

DESIGNING A LEARNING ENVIRONMENT TO PROMOTE MATH DISCOURSE

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An Environment for Online Math Discourse

This paper reports on the design of a virtual learning environment that integrates:

- **synchronous** and **asynchronous** media
- with **multi-user** dynamic-math visualization
- and exploration toolbox.

This **VMT-with-GeoGebra** environment is designed to support the production of significant math discourse among small online groups of teachers or of students.

[Collaborative e-learning of math]

Small teams of students drawing & chatting

VMT Feedback

We were very impressed in the approach that divided the figure into the horizontal lines and the vertical lines and the quicksilver with which Quicksilver filled out the approach. It seemed as though you also were paying attention to each other's work and quickly reached agreement. You handled the technology of the chat software and the work activity.

We also noticed two places in the chat where some kinds of conversations did not happen. There was a point where 44 was posted as the number of sticks and 40 was offered as a correction. There was no discussion of how 44 was calculated. At another moment, Quicksilver posted an explanation of the pattern of growth that was not discussed.

There was a sense in which you indicated that your work was done when you had at least one answer for the questions in the problem. For the next step we will encourage you to think to one about the different approaches and the problems that

top/bottom $2n(n+1)$

Derived from
 $(1+70)/2 + 70 \cdot 2$

$$\sum_{n=1}^n = \frac{n(n+1)}{2}$$

Azrx
Gerry
Quicksilver
bwang8

■■■■■■■

bwang8 5/16/06 7:14:29 PM EDT: The equation would still be the same, right?

■■■

Quicksilver 5/16/06 7:14:46 PM EDT: I think so

bwang8 5/16/06 7:14:47 PM EDT: because there are the same number of cube each level

Quicksilver 5/16/06 7:14:50 PM EDT: but lets explain that

Quicksilver 5/16/06 7:14:58 PM EDT: bcuz that was in the feedback too

■

Azrx 5/16/06 7:15:13 PM EDT: It would make sense to me that the formula is the same.

■

Azrx 5/16/06 7:15:23 PM EDT: But I don't see why it should either.

■■■

Azrx 5/16/06 7:15:31 PM EDT: I can't find a specific explanation.

■■■

I mean there should technically be some changes, right?

Azrx is typing

Speed: 1

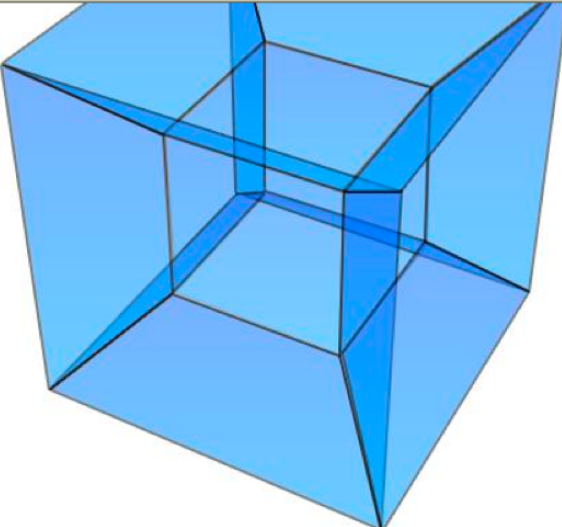
The Virtual Math Teams environment

ConcertChat Session Player - Room : channel:OID::1147211767857

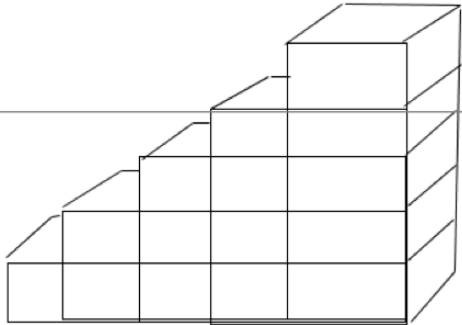
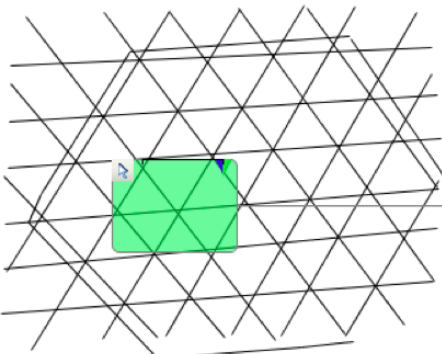
Whiteboard:

on your own solve them. Use the arrow by using the in order to

Last whiteboard action by 137 (5/16/06 7:17:47 PM EDT)



squares:
 $n(n-1)/2$
diamond:
 $n^2+(n-1)^2$



Current users:

137
Jason
nan
qwertyuiop

Chat: (0)

does everyone know what other people are doing?

- 137 5/16/06 7:14:25 PM EDT: Yes?
- qwertyuiop 5/16/06 7:14:25 PM EDT: no-just making triangles
- 137 5/16/06 7:14:33 PM EDT: I think...
- Jason 5/16/06 7:14:34 PM EDT: yeah
- nan 5/16/06 7:14:46 PM EDT: good 😊
- qwertyuiop 5/16/06 7:14:51 PM EDT: triangles are done
- 137 5/16/06 7:15:08 PM EDT: So do you want to first calculate the number of triangles in a hexagonal array?
- qwertyuiop 5/16/06 7:15:45 PM EDT: What's the shape of the array? a hexagon?
- 137 5/16/06 7:16:02 PM EDT: Ya.
- qwertyuiop 5/16/06 7:16:15 PM EDT: ok...
- Jason 5/16/06 7:16:41 PM EDT: wait-- can someone highlight the hexagonal array on the diagram? I don't really see what you mean...
- Jason 5/16/06 7:17:30 PM EDT: hmm.. okay
- qwertyuiop 5/16/06 7:17:43 PM EDT: oops
- Jason 5/16/06 7:17:44 PM EDT: so it has at least 6 triangles?
- Jason 5/16/06 7:17:58 PM EDT: in this, for instance

Message:

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Multi-User GeoGebra

- **GeoGebra for small groups of students**
- **Engage in dynamic geometry together**
- **Drag and explore together**
- **Chat about actions and noticings**
- **Construct and investigate collaboratively**
- **Share and test each other's hypotheses**
- **Explain and prove to each other**
- **Build on each other's custom tools and constructions**

Exploration & Discourse: VMT-with-GeoGebra Construction & Chat

TabbedChat: vmt (CID:1261090089485)

File Edit Chat GG File GG Edit GG View GG Options GG Tools GG Window GG Help

Material: Whiteboard GeoGebra Summary

Algebra View Graphics View

Free Objects

- A(-0.4, 9.58)
- B(2.38, 1.76)
- C(9.24, 0.66)
- D(8.22, 6.4)

Dependent Objects

- E(3.91, 7.99)
- F(0.99, 5.67)
- G(5.81, 1.21)
- H(8.73, 3.53)
- a = 8.3
- b = 6.95
- c = 5.83
- d = 9.19
- e = 3.73
- f = 6.57
- g = 3.73
- h = 6.57
- i = 13.13
- poly1 = 48.41
- poly2 = 24.21

Auxiliary Objects

EFcH, here

vmt 6:03:26 PM EST: interesting ... I dragged corner A around and watched how the areas of poly 1 and e changed

Gerry 6:04:05 PM EST: yeah, i c

Gerry 6:04:30 PM EST: poly2 seems to always be about half of poly1

vmt 6:05:14 PM EST: I bet that is always true because it is built from the midpoints of poly1

vmt 6:08:11 PM EST: Look! I connected A and C -- that forms two sets of similar triangles. I bet that if we made triangles DEH and DAC that DEH would be 1/4 the area of DAC because its b and h are 1/2

Gerry 6:10:00 PM EST: Cool! That proves it. If we draw BD, we will have 4 triangles each with a quarter the area of half the quadrilateral! Very elegant

Message: Nice work, partner! Thanks for explaining it to me.

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Virtual Math Teams Environment

- **An integrated online environment for small teams of students to do math together**
- **Combines text chat with drawing spaces and spaces for storing ideas and findings**
- **Teachers can configure chat rooms for different topics and tools**
- **Lobby, wiki, multiple tabs for constructions, activity topic, help pages**

The VMT Lobby

Virtual Math Teams 3.0-Dev.03

Welcome Professor

- ☐ New to VMT?
- ☐ List of All Rooms
- ☐ My Profile
- ☐ My Teammates
- ☐ My Rooms
- ☐ Messages
- ☐ Manage Activities

- [VMT Help](#) Pages
- [VMT Sandbox](#) Room
- [VMT Lounge](#) Room
- [VMT Wiki](#) Pages
- [VMT Replayer 2.2](#)
- [VMT Replayer 3 Dev 2 \(12/22 - 1/30\)](#)
- [VMT Replayer 3 - current](#)
- [Logout](#)

View Chat Rooms as

Math Subject Tree Tabular List

Filter Chat Rooms By...

Project: iSchool Last Activity: Show All

▼ **Geometry** (2 Topics)

- ▶ **Activity1** (9 Rooms, 0 Active)
- ▼ **Activity2** (0 Rooms, 0 Active)
 - ▶ **Lucky Number 1**
 - ▼ **Team Bee 1**

Username	# of Messages	Last Active
at373	10	Feb 1, 2012 19:50
cbartizek	25	Feb 1, 2012 19:56
charlie_mcmichael	141	Feb 1, 2012 19:50
gerry	12	Jan 31, 2012 23:19
professor	8	Feb 1, 2012 19:57

Students find chat rooms with activities

Teachers overview student work

Researchers, teachers, students access chat logs

The VMT Chat Room

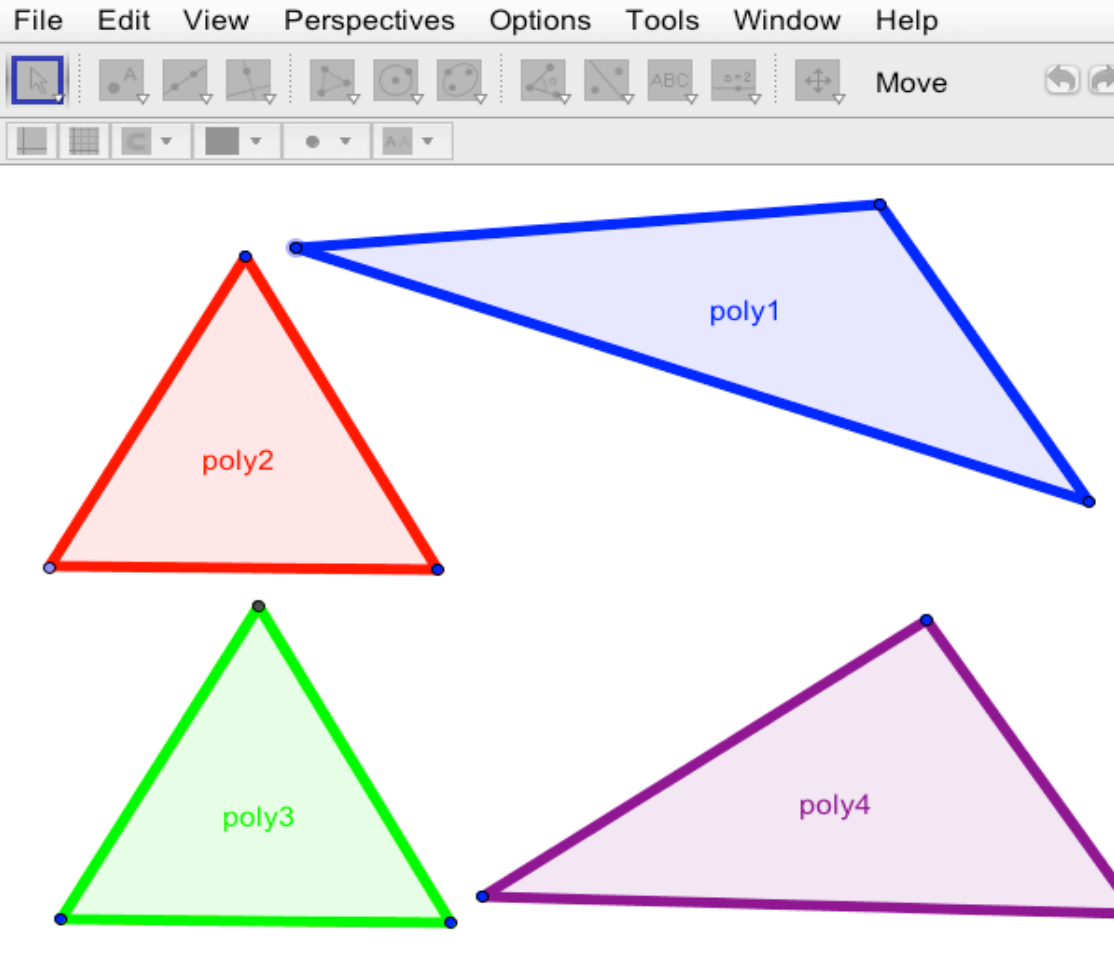
TabbedChat: Professor (CID:1328585967654)

File Edit Chat GeoGebra

Material: +

GeoGebra Summary A B C D Shared Whiteboard Topic Wiki

File Edit View Perspectives Options Tools Window Help



poly2

poly1

poly3

poly4

Refresh View Take Control nobody has control

Current users:
Professor

Chat: (1)

- Professor leaves the room 2/6/12 11:03:57 PM EST
- Professor joins the room 2/6/12 11:24:03 PM EST
- Professor leaves the room 2/6/12 11:25:12 PM EST
- Professor joins the room 9:14:50 PM EST

Professor 9:18:40 PM EST: I moved the blue triangle

Professor 9:18:52 PM EST: So it seems to be generic

Professor 9:19:00 PM EST: or scalene

Professor 9:19:18 PM EST: I think there is an equilateral

Professor 9:19:27 PM EST: and a right triangle

Professor 9:19:38 PM EST: and probably an isosceles

Professor 9:20:03 PM EST: Does everyone else agree with my conclusion?

Message:
How do you think these were constructed?

Turn Taking for Multi-User Control

The screenshot displays the GeoGebra software interface. At the top, there is a menu bar with options: File, Edit, Chat, GG File, GG Edit, GG View, and GG Options. Below the menu bar, a 'Material:' section contains buttons for Whiteboard, GeoGebra, Summary, Topic, and Help. The main toolbar features a hand icon, a 'Take Control' button, a 'Release Control' button, a mouse cursor icon, a vertical dotted line, a point 'A' icon, and a line icon. The 'Algebra View' panel on the left lists objects in three categories: Free Objects (A(-3.02, 0.56), B(0.22, 1.36), C(0.12, -4.42)), Dependent Objects (F(2.74, -1.4), G(-0.58, 4.6), a = 3.34, b = 0, c: $(x - 0.22)^2 + (y - 1.36)^2$, g: $-3.24x - 0.8y = -1.8$), and Auxiliary Objects. The 'Graphics View' panel on the right shows a coordinate plane with a circle, a line, and points A, B, C, and G. Two black arrows point from the top right towards the 'Take Control' and 'Release Control' buttons.

Material:

Whiteboard GeoGebra Summary Topic Help

Take Control

Release Control

Algebra View Graphics View

Free Objects

- A(-3.02, 0.56)
- B(0.22, 1.36)
- C(0.12, -4.42)

Dependent Objects

- F(2.74, -1.4)
- G(-0.58, 4.6)
- a = 3.34
- b = 0
- c: $(x - 0.22)^2 + (y - 1.36)^2$
- g: $-3.24x - 0.8y = -1.8$

Auxiliary Objects

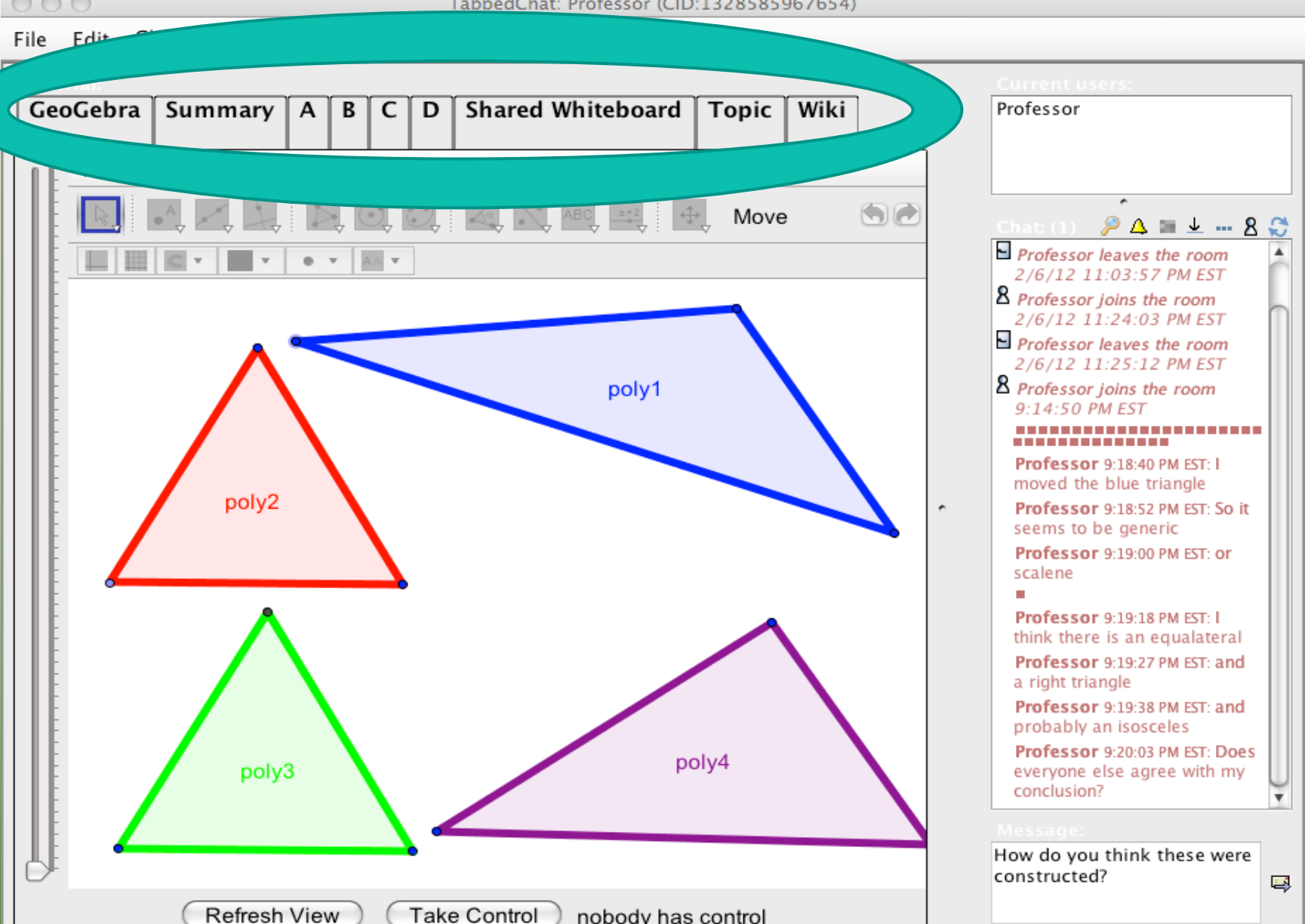
Last whiteboard action by Gerv (1:57:11 PM EDT)

Shared & Personal Spaces; Synch/Asynch

TabbedChat: Professor (CID:1328585967654)

File Edit

GeoGebra Summary A B C D Shared Whiteboard Topic Wiki



poly2

poly1

poly3

poly4

Refresh View Take Control nobody has control

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The VMT Wiki

Gerry my talk my preferences my watchlist my contributions log out

article discussion edit history move watch

Probability

Here are a set of challenges related to probability problems. **You can contribute** by adding your ideas about applying a strategy to a problem (adding content to a P#S# page), proposing a new strategy (adding a new column) or adding a new challenge (row).

Probability Strategies & Problems	S1. Drawing balls from a jar	S2. Solve Complementary Problem	S3. Enumerate & Organize your cases	S4. Use a Tree Diagram	S5. New Strategy
P1. The sock drawer	P1S1	P1S2	P1S3	P1S4	P1S5
P2. Box with three cards	P2S1	P2S2	P2S3	P2S4	P2S5
P3. Seating arrangements	P3S1	P3S2	P3S3	P3S4	P3S5
P4. Baseball_World_Series	(P4-S1 Example)	(P4-S2 Example)	(P4-S3 Example)	(P4-S4 Example)	P4S5
P5. Duck hunters	P5S1	P5S2	P5S3	P5S4	P5S5
P6. Clock hands	P6S1	P6S2	P6S3	P6S4	P6S5
P7. Length of Random Chords	P7S1	P7S2	P7S3	P7S4	P7S5
P8. New Problem	P8S1	P8S2	P8S3	P8S4	P8S5

If you need them, here are some [resources for probability](#)

Categories: [ProblemSolving](#) | [VMT](#)



navigation

- [VMT Lobby](#)
- [Wiki Main Page](#)
- [Recent changes](#)
- [Help](#)

search

Go

Search

toolbox

- [What links here](#)
- [Related changes](#)
- [Upload file](#)
- [Special pages](#)
- [Printable version](#)
- [Permanent link](#)

Curricular Activities

- **Based on US Common Core Standards**
- **Stress noticings and conjectures**
- **Promote math discourse**
- **Encourage collaboration**
- **Include individual reflection and group discussion**
- **Structured, guided collaborative learning, leading to open-ended creative exploration**

More Exploration, Less Instruction

TabbedChat: Professor (CID:1328585967654)

File Edit Chat GeoGebra

Material: GeoGebra Summary A B C D Shared Whiteboard Topic Wiki

File Edit View Perspectives Options Tools Window Help

poly1

poly2

poly3

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Message: How do you think these were constructed?

Reflection on Math Discourse

- **Access to VMT Chat logs in convenient formats**
- **VMT Wiki pages for sharing findings**
- **VMT Replayer to review action in detail: drawing and chat coordinated in playback mode**

The VMT Re-Player

Material: GeoGebra

Current users: tutorA

Chat (0)

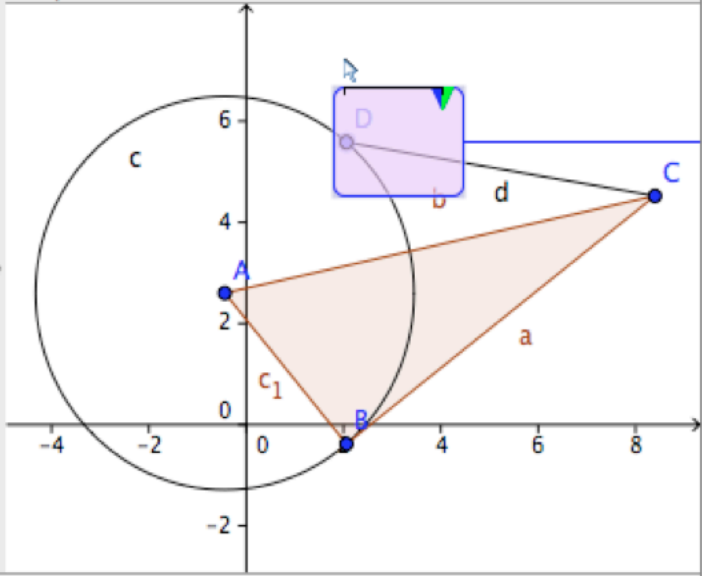
Free Objects

- A(-0.44, 2.6)
- B(2.06, -0.38)
- C(8.4, 4.52)
- D(2.06, 5.58)

Dependent Objects

- a = 8.01
- b = 9.05
- c: $(x + 0.44)^2 + (y - 2.6)^2$
- c₁ = 3.89
- d = 6.43
- poly1 = 15.57

Auxiliary Objects



Algebra View

Graphics View

Input:

Speed: 1

Time to previous: -0:22 (Message by: tutorA)

Current action at: 4:39:56 PM (Awareness info)

Time to next: 0:00 (Message by: tutorA)

Chat:

- Gerry joins the room 1/19/10 11:31:44 AM EST
- Gerry leaves the room 1/19/10 11:33:58 AM EST
- Gerry joins the room 1/19/10 11:34:43 AM EST
- Gerry leaves the room 1/19/10 11:44:24 AM EST
- Gerry joins the room 1/20/10 1:06:34 PM EST
- Gerry leaves the room 1/20/10 1:17:34 PM EST
- baba joins the room 1/20/10 4:28:55 PM EST

Message:

Here is point D on the circle and on line segment CD. Try to drag this point and watch the circle

The VMT Chat Log

Time of Posting	andicat	Annie	jr6g	loretta
14:45:03			i don't know how to do a perpendicular	
14:45:16				should we do
14:45:20	i need my tool!			
14:45:24		So, Jen, what do you think would go into a		
14:45:37			a 90degree angle and	
14:45:40	i created a tool to make a perpendicular			
14:46:01				can we use the built in tool to do
14:46:13		I'm thinking that we can use the built-in perpendicular tool.		
14:46:39	oh - didn't know that			
14:46:42				its under the intersect point
14:46:47			the perpendicular tool is under the fourth	
14:46:48	i thought it was only something we created			

Professional Development

- **Special courses for math teachers**
- **Full credit toward degree and certification**
- **Flexible online offering**
- **Includes synchronous contact with other teachers in small groups**
- **Prepares for use of technology and curriculum in classrooms**
- **Teachers try VMT-with-GeoGebra and plan for its use by their students**

Activities for Dynamic Geometry

- **Basic geometry from US Common Core State Standards and Math Practices**
- **Main propositions from Book 1 of Euclid**
- **Relationships needed for problem solving**
- **How to construct dynamic-geometry objects**
- **How to design dependencies**
- **How to create custom construction tools**
- **Euclidean construction and transformations**
- **Open-ended explorations and inquiry**

Some Case Studies of VMT

- **how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face;**
- **how the structure of text chat interaction differs from spoken conversation;**
- **how the media of graphical diagrams, textual narratives and symbolic representations can be intimately interwoven to build deep math understanding;**
- **how deictic referencing is important to establishing shared understanding;**
- **how students co-construct a joint problem space and accomplish collaborative meaning making and knowledge building;**
- **how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and**
- **how a methodology based on interaction analysis can be used for a science of group cognition.**

For further information

- **<http://GerryStahl.net>**
- Stahl, G. (2009). *Studying Virtual Math Teams*.
- Stahl, G. (2012). *Designing a learning environment for promoting math discourse*. http://GerryStahl.net/pub/icme_design.pdf
- Stahl, G. (2012). *Evaluating significant math discourse in a learning environment*. http://GerryStahl.net/pub/icme_discourse.pdf
- Powell, A. B., & Dicker, L. (2012). *Toward collaborative learning with dynamic geometry environments*.
- **Slides:** <http://GerryStahl.net/pub/icme2012.ppt>

For further information

- **Gerry@MathForum.org**
- **PowellAB@rutgers.edu**
- **<http://vmt.mathforum.org/vmt/courses.html>**
- **<http://vmt.mathforum.org/VMTLobby>**