, 1999). Also, anthropologists have elaborated methodological tools for describing mathematical activities in a succession of activities (see for example, Saxe et al., 2009 for a methodology for describing the 'travel of ideas' through observation of the transformation of form and function of artifacts).

Finally, researchers in mathematics education have adopted a design research approach to elaborate successions of activities in which they used the methodological tools developed to envisage desirable outcomes and values (Cobb, Sophian, et al., 2001; Hershkowitz et al., 2002). Among the central desirable practices, talk practices (e.g., Sfard, 2008) and collaborative practices (e.g., Hoyles & Healey, 1995). In another tradition, scientists from the Computer Supported Collaborative Learning (CSCL) community also shares talk and collaboration practices as settings that promote learning. CSCL scientists have recognized that these practices are not easy to trigger and have subsequently elaborated computerized tools that afford desirable practices (Stahl, Koschman & Suthers, 2006). For example, Suthers (2003) has shown that computerized tools provide *representational guidance* that deeply affects collaborative practices. The CSCL community has developed an impressive number of environments for triggering collaborative and argumentative practices.

This symposium is intended to present perspectives from the two communities in order to understand in depth some key issues in mathematics education. The three contributions of the symposium focus on successive activities that include collaborative and argumentative practices. This kind of focus invites in-depth analyses of learning processes that involve meaningful changes. The following themes emerge from the three contributions:

1. The transformational character of argumentation/collaboration in successive activities
2. Novel ways to mediate the social production of meaning: In Stahl's contribution, these are the multiples channels and mathematical representations that enable the synchronous coordination of actions among dyads; For Rasmussen, Zandieh and Wawro, these are the brokering moves of the teacher; For Hershkowitz, Schwarz & Azmon, these are distinctive talk patterns that propagate in individual and unguided activities.
3. The central role of the design of the learning unit towards productive interactions.

Each of the contributions provides a methodology for a fine-grained follow-up of the unfolding of production of mathematical meaning. Stahl extends discourse analysis methods to synchronous multi-channel communication; Rasmussen, Zandieh and Wawro adopt a cultural-historical activity theory approach to study the transformations of boundary objects between activity systems; Hershkowitz, Schwarz and Atzmon integrate between interactional analyses with inferential statistics to study the effects of kinds of structuring adopted by teachers during their interactions with students on the ways students engage in these interactions and on their argumentative ability in the collective discourse and individual tasks.

References

Cobb, P., & Bauersfeld, H. (1995). *The emergence of mathematical meaning*. Hillsdale, NJ: Erlbaum.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences, 10*(1&2), 113–163.
Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175–190.
Hershkowitz, R. & Schwarz, B. B. (1999). The emergent perspective in rich environments: Some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics, 39*, 149-166.
Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.) *The Emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Erlbaum.
Saxe, G. B., Gearhart, M., Shaughnessy, M., Earnest, D., Cremer, S., Sitabkhan, Y., Platas, L. & Young, A. (2009). A Methodological Framework and Empirical Techniques for Studying the Travel of Ideas in Classroom Communities, In B. B. Schwarz, T. Dreyfus & R. Hershkowitz (Eds.), *Transformation of Knowledge through Classroom Interaction*, (pp. 203-222). London, UK: Routledge.
Schwarz, B. B., Hershkowitz, R. & Prusak, N. (2009). Argumentation and Mathematics. In C. Howe & K. Littleton (Eds.), *Educational Dialogues: Understanding and Promoting Productive Interaction* (pp. 115-141). Routledge.
Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses and mathematizing. Cambridge, UK: Cambridge University Press.
Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), Cambridge handbook of the learning sciences (pp. 409-426). Cambridge, UK: Cambridge University Press.
Suthers, D. D. (2003). Representational guidance for collaborative inquiry. In: J. Andriessen, M. Baker, and D. Suthers (Eds.), *Arguing to Learn: Confronting Cognitions in Computer-Supported Collaborative Learning environments* (pp. 27-46). Kluwer Academic Publishers.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27, 458–477.

Computer Mediation of Collaborative Mathematical Exploration

Gerry Stahl

Two central principles of contemporary learning science theory (Stahl, Koschmann & Suthers, 2006) are:

1. Student learning involves construction of meaning by the learners.
2. Learning often takes place originally inter-subjectively.

For learning in the discipline of mathematics, these principles imply:

- Student exploration of mathematical issues should play a significant role in math education.
- Collaborative approaches should be incorporated, in which small groups of students build mathematical understanding together.

The field of computer-supported collaborative learning (CSCL) proposes that these aims could be furthered through the use of networked computers. Online collaboration environments can mediate activities designed to promote math learning in multiple ways. They can provide a workspace, representations of math objects and tools for manipulating the representations, to provide stimulating experiences of math exploration, discovery and meaning making. They can also provide communication media to support productive collaboration and the sharing of knowledge by individuals, small groups and larger communities, such as classrooms and communities of people interested in math.

The Virtual Math Teams (VMT) Project (Stahl, 2009c) is developing an online environment for small groups of students to explore mathematics together. As a result of a design-based research effort at Drexel University since 2003, our software system has evolved to integrate synchronous text chat, shared whiteboard, asynchronous portal and community wiki. Analysis of student interactions in this environment have documented processes of group cognition (Stahl, 2006), which move between graphical, narrative and symbolic modes of collaborative mathematical meaning making (Çakır, Zemel & Stahl, 2009).

We have found that mathematics can be accomplished collaboratively, even by small groups of novice math students helping each other, building sequentially on each other's moves and exploring together, even across sessions (Sarmiento & Stahl, 2008). Virtual math teams can engage in graphical constructions, verbal accounts and symbolic derivation, using conventional methods of interaction adapted to online media, such as methods of social acknowledgment, proposal offerings, deictic references, questioning and agreement. Issues of presence, orientation and embodiment—so important in face-to-face mathematical discourse—remain critical in virtual media, although in transformed ways.

Our approach to computer-supported collaborative mathematics learning is based on the theory that math cognition is at base a matter of math exploration (Livingston, 1999; Lockhart, 2009) and math discourse (Sfard, 2008; Stahl, 2008). Of course, math discourse involves drawings and symbolic expressions as well as terms, propositions and arguments in natural language. According to (Netz, 1999), the origin of mathematical deduction in ancient Greece was a function of the development of labeled diagrams and a specialized math dialect used for asynchronous collaboration. Consider the flowering of Western mathematics through the exchange of text and diagrams on clay tablets and papyrus documents among Euclid's colleagues around the Mediterranean; now imagine analogous digital versions of this within online global communities equipped with specialized computer media and tools.

Based on theories of embodied cognition, distributed cognition, discursive cognition and mediated cognition, we are currently extending the VMT virtual learning environment to support dynamic mathematics. We are integrating the first multi-user dynamic geometry system into VMT. This is a port of the single-user GeoGebra system, which provides coordinated symbolic, graphical and spreadsheet representations of geometric constructions. This will allow online teams of math students to propose, explore and solve problems from algebra, geometry, matrices, trigonometry, conics and elementary calculus. The VMT system is instrumented to capture the entire interaction of these groups for replay and analysis by students, teachers and researchers—allowing the students themselves, their teachers and educational researchers to observe and reflect on actual collaborative math processes.

The design of the VMT software supports scripting of educational activities integrating individual, small-group and community meaning making (Stahl, 2009a). The synchronous textual chat and graphical workspaces are associated with asynchronous wiki pages and web browsers. Curriculum units (Powell, Lai & O'Hara, 2009)—which can be tuned by teachers to classroom circumstances—guide students to explore sequences of open-ended mathematical topics, to pose their own questions, to reflect on multiple problem-solving paths, to bring in mathematical terminology and principles, to summarize their argumentation and to post accounts of their work for other student groups.

As learning-science researchers, we are interested in the details of how math learners make meaning within various learning environments. The VMT system, with its logging and replay facilities, makes visible and persistent the details of the interactions by means of which small online groups co-construct their understandings of mathematical problems and situations. In this symposium presentation, we will take a more detailed and comprehensive look at the interaction data that we began to analyze in CSCL 2007, CSCL 2009 and ICCE 2009 (Koschmann, Stahl & Zemel, 2009; Stahl, 2007; Stahl, Zemel & Koschmann, 2009). This is data from Team B during VMT Spring Fest 2006. Three students spent four hours online together discussing, exploring and reflecting upon a number of related pattern problems. We approach this data with a form of interaction analysis based on conversation analysis (Stahl, 2009d). Our new look at the data will extend sequential analysis to consider longer sequences than adjacency pairs—including group-cognitive moves and larger-scale thematic work. It will also look at a more detailed scale to identify different kinds of references—indexical, temporal, semantic and deictic, for instance—that link a given utterance to multiple past postings, drawings, events, resources and future potentialities.

Although chat postings often appear to be chaotic and inscrutable (Stahl, 2009b), it is possible to reconstruct their implicit response structure and to recreate the indexical relations that underlie their ubiquitous deictic references. We can then build up an explicit analysis of the semantic and linguistic structure that was implicit in the student collaboration. (This corresponds to the tacit understanding that participants maintain of the on-going interaction.) From the detailed relationships among lexical elements of the chat postings, we can identify pairs of initiating proposals and responses to them. These can be seen as communicative methods by which the students achieve group-cognitive moves that advance their larger discussion themes. In this way, among others, we analyze the mediation (within computer-based media) of collaborative mathematical exploration. By observing at this unprecedented level of detail the methods by which students actually coordinate their mathematical work and their shared understandings, we can see how collaborative mathematics can take place in a setting like VMT. We observe and analyze how: students co-construct, communicate, negotiate and make shared sense of math representations (graphical, narrative and symbolic); decide as a group what to do next; inquire about things they do not understand; agree on their joint findings. An understanding of student methods for doing such things deepens our sense of how to design technology to support collaborative math exploration and how to guide, script or scaffold it pedagogically. It also advances our understanding of collaboration as sequential interaction, math exploration as discourse and learning as a social process.

References

Çakır, M. P., Zemel, A., & Stahl, G. (2009). The joint organization of interaction within a multimodal CSCL medium. *International Journal of Computer-Supported Collaborative Learning, 4*(2), 115-149. Available at http://dx.doi.org/10.1007/s11412-009-9061-0.

Koschmann, T., Stahl, G., & Zemel, A. (2009). *"you can divide the thing into two parts": Analyzing referential, mathematical and technological practice in the VMT environment.* Paper presented at the international conference on Computer Support for Collaborative Learning (CSCL 2009), Rhodes, Greece. Available at http://GerryStahl.net/pub/cscl2009tim.pdf.

Livingston, E. (1999). Cultures of proving. *Social Studies of Science, 29*(6), 867-888.

Lockhart, P. (2009). *A mathematician's lament: How school cheats us out of our most fascinating and imaginative art forms*. New York, NY: Belevue Literary Press.

Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

Powell, A. B., Lai, F. F., & O'Hara, K. (2009). *Supplemental curriculum unit for online, collaborative problem solving in VMT*.

Sarmiento, J., & Stahl, G. (2008). *Extending the joint problem space: Time and sequence as essential features of knowledge building.* Paper presented at the International Conference of the Learning Sciences (ICLS 2008), Utrecht, Netherlands. Available at http://GerryStahl.net/pub/icls2008johann.pdf.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.

Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Available at http://GerryStahl.net/mit/.

Stahl, G. (2007). *Meaning making in CSCL: Conditions and preconditions for cognitive processes by groups.* Paper presented at the international conference on Computer-Supported Collaborative Learning (CSCL '07), New Brunswick, NJ: ISLS. Available at http://GerryStahl.net/pub/cscl07.pdf.

Stahl, G. (2008). Book review: Exploring thinking as communicating in CSCL. *International Journal of Computer-Supported Collaborative Learning, 3*(3), 361-368. Available at http://dx.doi.org/10.1007/s11412-008-9046-4.

Stahl, G. (2009a). Designing a mix of synchronous and asynchronous media for VMT. In G. Stahl (Ed.), *Studying virtual math teams* (pp. 295-310). New York, NY: Springer. Available at http://dx.doi.org/10.1007/978-1-4419-0228-3_16.

Stahl, G. (2009b). Meaning making in VMT. In G. Stahl (Ed.), *Studying virtual math teams* (pp. 505-527). New York, NY: Springer. Available at http://dx.doi.org/10.1007/978-1-4419-0228-3_26.

Stahl, G. (2009c). *Studying virtual math teams*. New York, NY: Springer. Available at http://GerryStahl.net/vmt/book.

Stahl, G. (2009d). Toward a science of group cognition. In G. Stahl (Ed.), *Studying virtual math teams* (pp. 555-579). New York, NY: Springer. Available at http://dx.doi.org/10.1007/978-1-4419-0228-3_28.

Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), *Cambridge handbook of the learning sciences* (pp. 409-426). Cambridge, UK: Cambridge University Press. Available at

http://GerryStahl.net/cscl/CSCL_English.pdf in English,
http://GerryStahl.net/cscl/CSCL_Chinese_simplified.pdf in simplified Chinese,
http://GerryStahl.net/cscl/CSCL_Chinese_traditional.pdf in traditional Chinese,
http://GerryStahl.net/cscl/CSCL_Spanish.pdf in Spanish,
http://GerryStahl.net/cscl/CSCL_Portuguese.pdf in Portuguese,
http://GerryStahl.net/cscl/CSCL_German.pdf in German,
http://GerryStahl.net/cscl/CSCL_Romanian.pdf in Romanian.

Stahl, G., Zemel, A., & Koschmann, T. (2009). *Repairing indexicality in virtual math teams.* Paper presented at the International Conference on Computers and Education (ICCE 2009), Hong Kong, China. Available at http://GerryStahl.net/pub/icce2009.pdf.

Brokering as a Mechanism for the Social Production of Meaning

Chris Rasmussen, Michelle Zandieh, Megan Wawro

A pressing concern in mathematics education is to reveal processes by which inquiry-oriented classrooms enable learners to explore and develop their own reasoning powers while simultaneously connecting them with the collected wisdom and conventions of the discipline (Cobb & Bauersfeld, 1995; Lampert, 2001). A teacher's role in this process is one that often comes with considerable tension. For example, in her work with elementary school students, Ball (1993) posed the tension in the following way: "How do I create experiences for my students that connect with what they now know and care about but that also transcend their present? How do I value their interests and also connect them to ideas and traditions growing out of centuries of mathematical exploration and invention?" (p. 375). Research in inquiry-oriented undergraduate mathematics classroom reveals similar tensions regarding the role of the teacher and other students in the social production and uptake of ideas (e.g., Wagner, Speer, & Rossa, 2007).

In this report we address the aforementioned pressing concern by identifying the brokering moves of the teacher and some students in an undergraduate mathematics class that functioned as a mechanism for the social production of meaning. From an individual cognitive point of view, there are well-established mechanisms that describe how individuals build ideas. From a social point of view, however, mechanisms that describe how ideas are interactively constituted are less developed. Such mechanisms are significant because they address the complex job of teaching and specific teacher moves that promote the social construction of meaning (Rasmussen & Marrongelle, 2008).

In his seminal work on communities of practice, Wenger (1998) highlights how brokering requires the ability to "cause learning" by introducing into one community elements of practice from a different community (Wenger, 1998, p. 109). We adapt Wenger's work to the classroom and consider three different communities: the broader mathematical community, the local classroom community, and the various small groups that make up the local classroom community. The brokers in these communities are the teacher and specific students in the class. A broker, by definition, is someone who has membership status in more than one community. For example, in our case the teacher is a member of the broader mathematics community, the classroom community, and a peripheral member of each of the small groups that make up the classroom community.

Data for the analysis is drawn from classroom videorecordings collected during a 15-week classroom teaching experiment (Cobb, 2000) conducted in an undergraduate differential equations course. Focusing on a one week sequence that led to the reinvention of a sophisticated inscription known by experts as a bifurcation diagram (Rasmussen, Zandieh, & Wawro, 2009), we identified different types of brokering moves that contributed to the social production of meaning. Each of these brokering moves highlight the view that teaching and learning mathematics is a cultural practice, one that is mediated by and coordinated with the broader mathematics community, the local classroom community, and the small groups that comprise the classroom community.

Close analysis of the function of students' and the teacher's discursive contributions resulted in the identification the following three broad categories of broker moves: creating a boundary encounter, bringing participants to the periphery, and interpreting between communities. In the first brokering move category, creating a boundary encounter, a boundary encounter refers to direct as well as indirect encounters between communities. Moreover, boundary encounters involve boundary objects that serve as an interface between different communities. A broker, by virtue of his or her membership in more than one community, is in a position to bring forth boundary objects that can facilitate encounters between communities. For example, we identified instances when the teacher functioned as a broker between the classroom community and the mathematical community by introducing and constituting tasks that engaged learners in indirectly encountering the mathematical community via their participation in mathematizing (Rasmussen, Zandieh, King, & Teppo, 2005; Schwarz, Dreyfus, & Hershkowitz, 2009). The task, as it was constituted, functioned as a boundary object. Creating a boundary encounter was not limited to the teacher, however. The presentation will detail how one small group's presentation of their work on a task resulted in their creation of an inscription that was new to other students in the class and mathematically quite sophisticated. As such, this inscription functioned as a boundary object between the small group community who created it and the classroom community.

We refer to the second brokering category as bringing participants to the periphery. We follow Wenger (1998) in distinguishing between the terms boundary and periphery. The term boundary is more closely aligned with possible discontinuities between communities, whereas the term periphery is more closely aligned with possible continuities between communities. As such, the brokering move of bringing participants to the periphery is one in which the broker helps members of one community move along a continuum toward another community. For example, brokers are able to draw specific ideas out from a small group community so that these ideas become more accessible to the classroom community. Conversely, brokers are in a position to encourage and promote engagement and reflection by the whole class community in ideas that were put forth by a particular small group community. Our analysis revealed that the teacher was more likely than students to function as broker in this capacity.

The third brokering category we identified, interpreting communities, is one in which the broker makes explicit connections between two communities with respect to how ideas are construed, notated, related, or labeled. In comparison to the first brokering category, creating boundary encounters, this third type of brokering move occurs when a broker takes specific steps to fulfill or realize the opportunities that the creating boundary encounter moves offered. Our examples of this type of brokering move involve both the teacher and specific students as brokers as they interpret between the mathematical community and the whole class community and between the whole class community and various small group communities.

The presentation will illustrate and further clarify each of these types of brokering moves. Implications of these brokering moves for teaching and professional development will also be discussed.

References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*(4), 373-397.

Cobb, P., & Bauersfeld, H. (1995). *The emergence of mathematical meaning*. Hillsdale, NJ: Erlbaum.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–334). Mahwah, NJ: Erlbaum.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Rasmussen, C. L., & Marrongelle, K. A. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education 37*, 388-420.

Rasmussen, C., Zandieh, M., King, K. & Teppo, A. (2005). Advancing mathematical activity: A view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7*, 51–73.

Rasmussen, C., Zandieh, M., & Wawro, M. (2009). How do you know which way the arrows go? The emergence and brokering of a classroom mathematics practice. In W.-M. Roth (Ed.), *Mathematical representations at the interface of the body and culture* (pp. 171-218). Charlotte, NC: Information Age Publishing.

Schwarz, B. B., Dreyfus, T., & Hershkowitz, R. (2009). The nested epistemic actions model for abstraction in context. In B. B. Schwarz, T. Dreyfus & R. Hershkowitz (Eds.), *Transformation of Knowledge through Classroom Interaction* (pp. 11-41). London, UK: Routledge.
Wagner, J., Speer, N., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior, 26*, 247-266.
Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge, UK: Cambridge University Press.

Distinctiveness of teachers' discourse patterns and their impact on students' emergent and subsequent argumentative activities

Rina Hershkowitz, Baruch Schwarz, and Shirly Azmon

Although many researchers have recommended guidance practices involved with intensive interactions, argumentation and collaboration as effective ways to foster learning gains in classroom (e.g., Webb, 2009), systematic research on influencing student interaction and learning gains through teacher discourse is under-represented. The goal of this research is twofold. We first aim at uncovering patterns of teacher-led talk in the course of a learning unit. We then aim at investigating whether those patterns impinge on the students' commitment to argumentation that emerged in the talk itself, as well as on characteristics of their individual arguments in a consolidation phase, at the end of the learning unit.

Four teachers, in four different schools, and their Grade 8 students participated in the study. The teachers were invited to teach a learning unit in probability carefully designed to encourage productive argumentation. The teachers were left free to choose the way to manage their lessons: we did not stipulate social settings (individual work, [un]guided small group problem-solving, teacher-led discussions, etc.) and did not provide scripts for teachers' structuring of interactions with learners. All lessons were observed, video documented and transcribed. Three episodes in three different lessons where chosen for each teacher for micro-analysis. Each episode was similar for all four teachers. This similarity between the four teachers concerned not only the problem situations involved but comparable length of time of the analyzed episode. Another criterion for the choice of the episodes was episodes that focused on content whose mastery was checked in an individual activity at the end of the learning unit.

A major finding of this study – supported by qualitative and quantitative analysis, is that *teacher-students interactions were governed by patterns*: These patterns were distinctive and relatively stable, for each teacher, beyond the different tasks and lessons that constituted the learning unit. The distinctiveness of each pattern was characterized as follows:

The kinds of challenges initiated by the teacher. For example, the first teacher generally initiated challenges through open questions that invite the expression of elaborated explanations and dialectical moves. In contrast, the second teacher did not challenge students but rather asked closed questions whose answers were clear and expectable.

The patterns identified at a micro-level conveyed socio-mathematical norms at a meso-level: For example, the first teacher adopted a dialogic-dialectical talk (similar to the exploratory talk identified by Mercer, 1987) in which the diverse claims which were raised, were supported by examples or/and explanations and at the same time, were also challenged by examples or explanations in a rich social context. The construction of knowledge emerged from this tension. In the second class, the talk is closer to a cumulative "traditional" talk governed by IRE patterns (Cazden, 2001) in which initiative is almost exclusively in the teacher's hands. This difference between the two teachers is exemplified in following data; while Teacher 1 dedicates 63% of her utterances, in all three episodes, to encourage argumentation (explanation of claims, arguing, raising dialectical dialog, etc…) and 11% to ask for claims only (short answers, declaring facts), Teacher 2 dedicates only 14% of her utterances to encourage argumentation and 49% to ask for claims. Beyond these data, we identified distinctive patterns involving teachers and students across all teacher-led discussions. These patterns and their validation will be presented in the symposium.

Delegation of responsibilities. The socio-mathematical norms that developed in each class impinged at a macro-level concerning the responsibility of students to their knowledge constructing. For example in the class of the first teacher, the challenges of the teacher led students to feel responsibility to provide elaborated explanations. 90%) of the claims expressed by the students in the three episodes were reasoned and eventually turned to full-fledged arguments. In the second class, the elaboration of explanations was not under the responsibility of the students but of the teacher. Only 18% of the students' claims, in the three episodes in class 2, were reasoned. Interactions appeared as chains of short questions and short claims as answers (70% of the students utterances in class 2, were claims and 46% in class 1) punctuated by social validation of correctness. Quite naturally, correctness was valorized rather than processes that led to the (correct) result.

The transformatory character of argumentation. The enactment of dialogic-dialectical talk (like for the first teacher) in a succession of tasks designed to encourage knowledge construction, led students to discuss mathematical principles under the orchestration of the teacher and to co-construct them with the teacher and when left alone in small groups. It turns then that there are strong bonds between teacher-led dialectic argumentative talk and subsequent co-construction of mathematical principles/concepts.

Adaptive planning as an expression of dialogic-dialectic norms. When the teacher aims at the construction of mathematical principles, it impinges on the structure of the lesson. The order of tasks, within the learning sequence, is only a suggestion that the teacher adopts or does not adopt according to her pedagogical considerations. The more confident the teacher is in her contents' knowledge and pedagogical knowledge, the more flexible she is in planning the activities of her students. In contrast, sticking in an orthodox way to a sequence of tasks and to proposed social settings (like for the second and third teachers) suggests that some teachers view educational goals in a bureaucratic way that preserves the authoritative status of the teacher in the class.

A second major finding suggests that the impact of teacher-led argumentative talk in the classroom, on subsequent individual argument elaboration is deep and subtle. We checked the quality, frequency and correctness of the explanations which were given by her students to support their claims at the end of the learning unit. In this posttest a high percent of students in all classrooms answered to the questions on a claim level, and gave explanations as required in the test. While more than half of students in the first class (in which the patterns of interaction between the teacher and the students were dialogical/dialectical) constructed right arguments (including right reasons), less than third does it in the other classes. In addition, in another test item it appeared that answers that interacted with the first teacher, arguments were richer as they contained more idea units per argument.

In the presentation we will present quantitative data and conclusions will be drawn.

The present research suggests the importance of the mediation of the teacher. This mediation seems to be more productive when the teacher acts as an agent that negotiates meanings with students (Hershkowitz & Schwarz, 1999). The argumentative patterns that characterize talk can either give birth to meaningful constructions or to senseless artifacts. Our findings suggest the importance of in-service teachers' programs focusing on the animation of classroom discussions for the sake of the promotion of mathematical reasoning and for delegating responsibility to students on their learning.

References

Cazden, C. B. (2001). *Classroom discourse*: *The language of teaching and learning*. Portsmouth, NH: Heinemann.

Hershkowitz, R., & Schwarz, B. B. (1999). Reflective processes in a technology-based mathematics classroom. *Cognition and Instruction*, Vol.17 (1), pp. 65 – 91.

Mercer, N. (1997). *The guided construction of knowledge- talk amongst teachers and learners.* Clevedon. Philadelphia. Adelaide.

Schwarz, B. B., Hershkowitz, R. & Prusak, N. (2009). Argumentation and Mathematics. In C. Howe & K. Littleton (Eds.), *Educational Dialogues: Understanding and Promoting Productive Interaction* (pp. 115-141). Routledge. Routledge.