

The Structure of Collaborative Problem Solving in a Virtual Math Team

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ABSTRACT

To develop a science of small-group interaction in groupware, we need a method for analyzing the structure of computer-mediated discourse. Conversation analysis offers an analysis of conversational talk in terms of a fine structure of adjacency pairs and offers some suggestions about longer sequences built on these pairs. This paper presents a case study of students solving a math problem in an online chat environment. It shows that their problem-solving discourse consists of a sequence of exchanges, each built on a base adjacency pair and each contributing a move in the solution process.

Categories and Subject Descriptors

H.5.3 [Group and Organization Interfaces]: Computer-supported cooperative work, Evaluation/methodology, Theory and models – *interaction analysis, group cognition, computer-supported collaborative learning.*

General Terms

Design, Experimentation, Human Factors, Theory.

Keywords

Group Cognition, Virtual Math Teams, Interaction Analysis, Conversation Analysis, Adjacency Pair, Long Sequence.

1. SMALL-GROUP INFORMATION USE

Information, people, and technology converge in a practical way in online collaborative problem solving. My colleagues and I have been pursuing a research agenda aimed at investigating how to support online collaborative problem solving. We have focused on the domain of school mathematics—especially beginning algebra and geometry—where students learn formal techniques and tacit practices of solving abstract problems. We find that mechanisms of group problem solving are visible in this context.

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Our research—such as that reported in this paper—confirms that there are distinctive processes of information use in problem solving at the small-group unit of analysis, which should not be reduced to either the individual psychological level or the larger social community level—despite the fact that groups are physically composed of individuals and that they are embedded in socio-historical contexts. While an approach methodologically focused on the group unit of analysis is in line with current post-cognitive theories it is rarely carried out consistently.

We developed the Virtual Math Teams (VMT) environment and invited students to work in online groups for up to four hour-long sessions. We presented challenging problems for them to explore together and encouraged them to pursue their own questions. The environment was instrumented to capture a complete and accurate record at the group unit of analysis—i.e., all text-chat postings, all drawing actions, and all social awareness messages that were displayed to the group. As researchers, we can replay the group interaction and view it as it appeared to the group or browse it in as much detail as needed for analysis.

Because we are pursuing design-based research to improve the VMT environment, we are not oriented toward theoretical hypotheses, statistical generalizations, individual mental representations, or socio-cultural influences—except to the extent that they manifest themselves in the group interaction. Rather, we try to understand the situated processes that take place at the group level of description in actual case studies. In particular, we look at the ways in which groups of math students use information and solve problems in our environment so that we can design improved socio-technical supports for their collaborative online problem solving.

We have tried a variety of research approaches in the VMT Project, including coding, statistical comparison, modeling, uptake analysis, conversation analysis, critical ethnography, and discourse analysis. In general, we have found the most insightful approach to involve adapting ethnomethodologically inspired conversation analysis (CA) to our context of online text chat by math students.

2. AN ANALYTIC METHOD

Recently, I have been trying to apply our CA approach in a systematic way to the analysis of VMT chat logs. Schegloff's [11] book on *Sequence Organization in Interaction* represents the culmination of decades of CA analysis. As indicated by its subtitle, it provides a useful primer in CA. My goal here is to extend the CA approach to short sequences of utterances to

analyze the larger scale interactions of group problem solving in VMT.

Schegloff's presentation highlights the central role of the adjacency pair as the primary unit of sequence construction according to CA. An adjacency pair is composed of two turns by two different people, with an interactional order, such as a question followed by an answer to the question. The simple two-turn pair can be extended with secondary adjacency pairs that precede, are inserted between or follow up on the base pair, potentially recursively. This yields "extensive stretches of talk which nonetheless must be understood as built on the armature of a single adjacency pair, and therefore needing to be understood as extensions of it" (p. 12).

These "extensive stretches of talk" are still focused on a single interaction of meaning making, and not a larger cognitive achievement like problem solving. However, both Sacks and Schegloff provide vague suggestions about the analysis of longer sequences. These suggestions have not been extensively developed within CA. This paper is an attempt to explore them in an online text-chat context.

Schegloff briefly takes up "larger sequence structures to which adjacency pairs can give rise and of which they may be building blocks ... such as sequences of sequences" (p. 12). One way in which a sequence (an extended adjacency pair) may be related to yet separate from a previous, completed adjacency pair "is that it implements a next step or stage in a course of action, for which the just-closed sequence implemented a prior stage" (p. 213). Note the two-way reference, with the second stage having the character of a next, but also the first stage having the character of a prior. This is analogous to the two parts of a simple adjacency pair:

Adjacency pair organization has (in addition to the backwards import just described) a powerful prospective operation. A first pair part projects a prospective relevance, and not only a retrospective understanding. It makes relevant a limited set of possible second pair parts, and thereby sets some of the terms by which a next turn will be understood—as, for example, being responsive to the constraints of the first pair part or not. (p. 16)

The adjacency pair structure was first discussed extensively by Sacks [7, II 521-569]. In these seminal lectures, he also briefly discussed long sequences. Here, his main point was to state that little is known about the structure of long sequences; that the analytic problem is in principle harder; and that, in particular, it is wrong to assume that an analysis at the level of adjacency pairs will be useful to understanding the co-construction of long sequences:

It turns out that one central problem in building big packages is that the ways the utterances that turn out to compose the package get dealt with as single utterances or pairs of utterances or triplets of utterances, etc., may have almost no bearing on how

they're to be dealt with when an attempt is made to build the larger package. (II p. 354)

The analyses provided by CA come primarily from the study of American adults conducting face-to-face, verbal, informal, social conversation, although some of the early data came from distance conversations by telephone and the field has broadened its sources considerably more recently. However, we must be careful when applying CA methods to online, text-based, learning-related discourse about mathematics by students. Along these lines, Schegloff warns about his presentation:

Note that this discussion is focused on conversation in particular. Because different organizations of turn-taking can characterize different speech-exchange systems [8, n. 11 729-731], anything that is grounded in turn-taking organization may vary with differences in the turn-taking organization. It is a matter for empirical inquiry, therefore, how the matters taken up in the text are appropriately described in non-conversational settings. [11, 15n]

As we have frequently argued [e.g., 13; 18; 20], we believe that adapting CA to computer-mediated communication offers the best prospects for analysis of interaction in socio-technical environments like VMT. The preceding review of the topics of adjacency pairs and long sequences indicates that it is an empirical question how well this proposed adaptation might work. We designed and conducted the VMT Project from 2003 to the present in order to produce a corpus of data that could be analyzed in as much detail as needed to determine the structure of group cognition, that is, of collaborative knowledge building through interaction at the group unit of analysis.

In looking at the VMT data corpus, the VMT research team has clearly seen the differences between online text chat and verbal conversation. The system of turn taking so important in CA [8] does not apply in chat. Instead, chat participants engage in reading's work [25], in which "readers connect objects through reading's work to create a 'thread of meaning' from the various postings available for inspection" (p. 274f). The first and second parts of an adjacency pair may no longer be literally temporally adjacent to each other, but they still occur as mutually relevant, anticipatory, and responsive. The task of reading's work—for both participants and analysts—is to reconstruct the threading of the underlying adjacency pair response structure [17].

In CA, adjacency pairs are related to both issues of timing (turn taking) and of sequentiality (response). In chat, they retain their importance solely as sequential, in order to maintain interaction in the absence of turn taking. We have tried to explore the larger sequential structure of problem-solving chat by using the CA notion of openings and closings [10]. VMT researchers looked at several math chats from 2004, which used a simple chat tool from AOL. We coded and statistically analyzed the fine-structure threading of adjacency pairs [1]. In addition, we defined long sequences based on when opening and closing adjacency pairs achieved changes in topic [26]. These long sequences were graphed to show their roles in constituting the chat sessions, but their internal sequential structures were not investigated.

My colleagues and I have subsequently conducted numerous case studies from the VMT corpus. We have been particularly drawn to the records of Team B and Team C in the VMT Spring Fest 2006. These were particularly rich sessions of online mathematical knowledge building because these teams of students met for over four hours together and engaged in rich explorations of interesting mathematical phenomena. However, partially because of the richness of the interactions, it was often hard for analysts to determine a clear structure to the student interactions. Despite access to everything that the students knew about each other and about the group interaction, it proved hard to unambiguously specify the group-cognition processes at work [5; 17; 22].

Therefore, in the following case study, I have selected a segment of Team B's final session, in which the structure of the interaction seems to be clearer. The interaction is simpler than in earlier segments partially because two of the four people in the chat room leave. Thus, the response structure is more direct and less interrupted. In addition, the students have already been together for over four hours, so they know how to interact in the software environment and with each other. Furthermore, they set themselves a straightforward and well-understood mathematical task. The analysis of this relatively simple segment of VMT interaction can then provide a model for subsequently looking at the more complex data and seeing if it may follow a similar pattern.

3. THE CASE STUDY

Three anonymous students (Aznx, Bwang, Quicksilver) from US high schools met online as Team B of the VMT Spring Fest 2006 contest to compete to be "the most collaborative virtual math team." They met for four hour-long sessions during a two-week period in May 2006. A facilitator (Gerry) was present in the chat room to help with technical issues, but not to instruct in mathematics.

In their first session, they solved a given problem, finding a mathematical formula for the growth pattern of the number of squares and the number of sticks making up a stair-step figure. They determined the number of sticks by drawing just the horizontal sticks together and then just the vertical ones (see Figure 1). They noticed that both the horizontals and the verticals formed the same pattern of $1 + 2 + 3 + \dots + n + n$ sticks at the n^{th} stage of the growth pattern. They then applied the well-known Gaussian formula for the sum of consecutive integers, added the extra n , and multiplied by 2 to account for both the horizontal and vertical sets of sticks.

In the second session, they explored problems that they came up with themselves, related to the stair-step problem, including 3-D pyramids. Here they ran into problems drawing and analyzing 3-D structures. However, they managed to approach the problem from a number of perspectives, including decomposing the structure into horizontal and vertical sticks.

In the third session, they were attracted to a diamond-shaped variation of the stair-step figure, as explored by Team C in the Spring Fest. They tried to understand how the other team had derived its solution. They counted the number of squares by simplifying the problem through filling in the four corners surrounding the diamond to make a large square; the corners turned out to follow the stair-step pattern from their original problem.

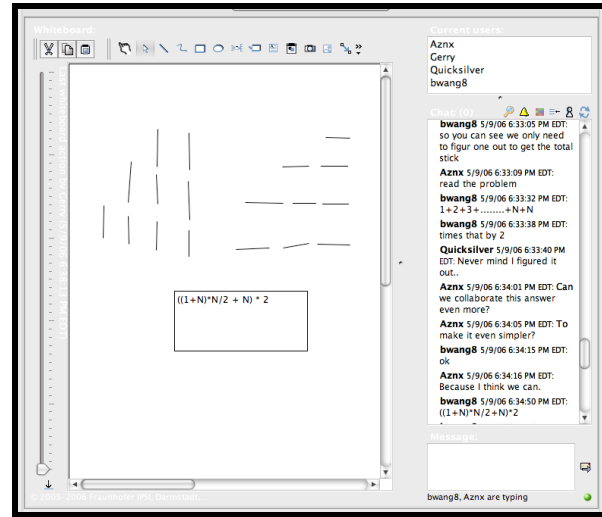


Figure 1. Screenshot of the VMT environment showing the pattern of horizontal and vertical sticks in the stair-step figure.

In the fourth session, they discovered that the other team's formula for the number of sticks was wrong. In the following, we join them an hour and 17 minutes into the fourth session, when one of the students as well as the facilitator had to leave.

4. PROBLEM-SOLVING MOVES

In this section of the paper, the interaction is analyzed as a sequence of moves in the problem-solving interaction between Bwang and Aznx, the two remaining students. Each move is seen to include a base adjacency pair (in bold face), which provides the central interaction of the move and accomplishes the focal problem-solving activity. The captions of the log excerpts indicate the aim of the move, according to the analysis.

Log 1. Open a Topic

LINE	TIME	AUTHOR	TEXT OF CHAT POSTING
1734	08.17.20	bwang8	i think we are very close to solving the problem here
1735	08.17.35	Quicksilver	Oh great...I have to leave
1736	08.17.39	Aznx	We can solve on that topic.
1737	08.17.42	Quicksilver	Sorry guys
1738	08.17.45	bwang8	oh
1739	08.17.46	Aznx	It shouldn't take much time.
1740	08.17.47	bwang8	ok
1741	08.17.50	Aznx	k, bye Quicksilver
1742	08.17.52	Quicksilver	Just tell me the name of the room
1743	08.17.52	bwang8	bye
1744	08.18.14	Gerry	The new room is in the lobby under Open Rooms
1745	08.18.44	Gerry	It is under The Grid World. It has your names on it
1746	08.18.49	Quicksilver	[leaves the room]
1747	08.19.00	Aznx	Alright found it.
1748	08.19.04	Aznx	Thanks.

In line 1734, Bwang states that the team is close to being able to solve the problem of the number of sticks in the n^{th} stage of the diamond pattern, suggesting that they might stay and finish it up. Note that this is the end of the last of the scheduled four sessions for the contest, despite some arrangements underway to allow the team to continue to meet.

Aznx responds in line 1736, indicating—and implicitly endorsing the suggestion—that the team could indeed continue to work on the current topic. This opens the topic for the group.

Quicksilver apologetically stresses that he must leave immediately. He just wants to know the location of the new chat room that the facilitator is setting up for the team to continue its math explorations on a future date. The facilitator supplies this information and everyone says goodbye to Quicksilver. We ignore this other activity in our current analysis, and focus on the problem-solving interactions.

Log 2. Decide to Start

1749	08.19.12	Aznx	I guess we should leave then.
1750	08.19.34	bwang8	well do you want to solve the problem
1751	08.19.36	bwang8	i mean
1752	08.19.39	bwang8	we are close
1753	08.19.48	Aznx	Alright.
1754	08.19.51	bwang8	i don't want to wait til tomorrow
1755	08.19.53	bwang8	ok

Aznx expresses uncertainty about how to proceed now that Quicksilver has gone and the facilitator has arranged things for the future. He questions whether he and Bwang need to go as well. Bwang then reiterates his suggestion that they could stay and finish solving the problem. He argues that it should not take much longer. Bwang directly asks Aznx if he wants to solve the problem now.

Aznx agrees by responding to Bwang's question in the affirmative. This effects a decision by the pair of students to start working on the problem right away. Bwang continues to argue for starting on the problem now—posting line 1754 just 3 seconds after Aznx' agreement, probably just sending what he had already typed before reading Aznx' response. Bwang then acknowledges the response.

Log 3. Pick an Approach

1756	08.19.55	Aznx	How do you want to approach it?
1757	08.20.14	bwang8	1st level have 1^4
1758	08.20.20	Gerry	You can put something on the wiki to summarize what you found today
1759	08.20.29	bwang8	2st level have $(1+3)^4$
1760	08.20.32	Aznx	bwang you put it.
1761	08.20.35	Aznx	for the wiki
1762	08.20.37	bwang8	ok
1763	08.20.42	Aznx	we actually did quite a lot today
1764	08.20.53	bwang8	3rd level have $(1+3+5)^4$
1765	08.21.05	bwang8	4th level have $(1+3+5+7)^4$
1766	08.21.10	Gerry	This is a nice way to solve it

Once a decision has been made to solve the problem, the question of how to approach the problem is raised in line 1756. Bwang immediately lays out his approach in lines 1757, 1759, 1764 and 1765. The approach is the same as they used in the first session: visualize just the vertical or just the horizontal sticks. The two sets follow the same pattern. In fact, the diamond is also symmetric left/right and top/bottom, so the vertical sticks can be divided left/right into two identical sets and the horizontal sticks can be divided top/bottom. This produces four identical sets of sticks (color-coded in Figure 2), each having rows of **1, 3, 5, 7, ...** sticks, up to **$(2n-1)$** for the n^{th} stage of the diamond pattern.

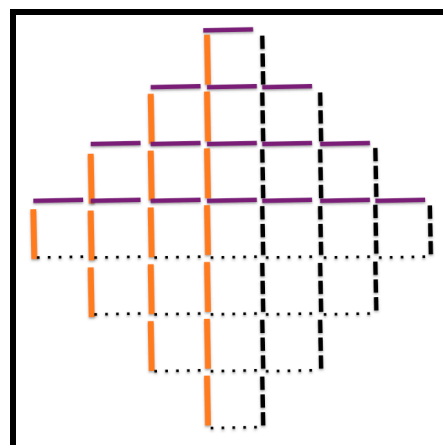


Figure 2. A representation (not from the data) of the diamond figure at stage $n=4$, color-coding the sticks in four identical (symmetric) sets.

Interspersed with this defining of the approach is a reminder from the facilitator to summarize the team's work on the Spring Fest wiki for other teams to see, motivating this with a word of encouragement about the team's work.

Log 4. Identify the Pattern

1767	08.21.12	Aznx	So it's a pattern of $+2s$?
1768	08.21.15	Aznx	Ah ha!
1769	08.21.15	bwang8	yes
1770	08.21.20	Aznx	There's the pattern!

Aznx has previously been oriented toward finding patterns of growth in the mathematical objects the group has been exploring. Often, someone will create a graphical representation of the object in such a way that it makes the pattern visible. Aznx will then formulate a textual description of the pattern. Then the group will work on a symbolic representation to capture the pattern in a mathematical formula. (See [2] for an analysis of the intertwining of graphical/visual, textual/narrative and symbolic/mathematical modes of interaction within the work of Team C.)

Here, in line 1767, Aznx describes the pattern as involving adding numbers that successively increase by **2**. The number of sticks in a given stage of the diamond shape is a sum of numbers that start at **1** and increase successively by **2**. When going from one stage to the next, one simply adds another number to this sum that is **2** more than the highest previous one.

Aznx presented his description as a question and Bwang affirmed it at the same time as Aznx posted line 1768. Aznx then emphasized that they had discovered the pattern.

Log 5. Seek the Equation

1771	08.21.39	bwang8	now we have to find a equation that describe that pattern
1772	08.21.49	Aznx	Hold on.
1773	08.21.51	Aznx	I know it.
1774	08.21.57	bwang8	what is it
1775	08.21.58	Aznx	But I'm trying to remember it. =P
1776	08.22.04	Aznx	and explain it as well.
1777	08.22.17	Aznx	try and think of it
1778	08.22.53	Gerry	Maybe Quicksilver can come back here tomorrow or next week to finish it with you
1779	08.23.01	Gerry	I have to go now
1780	08.23.05	Gerry	Bye!
1781	08.23.06	bwang8	ok
1782	08.23.07	bwang8	bye
1783	08.23.23	Gerry	[leaves the room]
1784	08.23.29	bwang8	ok
1785	08.23.32	bwang8	so
1786	08.23.37	bwang8	i think it is this
1787	08.23.53	Aznx	ok
1788	08.23.55	Aznx	i found it
1789	08.24.00	Aznx	n^2
1790	08.24.01	bwang8	(2*n)*n/2
1791	08.24.09	Aznx	or (n/2)^2

Bwang indicates that the next step in their work is to “find an equation that describes the pattern.” Aznx asks Bwang to let him state the equation, implicitly agreeing that this is the next step by trying to produce the equation.

Bwang asks Aznx to state the equation and Aznx expresses difficulty in formulating an adequate and accountable answer. After a half minute of silence with still no formulation from Aznx, the facilitator suggests that Aznx and Bwang might want to wait until a future time when the whole group can work together to finish the problem. The facilitator then says goodbye and leaves the chat room.

After more than a minute since Aznx posted anything, Bwang starts to preface the presentation of his own formulation. Eventually, Aznx joins back in. Simultaneously, Aznx and Bwang post their formulae. For Aznx, it is either n^2 or $(n/2)^2$. For Bwang, it is $2n(n/2)$.

Aznx has not given any indication of how he got his proposed formula. Bwang’s formula suggests the use of Gauss’ summation, which the students have used repeatedly in the past. According to this summation of an arithmetic sequence of integers, the result is the sum of the first and last member of the sequence times half the number of members. For a sequence of n members, $1 + 3 + 5 + \dots + (2n-1)$, the sum would be $[1 + (2n-1)]*(n/2)$. Adding the 1 and the -1 , yields Bwang’s formula, $2n(n/2)$. Note that the n^{th} odd integer can be represented by $(2n-1)$.

It is likely that Aznx used a similar method, working on his own during his prolonged silence, but got confused about the result when he simplified his expression. As Aznx shows next, Aznx’s first answer is equivalent to Bwang’s answer, once Aznx

simplifies it. His second answer is related to part of Bwang’s unsimplified answer.

Log 6. Negotiate the Solution

1792	08.24.14	Aznx	I'm simplifying
1793	08.24.30	Aznx	if u simplify urs
1794	08.24.35	Aznx	its n^2
1795	08.24.59	Aznx	bwang
1796	08.25.01	Aznx	you there?
1797	08.25.03	bwang8	so that's wrong
1798	08.25.07	bwang8	yeah
1799	08.25.08	bwang8	i am here

Aznx simplifies Bwang’s formula: $2n(n/2) = n^2$. This is the same as one of Aznx’ proposed formulae. When Bwang does not respond to this posting, Aznx wonders if Bwang is still present online.

Bwang was apparently already typing “so that is wrong” when he received Aznx’ question concerning his presence. This message in effect confirmed that Aznx’ second formula, $(n/2)^2$, is wrong and his first one, which agrees with Bwang’s, is correct.

Log 7. Check Cases

1800	08.25.11	Aznx	so
1801	08.25.13	Aznx	the formula
1802	08.25.22	Aznx	would be 4n^2?
1803	08.25.28	bwang8	let's check
1804	08.25.55	bwang8	Yes
1805	08.26.00	bwang8	it actually is
1806	08.26.02	Aznx	So we got it!

Going along with this, Aznx then multiplies their agreed upon formula by 4 because there were 4 sets of horizontal or vertical sticks, each numbering $1 + 3 + \dots$. Aznx poses his message as a question, soliciting confirmation from Bwang. By offering this next step in the symbolic representation, Aznx demonstrates that he understands where Bwang’s formula came from and he understands the larger strategy of approaching the problem that Bwang had proposed. In other words, Aznx demonstrates a level of mathematical competence and of shared understanding that he did not always display in the previous sessions.

Before being ready to answer whether $4n^2$ is actually the correct formula for the number of sticks, Bwang suggests that they first check if the formula works by testing it for a number of values of n and counting the sticks in drawings of diamonds at the corresponding n^{th} stage. A half-minute later, Bwang concludes that the formula does check out. He therefore answers Aznx’ question with confidence, perhaps mixed with a touch of surprise.

Aznx concludes that they got the solution for the number of sticks in the diamond pattern—a problem that Team C had posed for itself, but for which they had derived the wrong formula, without, however, realizing it. Team B had been shocked earlier to discover that the formula they had been struggling to understand from Team C had been wrong; that it did not check out for any values of n .

Log 8. Confirm the Solution

1807	08.26.02	bwang8	omg
1808	08.26.04	Aznx	yay!

1809	08.26.08	bwang8	i think we got it!!!!!!
1810	08.26.12	Aznx	WE DID IT!!!!!!
1811	08.26.12	bwang8	and it is so simple
1812	08.26.14	Aznx	YAY!!!!
1813	08.26.16	Aznx	i know
1814	08.26.17	bwang8	lol
1815	08.26.18	Aznx	lol

Their surprise and excitement is almost uncontrollable. They use every chat technique they know to express their joy. Their postings intertwine like a frenzied dance.

Log 9. Present a Formal Solution

1816	08.26.34	Aznx	So you're putting it in the wiki, right?
1817	08.26.37	bwang8	yes
1818	08.26.41	Aznx	Alright then.
1819	08.26.43	bwang8	ok
1820	08.26.53	Aznx	Give an email to Gerry, telling him that we got it. =)
1821	08.26.57	bwang8	ok
1822	08.26.59	Aznx	I meant Gerry
1823	08.27.04	bwang8	are you going to do it
1824	08.27.07	bwang8	or am i
1825	08.27.12	Aznx	You do it.
1826	08.27.14	bwang8	ok
1827	08.27.19	Aznx	Tell him that we both derived n^2
1828	08.27.29	Aznx	And then we saw that pattern
1829	08.27.37	Aznx	and we got the formula

Once the mathematical exploration is done, it is time to write up a report of ones findings. Professional mathematicians would do this in the form of a proof. When a group of mathematicians recently conducted an online collaborative analysis of a mathematical problem, it took them longer to write the publishable proof than it did to figure out the approach and solve it [4; 6]. Bwang posted the narrative shown in Figure 3 to the Spring Fest wiki.

We then move on to understand Team C's formula for summing up the total # of sticks in n -level diamond. We first tried to used the big square and then minus the extra corners, but the corners turns out to be to hard to calculate. Then we tried to simplify Team C's equation to help as find a lead, but we found out that their stick equation is wrong. We then decide to find out a whole new equation and tried to divide the sticks up into vertical and horizontal groups like we did before with all the other problems. The groups can be further divided into 2 equal parts. We found a pattern.

1st level: 1
 2nd level: 1+3
 3rd level: 1+3+5
 4th level: 1+3+5+7
 5th level: 1+3+5+7+9
 nth level: $(2*n)*n/2$

We then found out that each of these can be by calculated by $(2*n)*n/2$ which simplified into n^2 . n^2 can then be multiplied by 4 and get the total of sticks in a n th leveled diamond. The final equation is $4(n^2)$.

Figure 3. Wiki posting by Group B after session 4.

Log 10. Close the Topic

1830	08.27.44	Aznx	when should we meet again?
1831	08.27.49	Aznx	hat's your email?
1832	08.27.52	Aznx	we should keep in touch
1833	08.27.57	bwang8	yeah

Finally, Aznx and Bwang wrap up the conversational topic by exchanging email addresses and agreeing to meet again online with Quicksilver and pursue further mathematical adventures together.

5. THE SEQUENCE OF PAIRS

Within each of the preceding log excerpts, we have identified a base adjacency pair by means of which the work of a specific move in the problem-solving effort of the small group is interactively accomplished. In most cases, a question is posed and a response is then given to it.

As Schegloff [11] argues, an adjacency pair is itself a sequence. It embodies a temporal structure, with the first element of the pair projecting the opportunity and expectation of a response in the interactional immediate future. The second element constitutes an uptake of a first element that it implicitly references as in the interactional immediate past [23]. In engaging in the exchange of an adjacency pair, the participants in the interaction effectively co-construct an elementary temporal structure in which future and past are constituted.

In talk-in-interaction, as analyzed by conversation analysis, the immediacy of response is intimately related to the turn-taking structure of vocal conversation [8]. As discussed above, the completion of the adjacency pair can be postponed by insertion sequences, such as repairs of misunderstandings or clarification exchanges. The base adjacency pair can also be preceded by introductory exchanges, such as announcements of what is coming, or succeeded by follow-up exchanges or confirmations.

In chat-in-interaction, as seen in the preceding log extracts, adjacency pairs can be delayed by a more complicated response structure, in which multiple participants can be typing simultaneously and postings do not always directly follow the message to which they are responding. Thus, in Log 1, Quicksilver or Gerry can be initiating other topics in the midst of an interaction between Aznx and Bwang. Also, Aznx and Bwang can be typing to each other simultaneously as in Log 6, particularly if there has been an extended period of inactivity. This often makes textual chat harder to follow and to analyze than verbal conversation.

Nevertheless, it is generally possible to identify base adjacency pairs carrying the discourse along. In the previous section, we identified ten pairs. The discourse moves in the log excerpts (each including one of these base adjacency pairs) formed a problem-solving sequence:

- Log 1. Open the topic
- Log 2. Decide to start
- Log 3. Pick an approach
- Log 4. Identify the pattern
- Log 5. Seek the equation
- Log 6. Negotiate the solution
- Log 7. Check cases
- Log 8. Confirm the solution
- Log 9. Present a formal solution

- Log 10. Close the topic

The integrity of each of the ten moves is constructed by the discourse of the participants. Each move contains its single adjacency pair, which drives the interaction. In addition, there may be several utterances of secondary structural importance, which introduce, interrupt or extend the base pair; there may also be some peripheral utterances by other participants.

The analysis of this paper is an attempt to make explicit the structure of adjacency pairs and a problem-solving longer sequence that is experienced by the participants and is implicit in the formulation of their contributions to the discourse. This is in contrast to analytic approaches that to some degree impose a set of coding categories based on the analyst's research interests or on an a priori theoretical framework, rather than on the perspective of the participants as evidenced in their discourse.

Lines 1795 and 1796, for instance, show the power *for the participants* of the adjacency pairings. Here, Aznx has addressed a mathematical proposal to Bwang: “**If you simplify yours [expression], it is n^2 .**” After 24 seconds of inaction, Aznx cannot understand why Bwang has not replied, expressing agreement or disagreement with the first part of the proposal, for which Aznx expects a response. Because it is not a preferred move at this point for Aznx to reprimand Bwang for not responding, Aznx inquires if Bwang has disappeared, perhaps due to a technical software problem, which would not be anyone's fault. Two seconds later, we see that Bwang was typing a more involved response that implicitly accepted Aznx' proposal. Bwang then immediately explicitly accepts the proposal in line 1798, allowing Aznx to continue with the start of a new move with line 1802. Here we see Aznx and Bwang clearly orienting to the adjacency-pair structure of their discourse, in terms of their expectations and responses.

Aznx and Bwang co-constructed the longer (ten move) problem-solving sequence by engaging in the successive exchange of adjacency pairs. Sometimes one of the students would initiate the pair, sometimes the other. As soon as they completed one pair, they would start the next. This longer sequence also has a temporal structure. It is grounded in their present situation, trying to find a formula for the number of sticks in the diamond figure. It makes considerable use of resources from their shared (co-experienced) past during the previous four hours of online sessions. It is strongly driven forward into the future by the practices they have learned for engaging in problem solving, culminating teleologically in the presentation of a solution.

The problem-solving sequence analyzed in this paper—covering 100 lines of chat during 10 minutes—is not selected arbitrarily or imposed in accordance with criteria external to the interaction, but is grounded in the discourse as structured by the participants. The excerpted sequence is defined as a coherent conversational topic by the discourse of Aznx and Bwang. They open this topic with their interaction in Log 1 and they close it with the discourse move in Log 10 [10].

This case study provides an unusually clear and simple example of group cognition in a virtual math team. In earlier sessions, the students encountered many difficulties, although they also achieved a variety of successes and learned much about both collaboration and mathematics. At the beginning of their first session, they did not know how to behave together and showed rather poor collaboration skills. Bwang said very little in English, often simply producing drawings or mathematical expressions.

Aznx, at the other extreme, tried hard to engage the others, but seemed to have a weak mathematical understanding of what the others were discussing. At various points in the sessions, misunderstandings caused major detours and breakdowns in the group work. Moreover, from an analyst's perspective the interaction was often almost impossible to parse [17]. By contrast, in the final segment that is here reviewed, the interaction is focused on two participants; they work well together; they seem to follow each other well; and their work goes quite smoothly. The structure of the interaction is also relatively easy to follow.

It seems that Aznx and Bwang have substantially increased their skills in online collaborative mathematics. The level of their excitement—especially in the excerpt of Log 8—shows they are highly motivated. Log 10 indicates that they would like to continue this kind of experience in the future.

6. COLLABORATIVE MATHEMATICAL MEANING MAKING

Shared meaning is co-constructed as the discourse moves (the log excerpts based around adjacency pairs) build on each other to form the longer sequence of the discourse topic. This is a key level of analysis for understanding the workings of group cognition. Because these discourse moves are founded upon adjacency pairs, they essentially involve more than one participant, and therefore lend themselves to being vehicles for cognitive phenomena at the group unit of analysis. Through their sequential positioning and subtle forms of mutual referencing, they contribute to problem solving and other cognitive accomplishments. As an example, we can see how Team B solved their mathematical problem across Logs 5, 6 and 7.

In Log 5, we see that collaborative problem solving of a math topic—like most group meaning making—is an intricate intertwining of individual interpretation and shared meaning [13, Chapter 16]. Bwang (line 1771) states the goal for the dyad of finding an equation to describe the pattern of twos. Aznx immediately announces that he knows the equation (1773) and wants to provide it (1772), to which Bwang acquiesces (1774). However, Aznx has trouble coming up with an equation: remembering it, explaining it, thinking of it or finding it. After a while, Bwang gradually announces that he will provide the equation (1784-1786). Then they both propose equations. Throughout the online session, mathematical proposals originate from the understanding of individual students. In this excerpt, they negotiate about who is to make the proposal, and end up both doing so.

Then it is necessary in Log 6 to decide whose proposal will be adopted by the group as a basis for future work. Interestingly, Aznx reconciles their proposals by algebraically transforming Bwang's equation to be the same as one of Aznx' own (1792-1794). This circumvents the possibility that Bwang will reject Aznx' proposal, which he in fact does (1797). It also establishes a group solution whose meaning (derivation, use, form) is likely to be mutually understood since the solution was proposed by both.

Finally, in Log 7, Aznx takes a further mathematical step, multiplying the n^2 by 4 to account for the 4 symmetrical sets of sticks. However, he presents this final formula in question format (1800-1802), soliciting Bwang's agreement in order to establish the formula within their joint problem space. Bwang implicitly accepts Aznx's step and reinterprets the question as requiring a next step of checking the formula for values of n . Bwang

presumably checks several values and concludes that the formula works (1804-1805). Aznx summarizes, “**So we got it!**” Note his use of the pronoun, “we”, attributing the solving to the group.

The formula, $4n^2$, is a particularly meaningful expression in this chat, the triumphal culmination of four hours of mathematical exploration. It is a highly meaningful expression for the group, summarizing their analysis of the diamond pattern of sticks at every level of n . The students understand its meaning as a consequence of their participation in the group processes of drawing and discussing together a rich set of related mathematical phenomena. The shared meaning of the math expression is publicly available in the discourse and through its traces in the log; it was co-constructed through the contributions of individuals and is interpreted by those individuals—and later by analysts.

7. THE STRUCTURE OF GROUP COGNITION

The analysis of the case study in this paper provides a first analysis of the long-sequence-of-moves structure of collaborative mathematical problem solving in a virtual math team. This is a paradigmatic example of group cognition. The small group—here reduced to a dyad—solves a math problem whose solution had until then eluded them (and had escaped Team C as well).

The students accomplish the problem solving by successively completing a sequence of ten moves. Each of the moves seems almost trivial, but each takes place through an interaction that involves both students in its achievement. The moves are commonplace, taken-for-granted practices of mathematical problem solving. They are familiar from individual and classroom problem solving in algebra classrooms. They have also been encountered repeatedly by Team B in their previous four hours of collaborative problem solving [5].

Reviewing the sequence of the group’s ten moves presented in this paper, we can follow the mathematical solution process. After opening the topic of the sticks problem (Log 1) and deciding to work on it together (Log 2), the team picked an approach of looking at the number of sticks as being countable with the series $(1+3+5+7+\dots)*4$ (Log 3). This series is generated by counting the sticks in a visual representation of the diamond pattern at different values of n (Figure 2). This uses the approach from previous sessions of separating the horizontal and vertical sticks (Figure 1) and then dividing each of those groups into two symmetrical groups (Figure 3). The group then articulates a verbal description of this visual series as being “**a pattern of +2s**” (Log 4). Both students try to symbolize the pattern of the verbal description as an equation (Log 5) and they come to agreement on the formula as n^2 (Log 6), presumably based on the formula for summing integer series, familiar to them from previous sessions. They then check that their equation works for a number of stages of the diamond pattern (Bwang does this off-line during Log 7). Having solved the mathematical challenge as a group they celebrate the group achievement: “**WE DID IT!!!!!!**” (Log 8), decide to present their solution publicly (Log 9) and close the discourse topic (Log 10).

It is this sequence of moves that accomplishes the problem solving. The sequence has an inner logic, with each move requiring the previous moves to have already been successfully completed and each move preparing the way for the following ones. Of course, in working on a problem, problem solvers—even

professionals [4; 6]—often make mistakes and explore deadends. Team B’s wiki posting (Figure 4) documents that some of this had happened prior to the excerpt analyzed in this paper. Part of what contributes to the unusual clarity of our example is the simplicity of the sequence followed in the final segment.

The common assumption about mathematical problem solving is that information in the form of math facts and manipulations are what are most important. In our analysis of problem solving in a group context, math content and other information is simply, unproblematically included in individual postings. In fact, more often than not, it is implicitly used and understood “between the lines” of the text chat. Of course, this is only possible because the group had already co-constructed a joint problem space [5; 9; 24] that included this math content as already meaningful for the group. Rather, the important aspects of discourse engaged in collaborative math problem solving are matters of coordination, communication, explanation, decision making and perspective shifting (e.g., moving between visual, verbal and symbolic modes [2]). To some extent, these are interactional moves required by most group activities; to some extent, these are adapted to the nature of mathematical discourse.

In conclusion, the group-cognitive achievement of the solution to the group’s final problem was accomplished by a sequence of moves. Each move was mundane when considered by itself. The moves and their sequencing were common practices of mathematical problem solving. Each move was interactively achieved through the exchange of base adjacency pairs situated in the ongoing discourse. The problem solving was an act of group cognition structured as a sequence of these interactive moves.

While we cannot generalize from the analysis in this paper, it seems that this case study can serve as a perhaps unusually clear and simple model of the structure of group cognition in mathematical problem solving by a virtual math team. It shows the group cognition taking place through the co-construction of a temporal sequence of problem-solving moves. Each move is conducted on the basis of an interactional adjacency pair of chat utterances. While the fine structure adheres to the adjacency-pair system of interactional exchange, the larger problem-solving structure builds on these elements through a sequence defined by the topical moves of mathematical deduction.

More generally, this suggests a multi-layered hierarchical structure to discourse in virtual math teams [16]. Each layer is oriented to by the participant activities:

- a. **Group event:** E.g., Team B’s participation in the VMT Spring Fest 2006. The team meets together and gradually starts to act as a collaborative group.
- b. **Temporal session:** Session 4 of Team B on the afternoon of May 18, 2006. The participants agree when to break up a session, when to meet next, and then show up at the same time.
- c. **Conversational topic:** Determining the number of sticks in a diamond pattern (lines 1734 to 1833 of Session 4). We saw how Bwang and Aznx open the topic and later close it.
- d. **Discourse move:** A stage in the sequence of moves to accomplish discussing the conversational topic (e.g., lines 1767-1770). The team steps through the sequence of moves.
- e. **Adjacency pair:** The base interaction involving two or three utterances, which drives a discourse move (lines 1767 and 1769). Each initial utterance elicits a response.

- f. **Textual utterance:** A text chat posting by an individual participant, which may contribute to an adjacency pair (line 1767). The group members format their separate postings.
- g. **Indexical reference:** An element of a textual utterance that points to a relevant resource. In VMT, actions and objects in the shared whiteboard are often referenced. Mathematical content and other resources from the joint problem space and from shared past experience are also brought into the discourse by explicit or implicit reference in an utterance.

This multi-layered structure corresponds to the multiplicity of constraints imposed on small-group discourse—from the character of the life-world and of culture, which mediate macro-structure, to the semantic, syntactic and pragmatic rules of language, which govern the fine structure of utterances. An understanding of group cognition must concern itself primarily with the analysis of mid-level phenomena—such as how small groups accomplish collaborative problem solving and other conversational topics. This is a realm of analysis that is currently underdeveloped.

The preceding analysis illustrates the applicability of the notion of a long sequence as suggested by both Sacks [7] and Schegloff [11]. The sequence consists of a coherent series of shorter sequences built on adjacency pairs. This multi-layered sequential structure is adapted from CA to the essentially different, but analogous, context of groupware-supported communication and group cognition. Having seen that this kind of sequential structure exists in the relatively simple case we analyzed, we can now look for longer sequences in the traces of other acts of groupware-mediated group cognition.

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