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DESIGNING A LEARNING ENVIRONMENT TO PROMOTE MATH DISCOURSE

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Designing a software environment for online learning of mathematics in small collaborative groups requires innovation in multiple dimensions. There has to be generic support for collaborative learning at a distance and also special functionality for mathematical work and communication. We combine the Virtual Math Teams environment with a multi-user version of GeoGebra. We also develop curricular activities through an iterative process of evaluating the discourse that is stimulated by drafts of the activities in prototypes of the technology.
Significant mathematical discourse, virtual math teams, group cognition, learning environment.

INTRODUCTION

Mathematics education in the future faces enormous opportunities from the availability of ubiquitous digital networks, from innovative educational approaches based on theories of collaborative learning and from rich resources for interactive, online, dynamic math exploration.

The fact that more and more teachers and students are learning online—with distance education, online masters programs, home schooling, online high schools, etc.—makes the incorporation of virtual collaborative learning environments a growing need.

This paper reports on the design of a virtual learning environment that integrates synchronous and asynchronous media with an innovative multi-user version of a dynamic math visualization and exploration toolbox. This VMT-with-GeoGebra environment is designed to support the production of significant math discourse.

AN ONLINE MATH COLLABORATIVE-LEARNING ENVIRONMENT

The VMT-with-GeoGebra learning environment integrates two forms of technology to support math learning with collaborative and interactive tools:

- Computer-supported collaborative learning (CSCL) software and
- Dynamic mathematics (software that allows users to manipulate geometric diagrams, equations, etc.).

(a) CSCL provides virtual-learning environments in which teams of students can interact synchronously and asynchronously to build knowledge together. This student-centered approach has many advantages, including increased motivation, sharing of skills, engaging in significant discourse and practicing teamwork. The system reported here extends the Virtual Math Teams (VMT) environment, which has already been prototyped and tested (Stahl, 2009b).

(b) Dynamic math (such as Geometer's Sketchpad, Mathematica, Cabri or GeoGebra) has profoundly impacted math education (Goldenberg, 1995; Hoyles & Noss, 1994; King & Schattschneider, 1997; Laborde, 1998; Myers, 2009; Scher, 2002), with Geometer's Sketchpad and GeoGebra used in many US classrooms and globally. Yet, research on math education has not analyzed how students use dynamic math tools in sufficient detail (compare Cakir, Zemel & Stahl, 2009; Stahl, 2009b). GeoGebra (http://www.geogebra.org) is an open-source system for dynamic geometry, algebra and beginning calculus—including trigonometry, conics, matrices, graphing and Euclidean constructions. It offers multiple representations of objects in its graphics, algebra and spreadsheet views—which are all dynamically linked—making GeoGebra a particularly flexible tool for exploration. The VMT-with-GeoGebra system provides the first multi-user version of dynamic math, so that student teams can explore math collaboratively; it integrates the GeoGebra dynamic math tools into the larger VMT virtual collaborative-learning environment with text chat and wiki to support persistent discourses about math—that can be shared, reflected on and researched.[1]

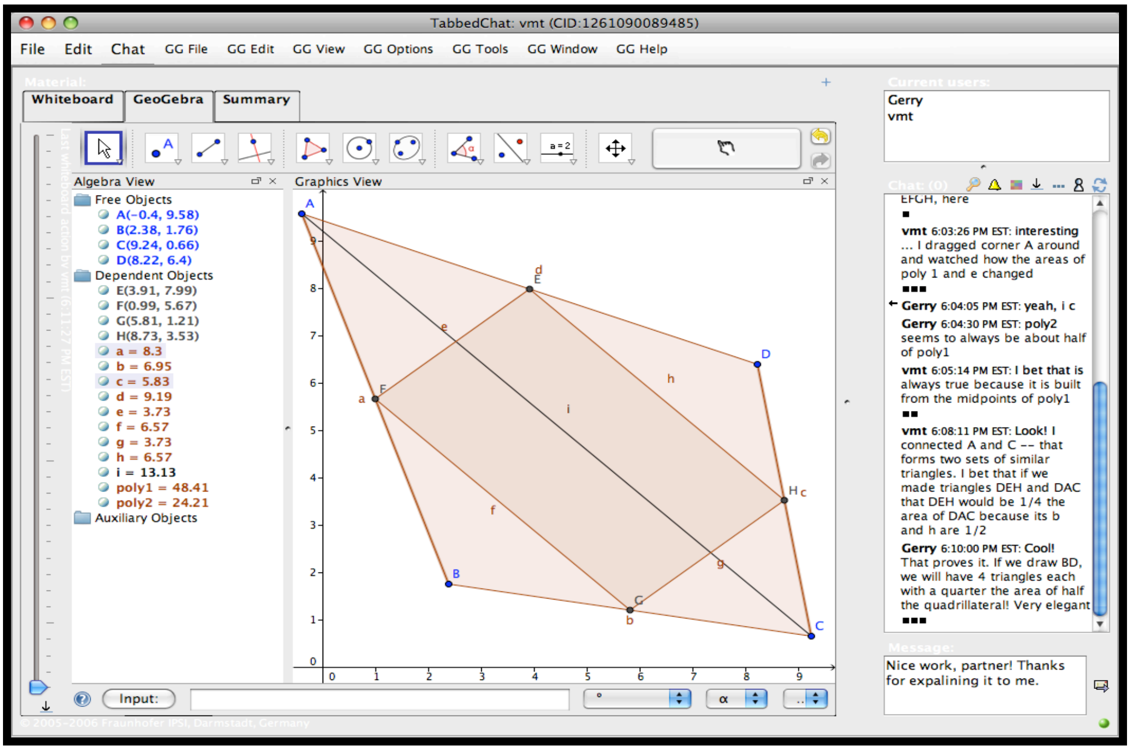

Figure 1. A demo (not real student interaction data) GeoGebra construction created and discussed collaboratively in a multi-user prototype of the learning environment, based on

[1] For a demo of the prototype system, go to http://vmt.mathforum.org/VMTLobby. Log in as "guest" with password "guest". The Lobby should open showing the List of All Rooms. Select Project "VMT Research". Click on "Apply filters". Open "Geometry". Open "Polygons". Click on "GeoGebra Demo Room" Eventually a JavaWebStart chat room should open. Explore its different tabs and functions.

the VMT system. The VMT system includes (not shown here): a Lobby with social networking and tools for teachers, integration with a wiki, and Web browsers.

The VMT-with-GeoGebra system grew out of the successful Virtual Math Teams (VMT) Project. The VMT Project developed an open-source virtual learning environment for math students between 2003 and 2010. The system integrated a social-networking portal, synchronous text chat, a shared whiteboard, an asynchronous wiki, a referencing tool, mathML expressions and a web browser. Student actions and chat postings are automatically logged; they can be replayed for reflection, assessment and analysis by students, teachers and researchers. Over a thousand student-hours of piloted usage were logged. A qualitative micro-analytic approach to interaction analysis was developed based on ethnomethodologically inspired conversation analysis (Garfinkel, 1967; Sacks, 1962/1995; Stahl, 2009a; 2009c; Zemel, Çakir & Stahl, 2009). A large number of publications have appeared from the project (see http://GerryStahl.net/vmt/pubs.html), including 2 books (Stahl, 2006; 2009b) and 8 doctoral dissertations (Çakir, 2009; Litz, 2007; Merges, 2010; Mühlpfordt, 2008; O'Hara, 2010; Sarmiento-Klapper, 2009; Wee, 2009; Zhou, 2010).

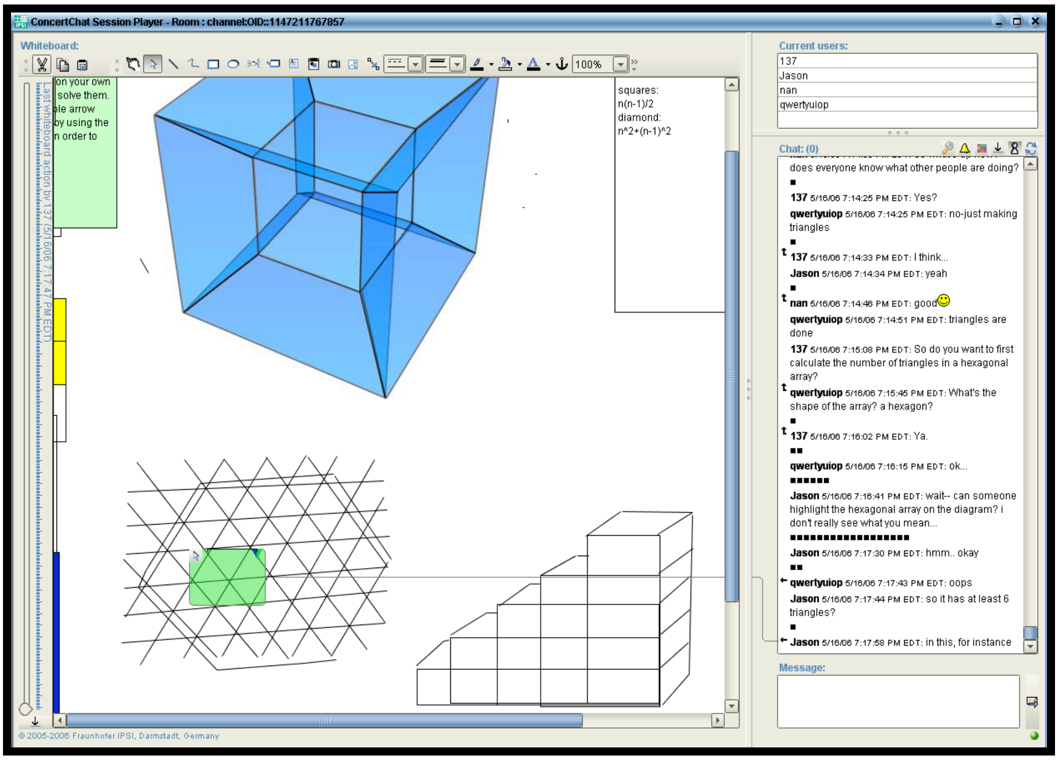

Figure 2. Image of actual student online collaborative work on patterns; a student points from a chat message to a smallest hexagon pattern composed of 6 triangles illustrating VMT's unique integration of chat and whiteboard with its deictic reference tool.

A DESIGN-BASED RESEARCH APPROACH

The VMT Project pioneered the study of online collaborative math discourse—both its nature and modes of computer support for it. The 28 studies in (Stahl, 2009b) present some of the most important of the 169 publications related to the project. They include a number of case studies of interactions in the VMT environment by middle-school, high-school and junior-college students, which analyze: how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face; how the structure of text chat interaction differs from spoken conversation; how the media of graphical diagrams, textual narratives and symbolic representations can be intimately interwoven to build deep math understanding; how deictic referencing is important to establishing shared understanding; how students co-construct a joint problem space; how collaborative meaning making and knowledge building are accomplished in detail; how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and how a methodology based on interaction analysis can be used for a science of group cognition. (See Figures 2 and 3.)

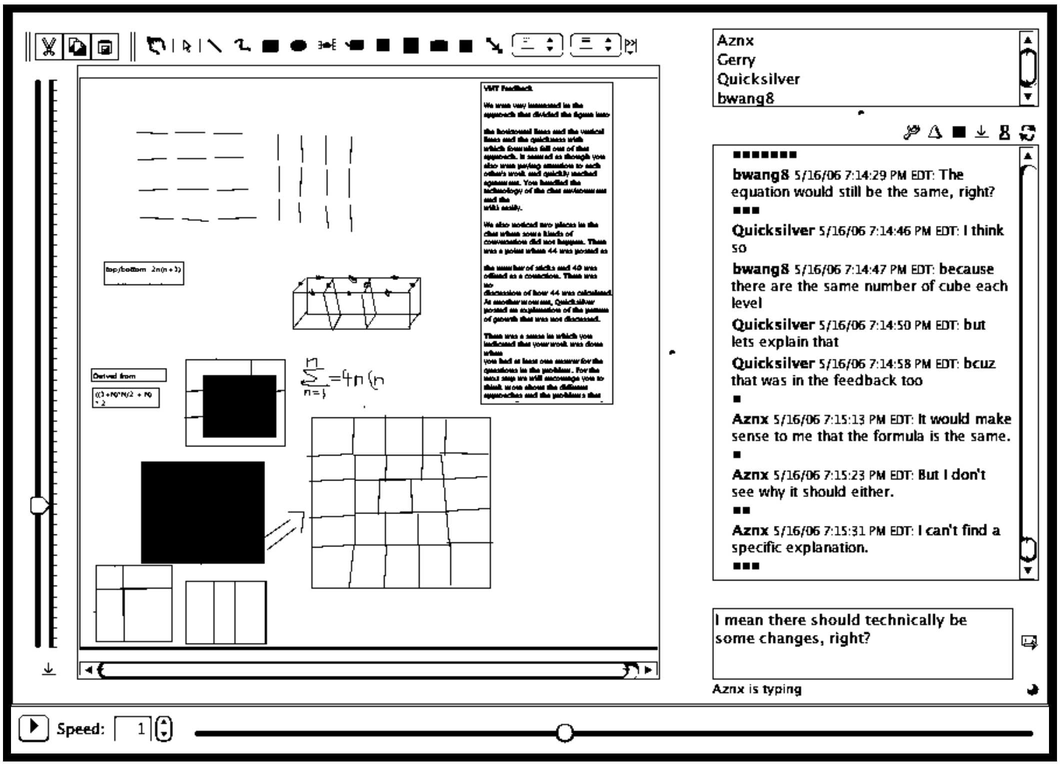

Figure 3 shows the Replayer tool interface across the bottom.

The VMT Project was structured as design-based research, with the technology, research and theory co-evolving through dozens of iterations. The VMT Project demonstrated both the practicality of the VMT-with-GeoGebra system and the need for it. While the VMT Project prototyped a rich cyber-learning environment and studied student interaction, it did not develop the range of supports that we know are needed for classroom use: robust software, problem sets, guidelines, etc. Furthermore, it did not include a dynamic-math component. The VMT-with-GeoGebra system extends the environment to cover these needs.

The VMT Project was widely recognized as an important example of synchronous support for online collaboration and was studied by several international researchers. The VMT Replayer allows complete replay of a user session, including all actions and system notices, as though the session was digitally video-recorded. The researcher's view is guaranteed to be identical to the user's view since it is generated from the same data as sent to a client computer. The log information is also made available in convenient textual or spreadsheet formats for student reflection and reporting as well as for researcher analysis.

TECHNOLOGY DEVELOPMENT

In the VMT-with-GeoGebra system, GeoGebra version 4 has been ported into the VMT system, making the dynamic math tools fully multi-user. GeoGebra is integrated as a tab in VMT (see Figure 1 above). GeoGebra is a particularly appropriate dynamic-math application for this project because its source code is freely available as open source, there is an active international development community to support on-going development, the application supports a wide range of math from algebra and geometry construction to calculus and 3-D, GeoGebra has won international prizes, and it has been translated into about 50 languages. Like all other dynamic-math applications, GeoGebra has until now only existed as a single-user application. While users can send their static constructions to each other, display screen images, or awkwardly include a view of the GeoGebra application within other environments through screen sharing (e.g., in Blackboard, Moodle, Elluminate, etc.), only one person can dynamically manipulate the construction. The port into VMT converted GeoGebra to a client-server architecture, allowing multiple distributed users to manipulate constructions and to all observe everyone's actions in real time. Every action in the GeoGebra tab is immediately broadcast by the server to all collaborating clients (and logged in detail for replay and research).

We have been exploring turn-taking mechanisms (see Figure 4) to avoid conflicts in the construction and modification of GeoGebra drawings; although it is important in synchronous chat to allow multiple users to type simultaneously, we have found that it is natural for a group to allow one member at a time to change a graphical construction and for group members to take turns editing and rearranging.

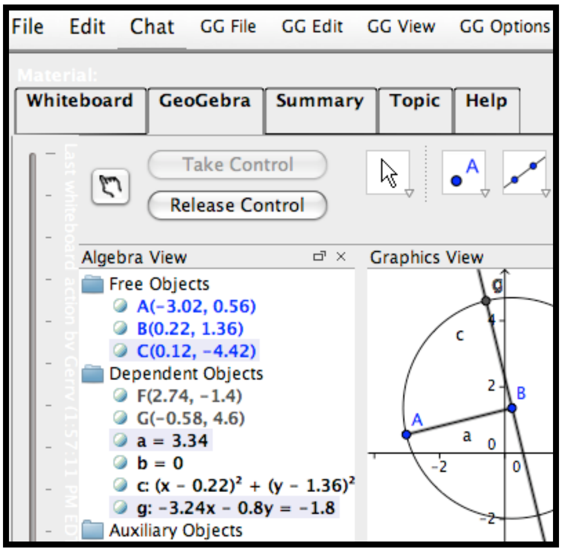

Figure 4. The GeoGebra tab with turn-taking button to avoid conflicts.

DESIGNING ACTIVITIES

The VMT-with-GeoGebra system is not a walk-up-and-use simple app. It requires orientation of students to its purposes and introduction to its functionality. The system therefore includes sets of Activities, which step students through interactions with each other, with the technology and with the mathematics. Each Activity stresses the use of the chat medium to support coordination and collaboration as well as to reflect on the mathematical actions engaged in and to investigate the relationships among the dynamic math objects. These Activities are correlated with math content presented in the U.S. *Common Core State Standards for Mathematics* and in selected math textbooks.

Math teachers are trained in the use of the VMT-with-GeoGebra environment by having them work in it on Activities in small groups of teachers, and reflect on their experiences and on how they might use the Activities in their classrooms.

These Activities have been designed to promote collaborative learning, particularly as exhibited in significant mathematical discourse about geometry. Collaborative learning involves a subtle interplay of processes at the individual, small group and class levels of engagement, cognition and reflection. Accordingly, the Activities are structured with sections for individual work, small-group collaboration and whole-class discussion. It is hoped that this mixture will enhance motivation, extend attention and spread understanding.

CURRICULAR GOALS

The goal of the set of Activities is to improve the following skills in math teachers and students:

1. To engage in significant mathematical *discourse*; to collaborate on and discuss mathematical activities in supportive small online groups
2. To collaboratively *explore* mathematical phenomena and dependencies; to make mathematical phenomena visual in multiple representations; and to vary their parameters
3. To *construct* mathematical diagrams – understanding and exploring their structural dependencies
4. To notice, wonder about and form conjectures about mathematical relationships; to justify, explain and *prove* mathematical findings
5. To understand core concepts, relationships, theorems and constructions of basic high-school *geometry*

The working hypothesis of the activities is that these goals can be furthered through an effective combination of:

1. Collaborative experiences in mathematical activities with guidance in collaborative, mathematical and accountable geometric *discourse*
2. *Exploring* dynamic-mathematical diagrams and multiple representations
3. Designing dependencies in dynamic-mathematical *constructions*

4. Explaining conjectures, justifications and *proofs*
5. Engagement in well-designed activities around basic high-school *geometry* content

In other words, the Activities seek a productive synthesis of *collaboration*, *discourse, visualization, construction, and argumentation* skills applied in the domain of beginning geometry. They operationalize "deep conceptual learning" of mathematics in terms of these measurable outcomes:

1. The quality and quantity of significant mathematical *discourse* in collaborative interactions
2. Group *explorations* of mathematical objects and representations, including noticing and wondering
3. *Constructions* of mathematical objects with dependencies
4. Explanations, justifications and *proofs* of conjectures
5. Engagement in significant mathematical discourse involving *geometric* notions of congruence, symmetry, dependencies, relationships, transformations and deduction

GEOMETRIC DEPENDENCIES

Our focus has centered increasingly on facilitating and supporting lessons in geometric dependency. GeoGebra allows one to construct systems of interdependent geometric objects. Students have to learn how to think in terms of these dependencies. They can learn through visualizations, manipulations, constructions and verbal articulations. These can all be modeled and these skills can be developed gradually.

Our concerns are incorporated in a focus on dependency as follows:

1. Increase the ability of math teachers and students to engage in significant mathematical *discourse* about geometric dependencies.
2. Provide math teachers and students with a coherent sequence of activities *exploring* mathematical dependencies.
3. Empower math teachers and students to *construct* their own mathematical dependencies among objects in a dynamic-mathematics environment, which they can use in the future as well
4. Increase the understanding of math teachers and students in why mathematical objects behave in the ways they are constrained to by their dependencies, possibly *proving* why the dependencies have specific consequences
5. Increase the understanding of math teachers and students in the content of basic high-school *geometry* dependencies, including how to discuss them, explore them, visualize them, prove them and extend them

We are now drafting and piloting versions of curricular activities designed to develop significant mathematical discourse focused on dependencies among geometric objects. Concomitantly, we are implementing software support for teachers and students to explore the dependencies and assembling materials for professional development to prepare teachers to enact this curriculum with their students.

CONCLUSION

Incorporation of GeoGebra in the VMT environment framework allows users to engage in text chat while manipulating geometric constructions. Importantly, users can graphically point from a chat posting to an area of the construction that they want to index (see Figure 2)—a handy support for math discourse that is unique to VMT. They can also scroll back and forth through the history of the GeoGebra construction, animating its evolution—a powerful way to explore many mathematical relationships. In addition, a complete record of the collaborative construction is available to the participants, their teachers and project researchers, allowing them all to analyze and reflect upon the complete interaction, including the construction actions synchronized with the chat. GeoGebra in VMT provides an exciting collaborative experience and a rich dataset for research on collaborative learning of mathematics. A set of carefully designed Activities introduces students to the VMT environment, the construction of objects in GeoGebra and the approach of dynamic geometry in a collaborative setting.

References

Çakir, M. P. (2009). *How online small groups co-construct mathematical artifacts to do collaborative problem solving.* Unpublished Dissertation, Ph.D., College of Information Science and Technology, Drexel University. Philadelphia, PA, USA.

Cakir, M. P., Zemel, A., & Stahl, G. (2009). The joint organization of interaction within a multimodal CSCL medium. *International Journal of Computer-Supported Collaborative Learning. 4*(2), 115-149.

Garfinkel, H. (1967). *Studies in ethnomethodology*. Englewood Cliffs, NJ: Prentice-Hall.

Goldenberg, E. P. (1995). Ruminations about dynamic imagery (and a strong plea for research). In R. Sutherland & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education.* (pp. 203-224). Germany: Springer Verlag.

Hoyles, C., & Noss, R. (1994). Dynamic geometry environments: What's the point? *Mathematics Teacher. 87*, 716-717.

King, J., & Schattschneider, D. (1997). Making geometry dynamic. In J. King & D. Schattschneider (Eds.), *Geometry turned on.* (pp. ix-xiv). Washington, DC: Mathematical Association of America.

Laborde, C. (1998). Visual phenomena in the teaching/learning of geometry in a computer-based environment. In C. M. V. Villani (Ed.), *Perspectives on the teaching of geometry for the 21st century.* (pp. 113-121). The Netherlands: Kluwer Academic Publishers.

Litz, I. R. (2007). *Student adoption of a computer-supported collaborative learning (CSCL) mathematical problem solving environment: The case of the math forum's virtual math teams (VMT) chat service.* Unpublished Dissertation, Ph.D., School of Computer and Information Sciences, Nova Southeastern University. Florida.

Merges, K. (2010). *Tracing knowledge transfer through a wiki in an online synchronous environment.* Unpublished Dissertation, PhD, The Graduate School of Education, Rutgers University. New Brunswick, NJ. Web: http://GerryStahl.net/pub/merges_dissertation.pdf.

Mühlpfordt, M. (2008). *Integration dualer interaktionsräume: Die verknuepfung von textbasierter synchroner kommunikation mit diskreten konstruktionswerkzeugen. (the integration of dual-interaction spaces: The connection of text-based synchronous communication with graphical construction tools [in German]).* Unpublished Dissertation, Ph.D., Fakultaet fuer Mathematik und Informatik, Fern Universitaet. Hagen, Germany.

Myers, R. Y. (2009). *The effects of the use of technology in mathematics instruction on student acheivement.* Unpublished Dissertation, Ph.D., Curriculum and Instruction, Florida International University. Miami, FL. Web: http://digitalcommons.fiu.edu/etd/136.

O'Hara, K. (2010). *Tracing students' mathematical indentity in an online synchronous environment.* Unpublished Dissertation, PhD, The Graduate School of Education, Rutgers, The State University of New Jersey. New Brunswick, NJ. Web: http://GerryStahl.net/pub/ohara_dissertation.pdf.

Sacks, H. (1962/1995). *Lectures on conversation*. Oxford, UK: Blackwell.

Sarmiento-Klapper, J. W. (2009). *Bridging mechanisms in team-based online problem solving: Continuity in building collaborative knowledge.* Unpublished Dissertation, Ph.D., College of Information Science and Technology, Drexel University. Philadelphia, PA, USA.

Scher, D. (2002). *Students' conceptions of geometry in a dynamic geometry software environment.* Unpublished Dissertation, Ph.D., School of Education, New York University. New York, NY. Web: http://GerryStahl.net/pub/GSP_Scher_Dissertation.pdf.

Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Web: http://GerryStahl.net/mit/.

Stahl, G. (2009a). *Keynote: How I view learning and thinking in CSCL groups.* Paper presented at the International Conference on Computers and Education (ICCE 2009). Hong Kong, China. Web: http://GerryStahl.net/pub/icce2009keynote.pdf.

Stahl, G. (2009b). *Studying virtual math teams*. New York, NY: Springer. Web: http://GerryStahl.net/vmt/book.

Stahl, G. (2009c). Toward a science of group cognition. In G. Stahl (Ed.), *Studying virtual math teams.* (ch. 28, pp. 555-579). New York, NY: Springer. Web: http://GerryStahl.net/vmt/book/28.pdf.

Wee, J. D. (2009). *Reinventing mathematics problem design and analysis of chat interactions in quasi-synchronous chat environments.* Unpublished Dissertation, Ph. D., National Institute of Education, Nanyang Techological University. Singapore.

Zemel, A., Çakir, M. P., & Stahl, G. (2009). *Understanding and analyzing chat in CSCL as reading's work.* Paper presented at the international conference on Computer Support for Collaborative Learning (CSCL 2009). Rhodes, Greece. Web: http://GerryStahl.net/pub/cscl2009zemel.pdf.

Zhou, N. (2010). *Troubles of understanding in virtual math teams.* Unpublished Dissertation, Ph.D., College of Information Science and Technology, Drexel University. Philadelphia, PA, USA.