DESIGNING A LEARNING ENVIRONMENT TO PROMOTE MATH DISCOURSE

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This paper reports on the design of a virtual learning environment that integrates:

- **synchronous and asynchronous media**
- with **multi-user** dynamic-math visualization
- and exploration toolbox.

This **VMT-with-GeoGebra** environment is designed to support the production of significant math discourse among small online groups of teachers or of students.

[Collaborative e-learning of math]
Small teams of students drawing & chatting
The Virtual Math Teams environment
Multi-User GeoGebra

- GeoGebra for small groups of students
- Engage in dynamic geometry together
- Drag and explore together
- Chat about actions and noticings
- Construct and investigate collaboratively
- Share and test each other’s hypotheses
- Explain and prove to each other
- Build on each other’s custom tools and constructions
Exploration & Discourse: VMT-with-GeoGebra
Construction & Chat

Chat (0)

vmt 6:03:26 PM EST: Interesting
... I dragged corner A around and watched how the areas of poly 1 and e changed

Gerry 6:04:05 PM EST: yeah, I c
Gerry 6:04:10 PM EST: poly2 seems to always be about half of poly1

vmt 6:05:14 PM EST: I bet that is always true because it is built from the midpoints of poly1

vmt 6:08:11 PM EST: Look! I connected A and C -- that forms two sets of similar triangles. I bet that if we made triangles DEH and DAC that DEH would be 1/4 the area of DAC because its b and h are 1/2

Gerry 6:10:00 PM EST: Cool! That proves it. If we draw BD, we will have 4 triangles each with a quarter the area of half the quadrilateral! Very elegant

Nice work, partner! Thanks for explaining it to me.
Virtual Math Teams Environment

• An integrated online environment for small teams of students to do math together
• Combines text chat with drawing spaces and spaces for storing ideas and findings
• Teachers can configure chat rooms for different topics and tools
• Lobby, wiki, multiple tabs for constructions, activity topic, help pages
The VMT Lobby

Virtual Math Teams 3.0-Dev.03

Welcome Professor

View Chat Rooms as
Math Subject Tree Tabular List

Filter Chat Rooms By...
Project
Last Activity

Apply filters Use default filters

Geometry (2 Topics)
- Activity1 (9 Rooms, 5 Active)
- Activity2 (1 Room, 0 Active)

- Lucky Num
- Team Bee_1

<table>
<thead>
<tr>
<th>Username</th>
<th># of Messages</th>
<th>Last Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>at373</td>
<td>10</td>
<td>Feb 1, 2012 19:50</td>
</tr>
<tr>
<td>chartizek</td>
<td>25</td>
<td>Feb 1, 2012 19:56</td>
</tr>
<tr>
<td>charlie_mcmichael</td>
<td>141</td>
<td>Feb 1, 2012 19:50</td>
</tr>
<tr>
<td>gerry</td>
<td>12</td>
<td>Jan 31, 2012 23:19</td>
</tr>
<tr>
<td>professor</td>
<td>8</td>
<td>Feb 1, 2012 19:57</td>
</tr>
</tbody>
</table>

Add to Favorites Save as JSON View Chat Log

Students find chat rooms with activities
Teachers overview student work
Researchers, teachers, students access chat logs
The VMT Chat Room

File Edit Chat GeoGebra

Materials:
GeoGebra Summary A B C D Shared Whiteboard Topic Wiki

File Edit View Perspectives Options Tools Window Help

Chat (1)
Professor leaves the room
2/6/12 11:03:57 PM EST
Professor joins the room
2/6/12 11:24:03 PM EST
Professor leaves the room
2/6/12 11:25:12 PM EST
Professor joins the room
9:14:50 PM EST

Professor 9:18:40 PM EST: I moved the blue triangle
Professor 9:18:52 PM EST: So it seems to be generic
Professor 9:19:00 PM EST: or scalene
Professor 9:19:18 PM EST: I think there is an equalateral
Professor 9:19:27 PM EST: and a right triangle
Professor 9:19:38 PM EST: and probably an iscosles
Professor 9:20:03 PM EST: Does everyone else agree with my conclusion?

Message:
How do you think these were constructed?
Turn Taking for Multi-User Control
The VMT Wiki

### Probability

Here are a set of challenges related to probability problems. **You can contribute** by adding your ideas about applying a strategy to a problem (adding content to a P#S# page), proposing a new strategy (adding a new column) or adding a new challenge (row).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1. The sock drawer</td>
<td>P1S1</td>
<td>P1S2</td>
<td>P1S3</td>
<td>P1S4</td>
<td>P1S5</td>
</tr>
<tr>
<td>P2. Box with three cards</td>
<td>P2S1</td>
<td>P2S2</td>
<td>P2S3</td>
<td>P2S4</td>
<td>P2S5</td>
</tr>
<tr>
<td>P3. Seating arrangements</td>
<td>P3S1</td>
<td>P3S2</td>
<td>P3S3</td>
<td>P3S4</td>
<td>P3S5</td>
</tr>
<tr>
<td>P4. Baseball_World_Series</td>
<td>(P4-S1 Example)</td>
<td>(P4-S2 Example)</td>
<td>(P4-S3 Example)</td>
<td>(P4-S4 Example)</td>
<td>P4S5</td>
</tr>
<tr>
<td>P5. Duck hunters</td>
<td>P5S1</td>
<td>P5S2</td>
<td>P5S3</td>
<td>P5S4</td>
<td>P5S5</td>
</tr>
<tr>
<td>P6. Clock hands</td>
<td>P6S1</td>
<td>P6S2</td>
<td>P6S3</td>
<td>P6S4</td>
<td>P6S5</td>
</tr>
<tr>
<td>P7. Length of Random Chords</td>
<td>P7S1</td>
<td>P7S2</td>
<td>P7S3</td>
<td>P7S4</td>
<td>P7S5</td>
</tr>
<tr>
<td>P8. New Problem</td>
<td>P8S1</td>
<td>P8S2</td>
<td>P8S3</td>
<td>P8S4</td>
<td>P8S5</td>
</tr>
</tbody>
</table>

If you need them, here are some resources for probability

Categories: ProblemSolving | VMT
Curricular Activities

- Based on US Common Core Standards
- Stress noticings and conjectures
- Promote math discourse
- Encourage collaboration
- Include individual reflection and group discussion
- Structured, guided collaborative learning, leading to open-ended creative exploration
More Exploration, Less Instruction
Reflection on Math Discourse

- Access to VMT Chat logs in convenient formats
- VMT Wiki pages for sharing findings
- VMT Replayer to review action in detail: drawing and chat coordinated in playback mode
The VMT Re-Player

Here is point D on the circle and on line segment CD. Try to drag this point and watch the circle.
<table>
<thead>
<tr>
<th>Time of Posting</th>
<th>andicat</th>
<th>Annie</th>
<th>jr6g</th>
<th>loretta</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:45:03</td>
<td></td>
<td>i don't know how to do a perpendicular</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:45:16</td>
<td></td>
<td></td>
<td></td>
<td>should we do</td>
</tr>
<tr>
<td>14:45:20</td>
<td></td>
<td>i need my tool!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:45:24</td>
<td></td>
<td>So, Jen, what do you think would go into a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:45:37</td>
<td></td>
<td>a 90degree angle and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:45:40</td>
<td></td>
<td>i created a tool to make a perpendicular</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:46:01</td>
<td></td>
<td></td>
<td></td>
<td>can we use the built in tool to do</td>
</tr>
<tr>
<td>14:46:13</td>
<td></td>
<td>I'm thinking that we can use the built-in</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>perpendicular tool.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:46:39</td>
<td></td>
<td>oh - didn't know that</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:46:42</td>
<td></td>
<td></td>
<td></td>
<td>its under the intersect point</td>
</tr>
<tr>
<td>14:46:47</td>
<td></td>
<td></td>
<td></td>
<td>the perpendicular tool is under the fourth</td>
</tr>
<tr>
<td>14:46:48</td>
<td></td>
<td>i thought it was only something we created</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Professional Development

- Special courses for math teachers
- Full credit toward degree and certification
- Flexible online offering
- Includes synchronous contact with other teachers in small groups
- Prepares for use of technology and curriculum in classrooms
- Teachers try VMT-with-GeoGebra and plan for its use by their students
Activities for Dynamic Geometry

- Basic geometry from US Common Core State Standards and Math Practices
- Main propositions from Book 1 of Euclid
- Relationships needed for problem solving
- How to construct dynamic-geometry objects
- How to design dependencies
- How to create custom construction tools
- Euclidean construction and transformations
- Open-ended explorations and inquiry
Some Case Studies of VMT

• how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face;

• how the structure of text chat interaction differs from spoken conversation;

• how the media of graphical diagrams, textual narratives and symbolic representations can be intimately interwoven to build deep math understanding;

• how deictic referencing is important to establishing shared understanding;

• how students co-construct a joint problem space and accomplish collaborative meaning making and knowledge building;

• how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and

• how a methodology based on interaction analysis can be used for a science of group cognition.
For further information

- http://GerryStahl.net
- Slides: http://GerryStahl.net/pub/icme2012.ppt
For further information

- Gerry@MathForum.org
- PowellAB@rutgers.edu
- http://vmt.mathforum.org/vmt/courses.html
- http://vmt.mathforum.org/VMTLobby