# Interaction Analysis of Student Teams Enacting the Practices of Collaborative Dynamic Geometry

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**Abstract:** This workshop investigates excerpts from the logs of a virtual math team as it engages in dynamic-geometry exploration during eight hour-long sessions in a chat room with a multi-user version of GeoGebra. It describes the display of mathematical reasoning by the team of three eighth-grade students discussing the geometric dependencies of several figures. It follows the meaning-making process of the team to study how the team enacts foundational practices of collaborative dynamic geometry. The workshop focuses on several targeted excerpts in order to develop insights into specific research questions within a larger effort of working out the team's learning trajectory. The workshop takes the form of a number of data sessions of collaborative analysis of detailed discourse and geometric activity.

Collaborative learning has a variety of advantages, both for the participants and for educational researchers. For the participants, it can bring together resources, perspectives and proposals that would not be available to them individually. It can mediate between individual cognition and community knowledge, building group knowledge and group practices that situate community resources and that can subsequently be personalized. Thus, it can provide a non-didactic, student-centered, group-constructivist experience, which can overcome some of the barriers to school mathematics instruction. For educational researchers, it can provide an intimate view of learning processes as they take place in the media of interaction, which can be captured for detailed study. Through careful design of educational environments, authentic learning experiences can be facilitated and documented. The collaborators display for each other their contributions to the group knowledge building and these displays can be observed by others. This workshop makes available displays of a group of students as they learn the fundamentals of dynamic geometry. In it, one can observe learning taking place longitudinally as the student team follows an eight-hour trajectory of mathematical topics.

## Focus of Analysis

The Virtual Math Teams (VMT) Project is an on-going research effort at Drexel and Rutgers Universities to develop a computer-supported collaborative-learning (CSCL) approach to mathematics education (Stahl, 2006; 2009; 2013b). The aim of the VMT Project is to iteratively refine an approach to online collaborative mathematics, including relevant theory, pedagogy and technology. The project has implemented the VMT online environment to support small groups of students working on a sequence of mathematical topics. The VMT software incorporates a multi-user version of GeoGebra, so students can construct and explore dynamic-geometry figures together. Guided by an emerging theory of group cognition, the project has evolved a constructivist sequence of dynamic-geometry activities (Stahl, 2012; 2013a; 2014). A version of these activities was tried in WinterFest 2013 with over a hundred public-school students.

The project is driven by continuous cycles of formative assessment focused on the following three primary goals:

a) To facilitate the engagement of student teams in *collaborative knowledge building* and group cognition in problem-solving tasks of dynamic geometry.
b) To increase the quality and quantity of *productive mathematical discourse* by the small groups of students.
c) To develop effective team practices in exploration, construction and explanation of the *design of dependencies* in dynamic geometry.

In order to work together effectively on mathematical topics in the VMT environment, a group of students must increase its ability to act as a team, to collaborate effectively, to use the tools and features of VMT and of GeoGebra, to decide how to approach stated tasks and to become proficient at analyzing, manipulating and constructing dynamic-geometry figures.

By "collaborative knowledge building" or "group cognition," we mean the goal of having the students work together and proceed through their session as a team—taking turns, checking for agreement and building on each other's contributions so that the meaning making takes place at the group unit of analysis (Stahl, 2006, Ch.21). Taking turns chatting or manipulating geometric figures and adopting interactional roles should contribute to maintaining joint attention as a group and shared meaning making, rather than to a division of tasks among individuals.

By "productive mathematical discourse," we refer to the quality of the text chat within the VMT environment by a team of students to the extent that it furthers their problem-solving efforts as defined by their current dynamic-geometry task and by accepted mathematical practices (CCSSI, 2011). Postings in the chat facility of VMT are often closely associated with the manipulation of geometric objects by the team in the GeoGebra tabs of the same software interface, and should support such manipulation through guidance, explanation and reflection. The goal of the project is to increase the ability of participating teams to engage in productive mathematical discourse over the lifetime of the teams as they take part in successive sessions (Stahl, 2009, Ch.26).

The focus on "the design of dependencies in dynamic geometry" signifies what we target as the core, underlying theme of mastering dynamic geometry. Figures in dynamic geometry must be constructed in ways that build in appropriate dependencies so that when points of the figures are dragged the dependencies are maintained. For instance, an equilateral triangle must be constructed in a way that defines the lengths of the three sides to always be equal; then, even when a vertex of the triangle is dragged to move, rotate or enlarge the triangle, all the sides remain equal to each other. Mastery of dynamic geometry can be defined in terms of the ability to identify effective dependencies in existing figures and to design the construction of such dependencies into new figures (Stahl, 2013b, Ch.5).

The disciplinary application of the VMT Project is on introducing teams of students (and teams of their teachers) to dynamic geometry. Dynamic geometry—in our view—differs from previous presentations of geometry in at least three significant features (Stahl, 2013b, p.63): dynamic dragging, dynamic construction and dynamic dependencies.

Our sequence of topics presented to students in Winter 2013 was intended to provide experiences in these three features. In this workshop, we will try to observe how the students experienced dynamic geometry in their usage of GeoGebra, guided by the instructions in the topics.

By "dynamic dragging," we refer to the multiple roles of the dragging of points and other geometric objects in dynamic geometry (Arzarello et al., 2002; Powell & Grisi-Dicker, 2012). Dragging is not just a way to arrange objects in a static configuration, but rather a way to investigate or confirm relationships in a figure that are invariant under dragging (Hölzl, 1996). For instance, when placing a new point at the intersection of two lines, a student should use the "drag test" to confirm that the point cannot be dragged away from that intersection and that if the lines are dragged the point will remain at the changing intersection. Dragging is also used to investigate conjectures, such as dragging a vertex of a triangle suspected of being equilateral to confirm that the side lengths and angle measures all change together to remain equal. Dynamic dragging represents a different paradigm than the commonsensical visual appearance of figures—requiring a difficult paradigm shift by students (Laborde, 2004).

By "dynamic construction," we mean that students construct geometric objects in ways that will maintain specified relationships dynamically, under dragging. For instance, an isosceles triangle should be constructed with the length of one side defined to be equal to that of another side, not just with the current lengths of the two sides numerically equal. It turns out that the constructions presented by Euclid can be used for dynamic construction. This is because Euclid's constructions establish relationships that hold for any location of their free points, not just for the particular locations shown in a diagram. Understanding geometric objects as the results of dynamic constructions provides insight into the necessity of geometric relationships.

"Dynamic dependencies" underlie the possibility of dynamic constructions, whose specified characteristics remain valid under dynamic dragging. A dynamic isosceles triangle ABC (Figure 1) maintains the equality of two sides, AB and AC, even when an endpoint (B) of one side (AB) is dragged to change its length, because there is a dependency of the length of the second side (AC) on the (dynamic) length of the first side. This dependency may be the result of a construction of point C as being defined to be on a circle centered on point A and defined by point B. As long as point C remains on this circle, no matter how any point is dragged, the lengths of sides AB and AC will remain equal because they are both radii of the same circle. The ability to design dynamic dependencies requires the development of a variety of group-cognitive, mathematical and group-agentic skills.

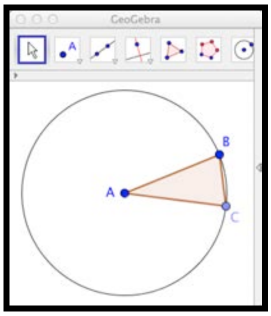

Figure 1. Dynamic isosceles triangle.

## Sequential Interaction Analysis

Methods of evaluating how small groups learn when interacting through computer-supported collaborative learning (CSCL) systems are not well established (Stahl, Koschmann & Suthers, 2006). In particular, the most common methods involve coding and aggregating utterances, which eliminates the sequential structure of the discourse; this can provide comparative measures of outcomes, but does not usually reveal the mechanisms or group processes involved. Such statistical measures do not show how specific support functionality is or is not effective in mediating productive group work, even when it computes a statistical effect of the functionality.

A primary concern for designers of educational interventions should be the extent to which groups using their approach are actually supported in the ways intended by the design of the intervention. Determination of what learning does and does not take place in the environment and the role of specific technical or curricular functionality in supporting or failing to support that learning is essential to re-design for subsequent iterations of the development cycle.

The VMT Project is a design-based research effort, which means that it undergoes cycles of design, implementation, testing, evaluation and re-design (Design-Based Research Collective, 2003). It has gone through countless design cycles during the past decade, evolving a CSCL environment integrating text chat and interactive graphics for small groups of students to learn mathematics together (Çakir & Stahl, 2013; Cakir, Zemel & Stahl, 2009; Stahl, 2008). In particular, the designers of the VMT environment have developed software, curricular resources, teacher-professional-development courses and best practices to introduce students to the core skills of dynamic geometry. Project staff members need periodic feedback on how their prototypes are succeeding in order to redesign for improved outcomes.

The question addressed by this workshop is: How well did students in the WinterFest 2013 iteration of the VMT Project learn the skills that the environment was intended to support? The point is not to come up with a rating of the success of this approach, as though the software, curriculum and pedagogy were in a final state. It is also not to compare how users feel or succeed when using VMT versus not using this support system. Rather, the aim is to observe just *how* teams of students learn targeted skills or how they fail to learn them within the designed environment. These observations should be concrete enough to drive future cycles of re-design.

In the beginning of 2013, the Math Forum sponsored a "WinterFest" in which teams of three to five students participated in a sequence of eight online sessions using the VMT environment. The groups were organized by teachers who had been through a semester-long teacher-professional-development course in collaborative-dynamic-mathematics education, offered by Drexel University and Rutgers-Newark. The VMT environment at that time included the first multi-user dynamic-geometry system, an adaptation to VMT of the open-source GeoGebra system (www.GeoGebra.org). The mathematical topics for the eight sessions were embedded in multiple tabs of VMT chat rooms for each of the sessions. The topics were developmentally designed to gradually convey an understanding of geometric dependencies.

Although design-based research is a popular approach to the development of educational software, especially in CSCL and Technology-Enhanced Learning, there is little agreement on how to evaluate trials in a way that contributes systematically to re-design. The theory of Group Cognition proposed that one could make collaborative learning—or group cognition—visible (Stahl, 2006, Ch. 18), based on Garfinkel's principles of ethnomethodological description. This is because meaning making is an intersubjective or small-group process, requiring group members to make their contributions visible to each other, and therefore also to researchers (Stahl, 2006, Ch. 16). As the editor's introduction to (Garfinkel, 2002) explains, "the sounds and movements that comprise social action are meaningful creations that get their meaning from the shared social contexts of expectation within which they are enacted…. Intended meanings, however, can only be shared if they can be successfully displayed before others in expected ways" (p. 57).

This workshop's analysis of the meaning-making process focuses on the sequential response structure (or "adjacency pairs") of utterances, which build on previous utterances and elicit further possible, anticipated or expected responses (Schegloff, 2007). The analysis re-constructs the web of situated semantic references: "The meaning of the interaction is co-constructed through the building of a web of contributions and consists in the implicit network of references" (Stahl, 2009, p. 523).

Most sequential analyses of conversation are limited to brief excerpts; this workshop's analysis of each hour-long session—especially considered in the context of the series of eight sessions—goes beyond the analysis of so-called "longer sequences" (Stahl, 2011) toward longitudinal analysis of collaborative learning across multiple sessions. We want to observe the collaborative learning of the team as it evolves during eight hours of intense, complex interaction.

Analysis of longer sequences is more important in studying geometry instruction than in most conversation analysis. While ethnomethodologically informed conversation analysis (Schegloff, 2007) is interested in how meaning is socially constructed in the momentary interaction, we are here concerned with (a) longer chains of meaning making and (b) how the meaning making changes from one time to a later time. The data required for such analysis is available in detailed spreadsheets and files for replaying sessions at www.gerrystahl.net/vmt/icls2014.

## The Display of Collaborative Learning

Learning is often conceived as a change in propositional knowledge possessed by an individual student. Opening up an alternative to this view, Vygotsky argued that students could accomplish epistemic tasks in small groups before they could accomplish the same tasks individually—and that much individual learning actually resulted from the earlier group interactions (Vygotsky, 1930/1978), rather than the group being reducible to its members as already formed individual minds. He conceived the group interactions as mediated by artifacts, such as representational images and communication media. More recently, educational theorists have argued that student processes of becoming mathematicians or scientists, for instance, are largely a matter of mastering the linguistic practices of the field.

Views of learning focused on individual minds require methodologies that test individual changes over time and interpret them in terms of some theory of mental processes that are not directly observable such as mental models, mental representations, cognitive change, cognitive convergence, cognitive conflict, etc. In contrast, a view of learning focused on group interaction can hope to observe processes of group cognition more directly. A reason for this is that in order for several students to collaborate effectively, they must display to each other what the group is planning, recalling, doing, concluding and accomplishing. These displays take place in the physical world through speech, gesture and action or in the virtual world through text chat and graphics. They are in principle visible to researchers as well as to participants.

In practical terms, it is often difficult for educational researchers to capture enough of what is taking place in group interactions to be able to reliably understand what is going on as well as the participants do. Capturing face-to-face collaborative interaction in an authentic classroom involves many problematic complications, including selecting video angles, providing adequate lighting, capturing multiple high-quality audio recordings, audio-to-text transcription and synchronization of all the data streams (Suchman & Jordan, 1990). In this paper, we present data that was automatically captured during an online chat involving three students. All of their communication and action that was shared within the group is available to us as analysts in exactly the same media and format as it appeared for the students, as well as in automatically generated textual logs. So none of the issues of selection, interpretation, partiality and representation of audio, gesture and other data are present here the way they are in face-to-face settings. In particular, all the representational graphical images and textual language shared by the group are available in detail to the researchers. We can use methods of interaction analysis or conversation analysis (Jordan & Henderson, 1995; Schegloff, 2007), adapted to our online math-education setting (Zemel & Çakir, 2009).

Of course, interpretation and analysis of meaning can still be controversial in our approach. However, the raw data is available and excerpts of it can be included easily in the presentation so that readers can see where interpretive decisions have been made and can judge for themselves the plausibility of the analysis. There are no hidden stages of imposing categories and theoretical perspectives on the presented data. The students speak in their own words. Furthermore, the data displays the work of the students as they are engaged together, rather than providing retrospective views of the students in reaction to questions when they are no longer engaged in their mathematical tasks or situated together online.

For some time, we have proposed the idea of focusing on the small group as the unit of analysis and foregoing any reliance on theories of mental processes in favor of observing the visible interactions (Stahl, 2006). We spent a decade developing an online environment which could support collaborative learning of mathematics and also be instrumented to capture group interaction (Stahl, 2009). Our research and theory now distinguish distinct learning processes at the individual, small-group and community units of analysis (Stahl, 2013b, Ch.8). Although we recognize that these processes are inextricably intertwined in reality, we focus methodologically in this workshop on the group unit of analysis, which is where individual learning, group becoming and community practices are often most visibly displayed.

The data consisting of the eight hours of online interaction by a particular team in WinterFest 2013 provide a rich source for analysis of collaborative learning of dynamic geometry. The learning of geometry has been a pivotal moment in the cognitive development of many people and of humanity generally, but also a difficult achievement for many people (Lockhart, 2009; Sinclair, 2008; Stahl, 2013b, Ch.1). This team of students develops many skills and practices important to collaborative learning and to doing mathematics.

Perhaps more importantly from a research perspective, the student team displays its learning in its chat discourse and in its geometric actions. In the displayed group learning, we can see how progress in collaboration, math discourse and dynamic geometry comes about. The goal of the following analysis is therefore to let the students' voices speak for themselves and to observe what the students display to each other. One qualification to this ideal is that making sense of math discourse often requires the analyst to be aware of the underlying mathematics of the topic being discussed (Livingston, 1986). For this reason, we will sometimes mention the intended lesson of a curricular resource in order to understand how the team took advantage of a learning opportunity. The WinterFest series of online sessions is designed to be an educational experience. Since there is typically no teacher present in the chat rooms during the collaborative sessions, the instructional role is largely "scripted" by the texts in the chat room tabs. The analysis will show how the team enacted those instructions—how successful the curricular design was.

The goal in this workshop is to observe the student displays of how a particular virtual math team learns to collaborate, discuss mathematics and engage in dynamic geometry during WinterFest 2013. We try to observe the learning as it takes place. The VMT instrumentation allows each session to be replayed, so one can see the same thing each student saw. By studying the interactions in which students display their emergent understanding to each other, one can see the collaborative learning taking place. This makes available to researchers not just occasional pre and post states, but the on-going problem-solving and knowledge-building processes as they unfold at the group unit of analysis.

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