

Exploring Interactive Learning for the Era of Climate Change

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Abstract

The world changed significantly about 70 years ago. The atomic bomb, population explosion, exponential growth of fossil-fuel usage with CO₂ emissions, urban/suburban sprawl and numerous other socio-economic transformations led to a dangerously increased influence of human behavior on worldwide natural systems. Public understanding now has to catch up with these changes so we can comprehend and moderate the new and potentially catastrophic processes. Learning researchers should urgently develop transformed approaches to understanding and teaching about this new world of climate change, in which natural and societal processes are inexorably entangled. This will require new conceptualizations of knowledge and new methods of education.

This essay reports on a research project that may indicate a direction for designing education in and about the era of climate change. By reviewing the project's empirical results, it suggests a direction for the future of interactive and intelligent learning. Specifically, it sketches focal points of a philosophy of computer-supported collaborative learning (CSCL) that emerged from research on teaching and learning dynamic geometry. As an illustrative case study of educating for the new age, it suggests that dynamic geometry as taught in the project can provide student collaborative knowledge building with a model of “dependencies” in interconnected systems, preparing groups of students to understand the interactions among human and natural systems in the present era.

Review of this research project exploring the development of mathematical cognition by student groups learning dynamic geometry in online teams elaborates a theory of learning and thinking as “group cognition” – thinking by means of social and semantic interaction within group discourse, mediated by physical and digital artifacts, rather than within individual minds. The recommended approach to learning centered on technologically supported group interaction aligns education with the multidisciplinary nature of science in the present, and suggests promising approaches to interactive and intelligent learning in this challenging epoch.

Keywords: Anthropocene, group cognition, dynamic geometry, dependency, shared understanding, group practices, virtual math teams

Learning in the Era of Climate Change

Learning in the future will require new ways of understanding interactions among countless actors: including human, animal, mineral, technological, computational and Earth-system agents. Some scientists denote the era of the past 70 years as the Anthropocene. Referring to the present geological epoch as the “Anthropocene” (from the Greek for human-centered new era) denotes the essential influence of human

behavior, industry and consumption upon major systems of the biosphere, including the land, oceans, vegetation, animals, sea life, insects, viruses and climate.¹

The current coupling and interpenetration of cultural and natural evolution (Donald, 1991; Donges, Winkelmann, Lucht, et al., 2017; Latour, 2017) requires more than simple mechanistic laws and equations like Galileo's and Newton's to comprehend, anticipate and influence; it involves thinking in terms of probabilistic formulations of subtle interdependencies (Kolbert, 2014; Krauss, 2021; Thomas, et al., 2020; Wiener, 1950). In particular, teaching and learning mathematics in our age should provide cognitive tools and perspectives for humanity to survive in this complex setting of global climate change and potential species extinction (Coles, 2017; Gomby, 2022).

In response to a major shift in reality, we need to re-conceptualize scientific analysis, including its mathematical and cognitive underpinnings (Cole, 2024; Griscom, Adams, Ellis, et al., 2017; Steffen & Morgan, 2021). New approaches to teaching and learning mathematics are required here as much as in particle physics (Boylan & Coles, 2017; Mikulan & Sinclair, 2017; Yoon, Klopfer, Anderson, et al., 2016). Physics has had to consider stochastic and non-linear processes, relativistic and quantum calculations, feedback and observer influences, field and gauge theories or conceptualizations like entropy, strings, entanglement, dark energy and alternative universes. Now our understanding of the everyday world needs to see how things are intertwined in surprising ways with exponential change, feedback loops and tipping points (Kemp, Xu, Depledge, et al., 2022; Lenton, Held, Kriegler & Schellnhuber, 2008; Steffen, Rockström, Richardson, et al., 2018).

This paper reports on a research project to develop a paradigmatic approach to teaching dynamic geometry as a way of conceptualizing *dependencies* among objects and for comprehending causal interconnections. This may suggest an approach to interactive/intelligent learning and computer-supported collaborative learning (CSCL) in the future: to support group exploration of core concepts of Anthropocene science. Just as Roschelle's (1992) early CSCL experiment modeled acceleration as a fundamental concept of Newtonian mechanics, the project reviewed here modeled dependency as a central notion of Euclidean geometry. Similarly, Anthropocene science relies on many relationships that lend themselves to computer modeling of core relationships that could be incorporated in contexts of educational collaboration software (Boylan & Coles, 2017; Hashem & Mioduser, 2011; Yoon, Goh & Park, 2018), in intelligent digital support and in interactive processes.

Understanding Dependencies for the Era of Climate Change

Geometric dependency can provide a conceptual steppingstone for grasping the relationships that are useful for analyzing, modeling and predicting phenomena and interdependencies in the complex systems of the contemporary world. Software was successfully designed in the research reported here to scaffold

¹ The term Anthropocene was proposed in 2000 by a number of geologists (Crutzen & Stoermer, 2000; Steffen, Broadgate, Deutsch, et al., 2015; Thomas, Williams & Zalasiewicz, 2020) and inspired a lively multidisciplinary literature (Dunker, 2020; Giddens, 2011; Latour, 2018; Moore, 2016; Robinson, 2020; Steffen & Morgan, 2021). However, in 2024 official organizations of geologists declined to name a new Anthropocene epoch, noting that the human race has always interacted with the environment. Also, some researchers argue the change was not geological, but was a shift in global social systems or in the rate of technology development (Haraway, 2016; Klein, 2014; Saito, 2022). Nevertheless, there undoubtedly was a qualitative change after WW II. Trends of CO₂ emission, global warming, sea-level rise and biodiversity loss all went exponential. The age of human-forced climate change is already perceptible in our warming biosphere (Dixon-Decleve & Gaffney, 2022; Kolbert, 2021; Wallace-Wells, 2020; Zhai, Pirani, Connors, et al., 2021). Despite the geology profession's technical decision, it remains useful to use the term Anthropocene to designate the significant and increasing human impact on the Earth system.

teams of teenage students to comprehend basic geometric dependencies; analogous new software for learning other domains may aid adult teams to analyze more complex system dependencies.

The research on groups of students learning dynamic geometry in an online collaboration application showed that it was possible to increase the groups' shared understanding of dependency relationships and their consequences. For instance, as discussed in this paper, longitudinal analysis of their online discourse showed how students exploring dynamic geometry were able to increasingly comprehend, articulate and apply dependency relationships among geometric objects.

In the paradigmatic example of constructing an equilateral triangle in dynamic geometry, students were increasingly able to articulate the differences between points that were free, constrained or dependent on each other. The dynamic geometry app facilitated this by allowing users to drag vertices and sides of triangles that had been constructed with implicit dependencies. Ever since Euclid, an equilateral triangle is constructed starting with points A and B "free" (arbitrarily placed on the plane). Then point C (which forms the equilateral triangle) must be at an intersection of the two circles of diameter AB that are centered at A and B. Consequently, the position of point C is "dependent" on the positions of points A and B. By contrast, a point D placed on side AC is only "constrained" to lie at any position upon line segment AC. Triangle ABC is dynamic in well-defined ways. It can be moved around, rotated and enlarged by dragging certain points. To predict how the triangle can and cannot be moved involves understanding which points are free, constrained or dependent – as groups of students in the research project gradually determined. In his first proposition, Euclid proved that the equality of the lengths of the three sides of equilateral triangle ABC is dependent upon the equality of the various radii of the circles centered on A and B used in constructing the triangle.

The mechanics of Galileo and Newton can also be understood in terms of simple dependency relations. For instance, the strength of gravity between two objects is directly proportional to the masses involved and inversely proportional to the square of the distance between them. However, the important dependencies in the sciences of climate change are more sophisticated. For instance, in ecological science the population of a species may vary according to multiple seasonal cycles, in response to changes in populations of predators or prey, according to rates of birth and mortality, or due to plague and famine. In world politics, conflict may be influenced by technological advances, weapon production, population migration, crop productivity, ideological beliefs and many other social, economic and geophysical phenomena. Furthermore, such dependencies often involve feedback loops and tipping points, where, for instance, crop failure may lead to migration, which then provokes conflict resulting in more hunger and increased migration, eventually perhaps resulting in political instability and irreversible social transformations.

Dependencies in geometry are more determinant, predictable, definable, observable. So, they can be easier to identify, explain and understand. In the research reported below, groups of students gradually established increasing shared understanding of dependencies and constraints. They developed their own ways of analyzing and discussing dependencies in their geometric constructions and they acquired deeper comprehension and usage of traditional mathematical language.

In the intellectual development of children, understanding of geometry in terms of dependencies represents an important and challenging advance over understanding in terms of visual shape ("looks equal") or numeric measurement ("equally long") (deVilliers, 2004; Sinclair, 2017). As Euclid showed, comprehending dependencies underlies insight into both practical construction and theoretical proof in geometry (Euclid, 350 BCE; Netz 1999). Similarly, systematic understanding of dependencies is foundational for understanding Anthropocene sciences.

An understanding of how students learn the concept of dependency in dynamic geometry with the support of interactive and intelligent software in settings of collaborative knowledge building can indicate promising approaches to preparing students and professionals with the group-cognitive skills and group practices needed in the Anthropocene. The major challenges of the new age require collaborative strategies based on shared understanding of complex interdependencies, not just actions by individuals. Knowledge-

building communities must be prepared with new cognitive interaction skills and supportive intelligent artifacts to participate effectively.

Modeling the Era of Climate Change with Dynamic Geometry

Teaching and learning mathematical thinking relevant to the era of climate change may be promoted by student exploration of dynamic geometry. This kind of interactive application allows students to investigate the structure and interrelationships of well-defined geometric elements and complexes. This can provide a primer and conceptual foundation for understanding dependencies in the intertwined contemporary world.

Dynamic geometry is a computer-based form of mathematics grounded on Euclidean geometry and implemented in popular applications such as GeoGebra and Geometer's Sketchpad (Hohenwarter & Lavicza, 2009; Sinclair, 2008). As an illustration, Figure 1 shows an equilateral triangle constructed in dynamic geometry with side lengths dependent upon circles with equal radii, as specified in Euclid's first proposition. Then an interior equilateral triangle was constructed with vertices equal distances from the vertices of the exterior triangle. Dragging around points of each triangle suggests that the two triangles both remain equilateral regardless of the positions of the specified points.

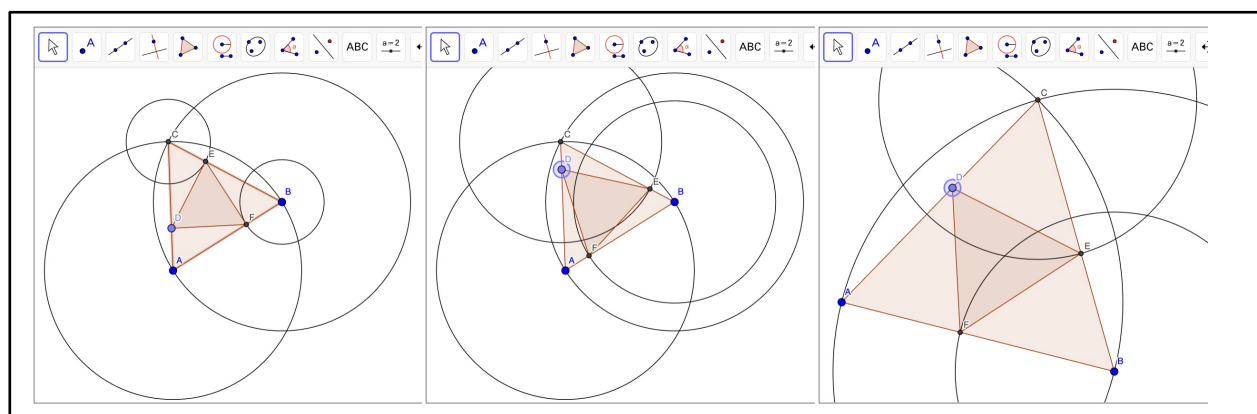


Figure 1. Inscribed equilateral triangles constructed in GeoGebra and dragged to different positions.

Dynamic geometry visualizes the generalization implicit in Euclidean geometry and the dependencies that underlie it by allowing points, lines and figures to be interactively dragged to alternative possible locations. Dependencies that persist despite such dragging reflect underlying causal relationships. They suggest which relationships still hold when locations are generalized from arbitrary initial positions of points to other possible positions.

While classical Greek proofs stress deduction, they implicitly assume the generality of their illustrative constructions which illustrate the proofs. Digital geometry, by contrast, allows points to be moved around visibly, rearranging related elements in order to maintain dependencies defined by the construction process. This allows a viewer to observe some of the generality of the construction, including effects (constraints) of the dependencies. The relevant dependencies are maintained by Euclidean constructions when carried out in dynamic geometry.

The implication of Euclid's (300 BCE) text in Figure 2 is that this construction works for any finite straight line and that the construction using the specific line AB in the accompanying diagram is an example of how to do the construction for any similar lines located elsewhere. If this construction is carried out in dynamic geometry as in Figure 1, then one can drag point A, point B and/or line AB to arbitrary other

positions and the constructed triangle ABC will still be equilateral. Such dragging, which is typical of dynamic geometry, displays graphically that the construction is valid for many lines AB – all those tested with different locations for end points A and B. It also displays the dependencies imposed by the construction that constrain the triangle to be equilateral: namely the two circles of radius AB, which ensure that the lengths of sides BC and AC are both equal to the length of line segment AB, and therefore the triangle's three sides are all equal to each other.

On a given finite straight line to construct an equilateral triangle.
 Let AB be the given finite straight line.

 Therefore, the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.
 Being what it was required to do.

Figure 2. Introduction and conclusion from Euclid's first proposition.

The same applies to Euclid's propositions that are proofs, rather than just constructions. They are presented as examples of how to conduct proofs for specific diagrams at specific locations, but are intended to be generalized to any diagrams with the same features (Netz, 1999). Euclid's static diagrams translate directly to dynamic-geometry constructions precisely because the constructions and proofs are designed to be generalizable to points or figures located anywhere on the infinite plane. They are tacitly built around the application of dependencies, such as the length of a line segment being dependent upon a circle of certain radius. These dependencies underlie the proofs, for which diagrams are constructed following Euclid's propositions. An understanding of dynamic geometry in terms of the design of dependencies provides insight into the design of geometric figures – insight that is not always fostered by a traditional presentation of deductive proof.

Becoming a skilled constructor of dynamic-geometry figures involves paying close attention to actions that establish dependencies among objects, such as dragging points to make sure that the software has defined those points at intended line intersections. Students must develop practices for accomplishing this. A student's growing explicit concern for establishing and checking effective dependency relationships quickly becomes habitual, a matter of assumed behavior that is henceforth carried out tacitly.

Viewing, understanding and manipulating constructions in terms of their interdependencies provides students with insight into why associated proofs work the way they do (deVilliers, 2004). It is because triangle ABC's sides were constructed by radii equal in length to segment AB that the three sides are always necessarily of equal length. Geometry can be viewed as the systematic study of dependencies that are designed into or discovered within complexes of simple objects like points, lines, angles, circles, polygons. The establishment and preservation of dependencies is fundamental to the logic of Euclid's propositions and to the mechanisms of dynamic geometry's software.

Euclid's propositions talk about points and lines being placed in the plane, but do not explicitly discuss the dependencies that are implicitly designed into the constructions. The dynamic-geometry software, on the other hand, must keep systematic track of these dependencies behind the scenes. When a point is moved, the software checks for any dependencies involving that point and immediately moves other points in ways that maintain the dependencies. The dynamic-geometry display thereby provides a dynamical model of a geometric structure that obeys sets of dependencies among its elements.

Students exploring dynamic geometry can learn to think about systems of interdependent elements, some of which are completely dependent upon the positions of others, some are constrained (e.g., to move only in a fixed circle around another point) and some are simply free to move anywhere (Hölzl, Healy,

Hoyles & Noss, 1994; Jones, 1996). Perhaps this kind of systems thinking can later be applied to evolutionary models of nature, such as a model of animal populations dependent upon climate, vegetation and interactions among species.

Educating students for the Anthropocene involves helping them to think and talk about systems of abstract (non-visible, underlying, theoretical) interdependencies. Global climate change is a high-level result of interactions at the molecular level: Combustion of wood, coal and gasoline release carbon dioxide into the atmospheric mix of gases. Some is absorbed by vegetation; some increases the ocean's acidity; and some causes a greenhouse effect in the atmosphere. The greenhouse effect raises average temperatures, melts arctic ice, modifies weather patterns, increases sea-level rise. These, in turn, alter the conditions for flora and fauna, potentially leading to species extinction. All these natural systems interact with each other and with human social systems – like fossil-fuel technologies – to feed back on each other, eventually passing tipping points.

While science in the Anthropocene is more complicated and multidisciplinary, it is still based on understanding dependencies, even if they are harder to compute (Krauss, 2021; Zhai, et al., 2021). Since the beginning of modern science, Euclidean geometry has been used to teach students how to think rigorously about dependencies. Today, we can see that dependency is the foundational concept in both dynamic geometry and environmental science. Thus, it is still necessary in the Anthropocene to prepare students for understanding the world by helping them to adopt practices required for understanding dependencies in geometry. This involves being acculturated in the traditions of mathematics.

Mediated Cognition in the Era of Climate Change

Since the Greek geometers, constructions and their proofs have been communicated among mathematicians and math students through carefully structured texts that reference associated diagrams. Understanding geometry involves reading/writing/speaking/thinking in the specialized language and recalling previous propositions. Mathematical cognition takes place in such inscriptions: sequential descriptive statements, illustrative figures and specialized symbol systems.

Geometric cognition is embodied in inscriptions – texts coordinated with labeled constructions (such as Figures 1 and 2 above). These are knowledge-building artifacts in the visible material world. Their meaning is shared and based on intersubjective language and cultural traditions. This meaning must be understood and interpreted by trained and capable individuals. Students have to learn how to make careful constructions, but also how to discuss these constructions and their designed dependencies with other people in the precise language of mathematics. These are skills requiring deep understanding and personal engagement, not just rote memorization of terminology and facts.

There is a subtle combination of individual, small-group and community cognition at work in the teaching and learning of mathematics. The knowledge of how to construct an equilateral triangle is expressed in an inscription of Euclid's first proposition. This inscription may be included in a geometry textbook or in a dynamic-geometry exercise. Its meaning is defined by the shared understanding of the mathematical community, including textbook authors, schoolteachers and – to a lesser extent – beginning geometry students.

When a small group of students explores one of Euclid's propositions, their group cognition consists of the shared meaning in their discourse – issuing from their multiple perspectives and their individual linguistic abilities to understand and contribute to the group interaction (Stahl, 2003). The group processes of collaborative learning involve individual capacities to participate effectively. However, while this individual cognition is required for group cognition, the group level cannot be reduced to a sum of individual contributions. The collaborative level is dominated by references, anticipations, goals, agreements, decisions and history of the group as such. Individuals in the group are typically not explicitly

aware of most of these factors and would not have been subject to them if not participating in the group interaction.

In general, high-level cognitive functions of individual human minds are developed first through small-group interactions and may be subsequently further developed as individual skills. Intellectual skills are mediated by socially transmitted language and tools. Mathematical cognition takes place through the terminology, practices, symbols and inscriptions adopted by the worldwide, historical community of practitioners.

The common focus on individual cognition in philosophy, psychology and educational theory is based on introspection by adults and on observation of skilled practitioners. As adults, we picture ourselves learning through solitary reading or silent reflection. However, if we observe infants and toddlers learning the basic skills for living in the physical and social world, we can see the central role of interaction with other people, such as parents and siblings. Vygotsky (1930, p.57) concluded that cultural development – including formation of concepts – occurs first on a social level. For instance, children in his studies “could do only under guidance, in collaboration and in groups at the age of three-to-five years what they could do independently when they reached the age of five-to-seven years.”

Vygotsky’s analysis of the development of the pointing gesture (p.56) provides a clear example of group cognition. The mother does not teach her infant how to point to what he wants; the meaningful gesture is not passively acculturated from existing culture. Rather, it is co-constructed by the participants situated in the setting as an intersubjective meaning-making interaction. The gesture develops as tacitly understood within the intimate mother/infant group and gradually becomes sedimented into a symbolizing artifact through repetitive habituation. The meaning of the pointing finger as a reference to some desired object is mediated by the whole situated interaction involving mutual recognition of agency, observed glances, bodily orientations and physical relations among the actors and intended objects. There is more going on here at the group level of analysis than the coordination of individual mental representations. Deixis, pointing or reference is a fundamental cognitive function. In Vygotsky’s analysis, we see how it develops as primarily a phenomenon of group interaction, rather than just individual mental mechanisms.

More generally, Vygotsky concluded that cognition is mediated by language and artifacts. He developed the foundations of a theory of “mediated cognition.” Cognition is not a matter of isolated mental functions that individuals develop internally, but a consequence of interaction with the social and physical world, including other people, physical artifacts and spoken language. This was a prescient step toward a conceptualization appropriate to the Anthropocene, in which phenomena are defined by their interactions with other agencies. To study mediated cognition, one must observe early learning in real-world social settings and see the embodied, intersubjective origin of cognition and learning. To stress the social basis of learning and cognition, we use the term “group cognition” as an alternative to the traditional focus on individual cognition and as a core foundational phenomenon for collaborative learning.

Networks and Groups in the Era of Climate Change

In human cultures — especially advanced technological ones — cognition is mediated by writings, symbol systems, drawings, maps, external memories, computational devices, automated processes, feedback signals, and so on. Cognitive accomplishments come about due to innumerable influences, determinants, factors and considerations. The causation is not mechanical, but dependent upon the nature of the agents and their interrelationships. Social interactions are matters of understanding, interpretation and ambiguity. Predictions can at best be probabilistic, taking into account tendencies and trends. Understanding human/nature interactions in the Anthropocene world requires similar analysis. The group in group cognition is not taken as a set of solipsistic minds, but as a cognitive ecology. Like a butterfly fluttering in the breeze, an emitted CO₂ molecule reflecting a sunray does not cause a storm, but may imperceptibly contribute to its likelihood or magnitude. Analogously, an interactive learning course may

sometimes foster group practices, which might eventually strengthen individual understandings or modify community norms.

For many reasons, causation can no longer be considered a simple effect of individual mental thinking determining physical action. First, cognition increasingly takes place within tools, such as sheets of paper, charts, calculators, computer models, spreadsheet analyses. Ideas are posed, worked out, communicated and preserved in these shared-meaning physical media in ways they could not be in pure thought (Donald, 2001). They are also discussed, shared, critiqued, developed and negotiated in small groups. Although experienced people can internalize some of these aids and alternative perspectives to incorporate them to some degree within their own minds, the embodied, mediated and interactional character of situated group cognition remains decisive.

Second, the consequences of individual human intentions and actions are not simple direct results of individual cognition. Latour (2014, p.7) points out that the central military outcome in Tolstoy's detailed presentation of *War and Peace* was not simply due to the commander's agency, but was influenced by innumerable peripheral actors and unanticipated circumstances. The details of a messenger's wanderings while delivering military orders, a cannonball's bouncing through the enemy's front line, a horse rearing in the Calvary line are examples multiplied countless times influencing events. Latour develops a new conceptualization of causation involving potentially huge networks of actors, both human and non-human. Technological artifacts, for instance, can embody inferred human intentionality, such as a spring door-closer trying to keep a door shut (Latour, 1988; Rabardel & Beguin, 2005; Simondon, Mellamphy & Hart, 1980).

Third, especially in the Anthropocene, human actions involve and affect natural phenomena. The causal relationships involved are complex and only partially understood by the actors. They may involve huge numbers of objects and intricate patterns of interaction, which are not precisely predictable. It is often not possible for people to know the ultimate consequences of their actions based on simple causal relationships; broader dependencies may have to be taken into account, particularly in the tightly integrated Anthropocene world.

As discussed above, dynamic geometry might provide a workspace for exploring systems of interdependent objects, where students can design dependencies into constructions of multiple objects and then can see the consequences of the dependencies through manipulation of the objects. This can offer an exploratory playground for groups of students to learn about the kinds of mathematical relationships that are important for understanding the contemporary world. Such cognitive models are needed in a world in which simplistic common sense is inadequate to understand our dynamic world systems. It may be possible to design interactive and intelligent learning environments to facilitate groups of students developing an understanding of ecological dependencies.

The design of computer software to support online collaborative learning was explored through experiments with a number of prototype computer-supported collaborative learning (CSCL) systems in *Group Cognition* (Stahl, 2006). One major finding from that research was that *the notion of "meaning making" or the "negotiation of meaning" needed to be better understood* than it had been in previous CSCL theory. Most earlier analyses of shared understanding were based on theories of individual cognition, perhaps coordinated by efforts of "common grounding" (Clark & Brennan, 1991). In the studies collected in *Group Cognition*, multiple analyses are provided of small groups adopting shared meanings of charts or mathematical problems through discourse, explicit agreement and subsequent tacit usage. The student groups are shown to construct shared knowledge through interaction, much as the mother and infant described by Vygotsky built their shared meaning of the pointing gesture.

The need for much more detailed analysis of meaning making and negotiation in collaborative learning led to a decade-long research effort: the Virtual Math Teams Project (VMT). This project involved designing and iteratively improving an online environment for small groups to explore and discuss mathematics together. Functionality was provided for both textual dialog (chat) and diagrams (whiteboard). Teams of

students were recruited through teachers and were provided with challenging mathematical problems, mainly of middle-school combinatorics and geometry curriculum.

Analysis of interaction in the VMT sessions was conducted at the levels of individual, small-group and classroom. Analogously to the mother/infant's pointing gesture in Vygotsky's analysis, meanings, artifacts, actions and knowledge can be created as the group cognition of online small groups in the VMT setting. The project's collaboration software, dynamic-geometry app and sequenced curriculum provide a setting in which the interaction of the group can evolve mathematical practices. The small-group learning can gradually spread to the members individually, as well as to the social context. Just as the mother and infant individually subsequently take frequent advantage of the intersubjectively understood pointing gesture, the students can apply their shared geometry habits together and eventually even use them in their personal individual cognition. Going in the other direction, geometric knowledge developed in the small group is aligned with the standards of the larger mathematical community through the automated constraints and feedback of the dynamic-geometry app, questioning by other students, the embedded curriculum and teacher guidance in the encompassing classroom. The VMT theory, pedagogy and technology were continually developed to support learning across such levels.

Teaching in and about the Era of Climate Change

Hosting education on computer devices not only allows the use of apps like dynamic geometry, but can also support collaborative learning beyond face-to-face settings with interactive and intelligent mechanisms. It can permit many forms of automated support, such as access to online information sources, AI commentary and archiving of activity transcripts. Furthermore, it can open new dimensions of social interaction and collective inquiry. Computer support must be designed to enhance individual and group cognition by students, rather than restricting their intellectual roles.

Unfortunately, most commercial collaboration software and social media are primarily designed to support the expression of narrowly directed individual thinking and hierarchical management. They reinforce individual opinions rather than stimulating collaborative thinking. The VMT Project experimented with systems of flexible computational support for collaborative interaction, negotiation of meaning, intersubjective consensus building. *Studying Virtual Math Teams* (Stahl, 2009a) includes reports of this research by about 40 academics from several countries. It motivates the project, analyzes the data of student interactions and draws implications for the science of CSCL and for future software design.

An important aspect of this research is that learning is primarily analyzed at the group level of analysis. It is studied as group cognition. There are few surveys or questions concerning individuals' ideas, reflections, representations or memories. Rather, the data for analysis of learning and knowledge building consists of automated transcripts of the small-group interactions. The VMT system is instrumented to capture all the chat discourse and whiteboard construction that took place. The data of group cognition includes discourse sequences consisting of proposals, responses, questions, answers, interpretations, acceptances and other chat postings or interjections that work together to anticipate, expand upon, accept or reject each other. The collection of reports includes examples of many approaches that were developed for analyzing this group-level data.

The VMT effort began to define a science of group cognition and to identify the characteristics and mechanisms of small-group-level cognitive phenomena, which can, for instance, contribute to the teaching and learning of mathematics. The computer technology involved in the project not only supports interaction and exploration by student groups, but also facilitates experimentation and analysis by the project researchers, software developers and curriculum developers.

The historical effectiveness of mathematical cognition requires a subtle interweaving of processes at the individual, small-group and community levels of analysis. Even a phenomenological analysis of

mathematical cognition in terms of individual subjectivity stresses the centrality of intersubjective concepts and associated shared inscriptions. Conversely, the functioning of cultural traditions like Euclidean geometry requires reactivation of insight by individuals. In considering the “crisis of the European sciences,” the philosopher Husserl (1936/1989) felt impelled to investigate “the origin of geometry.” As a phenomenologist, Husserl started from introspection on the experience of understanding a geometric proof and asked how an object of individual cognition like a geometric term could become an ideal object or universally recognized meaning (a concept). He described a multi-step process of group cognition in which people collaborated using geometric inscriptions (p.164). The insights into the necessity of proofs were “reactivated” by the individual participants as they shared the intersubjective meanings “sedimented” in their adopted mathematical language.

Design-Based Research in the Era of Climate Change

The VMT Project represented a systematic attempt to “translate” Euclidean geometry for the era of climate change by reactivating its meanings in settings of collaborative learning and by emphasizing the functioning of dependencies. A comprehensive description of this research in *Translating Euclid* (Stahl, 2013) includes chapters detailing interdisciplinary aspects of this effort, including: the project vision, history of geometry, guiding project philosophy, covered mathematics, developed technology, approach to collaboration, educational research, social theory, curricular pedagogy, analysis of practice and design-based-research methodology.

At this point, the VMT Project developed a unique multiuser version of GeoGebra (an open-source app for dynamic geometry) and integrated it into the online collaboration environment, so that groups could view and work on shared geometric constructions collaboratively in real time. It also iteratively tested curricula that scaffolded student groups to explore the basic concepts, propositions and dependencies of Euclidean geometry. Researchers analyzed the transcripts of group interaction in which meanings were negotiated, sedimented and tacitly reactivated in their group language and understanding.

The process documented in *Translating Euclid* was a continuous cycle of trial and adaptation. An intervention featuring interactive learning or intelligent support was tried with several groups of learners and the resultant knowledge-building interaction was analyzed. Then the technical support, the pedagogical approach, the curricular presentation, the analytic methodology or the learning theory was adjusted in response to the findings. This iterative approach to design and development is known as “design-based research.”

Cycles of analysis included consideration of social, psychological, mathematical, technological, semantic and pedagogical factors. Within this multi-dimensional consideration, focus centered on tracking the increasing group comprehension of dependency, as a central phenomenon of geometry and potentially of ecological and climate science. Software development was continuous, with major curriculum rewrite annually and analysis conducted in weekly data sessions involving multiple researchers. The theory of group cognition, the method of interaction analysis, successive curricula of digital geometry and software components are extensively documented in the publications reviewed here. The complexity of design-based research reflects the interrelated nature of phenomena and processes in the age of climate change.

Although the VMT software is designed for use by small groups of students collaborating online, the research project stresses the importance of integrating support for the individual students as well as for classroom efforts in addition to the collaborative learning. Group cognition necessarily includes interpretation and contributions from individual cognitive perspectives. It also benefits from a supportive classroom context. The theory of group cognition emphasizes this integration. It recommends that small-group collaborative learning be adopted in coordination with phases of individual and classroom learning. This provides multiple opportunities, formats and processes for the sedimentation of key concepts, the reactivation of mathematical insight and the sharing of knowledge and procedures. The VMT Project

included a strong teacher training and involvement emphasis to support the classroom level. The conceptualization of interaction among the levels of individual, small-group and community cognition is a characteristic kind of problem for scientific research in the Anthropocene.

Group Practices for Action in the Era of Climate Change

Because learning involves a mix of tacit understanding and explicit interpretation, it is perhaps best to conceive it in terms of “practices” rather than mental representations. In particular, collaborative learning can be analyzed as the adoption of group practices by the small group. These practices may be derived from pre-existing society-wide cultural practices, and they may be subsequently personalized as individual practices, but to be effective they must first be adopted by the small group and integrated into its activity and discourse.

The VMT Project was designed to study empirically how collaborative knowledge building takes place in a CSCL setting. *Constructing Dynamic Triangles Together* (Stahl, 2016) analyzes every chat posting by a particular small group of students who engaged in eight hour-long online sessions in the VMT Project using the collaborative version of dynamic geometry. It documents step-by-step and chat posting-by-posting how the group increased its understanding of dependency. The group adopted numerous practices that markedly increased their ability to identify, construct, manipulate and reason with geometric dependencies.

Through the close interaction analysis of their chat discourse and geometric manipulations, it becomes clear that the groups were collaboratively negotiating shared meanings and adopting these as group practices. About 60 distinct group practices are highlighted in the longitudinal analysis. Each of these is explicitly discussed in the group discourse and analyzed in the book. The variety of practices reviewed covers many of the basic needs of collaborative learning, dynamic geometry, computer support, design of dependencies and online interaction, including:

- Group collaboration practices
- Group VMT-tool-usage practices
- Group practices using chat and digital-geometry actions
- Group geometric-construction practices
- Group digital-geometric dragging practices
- Group dependency-related practices

For each practice, the group went through a process of (a) confronting a problem, (b) discussing action options, (c) agreeing on a path for going forward and then (d) proceeding with putting the practice into action. While this response to a problem initially required explicit discussion and group agreement, subsequently the group could tacitly proceed with the adopted solution without any discussion. The practice was adopted by the group and integrated into its behavior. The practice could have been derived from the larger social context, such as a teacher recommendation based on mathematical tradition or it could have been a suggestion from an individual student, but it had to go through the negotiation process by the group in order to become part of the group’s effective behavior or group cognition. More generally, Stahl (1993) had previously analyzed the interplay of explicit and tacit understanding in computer-mediated knowledge building, following this general procedure of reflection and acceptance.

The adoption of group practices is a way for groups to crystalize knowledge-building results of their group cognition. Human cognition is not a simple process of rational deduction that operates like the well-defined sequential operation of a computer program executing within a person’s head. Rather, it often takes place in group discourse – individual abilities contribute to shared cognitions from multiple perspectives and backgrounds, within complex shared situations. Articulated statements summon future responses by building on the past context in the present situation. Especially in instances where fundamental learning

takes place, there is a mix of individual, small-group and community processes, mediated by a complex historical world of influencing social factors and mediating physical artifacts.

While the cognitive behavior observed in the VMT Project was a mix of individual, small-group and classroom interactions, it is possible to distinguish phenomena at each of these levels of analysis, such as individual habits, group practices and classroom regularities. Although it may be possible to define other levels of analysis, these three are typical of school settings, in which individual students are graded, small groups of students may interact and teachers orchestrate classroom activities. Interpenetration of levels of analysis is a hallmark of Anthropocene science, spanning from atomic to global phenomena.

Group Cognition for Building Knowledge in the Era of Climate Change

The VMT analysis of group cognition in geometry education attempts to re-conceptualize the nature of mathematical cognition as embedded in Anthropocene social and physical complexes. Cognition takes place expressed in explicit dialog, hidden within tacit practices and preserved in persistent inscriptions. Knowledge building is mediated by and stored in physical knowledge artifacts (Damsa, 2014). These can be internalized or personalized in mental abilities and representations through memory and imagination, but they are not originally purely mental phenomena. Euclid's propositions exist in contemporary texts. Their meaning is not dependent upon the minds of Thales, Pythagoras or Euclid, but upon the current texts and accompanying figures, as well as upon the meanings and practices of the mathematical community today.

When a group of students collaborates on a dynamic-geometry problem in a system like VMT, their group cognition resides primarily in the shared software interface, which displays their group work, including both chat discourse and constructed figures as a kind of "joint problem space" (Teasley & Roschelle, 1993). From observation of these traces of shared work and interaction, researchers, teachers and the participants themselves can witness negotiation of meaning and mathematical reasoning without having to appeal to assumptions about individual mental events behind the scenes. Group cognition can be persistent and observable within physical knowledge artifacts such as textual inscriptions and computer transcripts. The learning of mathematics can be studied by analysis of the development of mathematical group cognition, such as occurred by teams of students using VMT.

The cognitive subject for CSCL is the interacting small group, not the individual student. The group is not defined by physical bodies or minds of several students, but by the semantic web of sequential linguistic utterances responding to each other and referencing innumerable diverse objects, such as math concepts, symbols, drawings, textual guidance, challenge statements, previously adopted group practices, past actions, future aims.

Group cognition is itself an Anthropocene conceptualization. Sciences and theories of the Anthropocene no longer support notions of independent organisms in environments, such as methodological individualism (Gibson, 1979; Winograd & Flores, 1986). They conceptualize agents as defined by intricate links, interactions and interdependencies. They focus on "complex nonlinear couplings between processes that compose and sustain entwined but nonadditive subsystems as a partially cohering systemic whole... self-forming, boundary maintaining, contingent, dynamic, and stable under some conditions but not others... not reducible to the sum of its parts, but achieve finite systemic coherence in the face of perturbations within parameters that are themselves responsive to dynamic systemic processes." (Haraway, p.36)

Analyses of group cognition do not consider the isolated thinker, but look at interactions among multiple agents embedded in rich worlds, especially socio-technical systems. They unfold over time and are subject to the ambiguities of interpreting meanings in shifting historical contexts. The analysis of group

cognition is a multidisciplinary undertaking; it often involves forms of conversation analysis, statistical analysis, educational psychology, semantics, video analysis, communication theory and software design.

Theoretical Investigations (Stahl, 2021b) brings together two dozen papers on various aspects of its philosophy of computer-supported collaborative learning. Starting with a meso-level analysis of software design that looks beyond a single app to its whole technological, digital infrastructure, the book goes on to consider technology in terms of its interaction with and adoption by students. This begins to shift CSCL to the kind of science appropriate to the Anthropocene, where minds and technologies increasingly work together. Other papers reprinted from the *International Journal of CSCL* in the first half of the volume consider semantic, visual, sequential, temporal and interactional aspects. Then, a pair of studies reflects on transforming national educational systems in Hong Kong and Singapore to feature collaborative learning.

The second half of the book presents microanalyses of VMT interaction data from small groups learning mathematics. It includes a wealth of examples of specific aspects of how group cognition unfolds. This includes detailed illustrations of groups constituting themselves as understanding each other intersubjectively, negotiating shared meaning, building group knowledge, solving problems, adopting practices, crafting knowledge objects, refining terminology and learning mathematics. The analyses reflect the situated nature of such group cognition within shared worlds of embodied and virtual existence – structured and defined by the ongoing interaction. Both successes and limitations of learning are showcased and evaluated.

These recent investigations of VMT data explicate core concepts of group cognition, such as: intersubjectivity, knowledge building, shared meaning making, negotiation of meaning, adoption of group practices, cognitive evolution, knowledge objects, referential resources, instrumental genesis and the co-experienced world. They look at how words and digital utterances in excerpts from VMT case studies weave together references to terms, objects and events in the past, present and future to create intersubjective meaning and shared knowledge. Elements of the theory of group cognition emerge from these empirical analyses. This collection of some of the theoretically most important analyses of empirical data from the VMT Project answers key hypotheses that motivated the founding of CSCL and provides an extensive analysis of group cognition. Considered as a whole, the volume points toward a multi-disciplinary science appropriate for the Anthropocene, which considers educational issues within a complex environment of interdependencies (see also Stahl, 2009b).

A Curriculum for the Era of Climate Change

The Virtual Math Teams Project provides an educational model for fostering group cognition of geometry. It developed and tested a dynamic-geometry curriculum for collaborative learning by small groups of teenage students. This can be used as one educational component of mathematical teaching and learning, to be integrated with individual and community learning in diverse educational social settings. The curriculum is designed to scaffold the adoption of group practices for exploring dependencies. The concept of “dependency” seems pivotal to comprehending both Euclidean geometry and Anthropocene science.

The VMT Project pursued a vision of students around the world learning mathematics collaboratively by communicating and exploring problems online within virtual math teams. The Covid Pandemic provoked rushed efforts around the world to provide educational resources for online pods of students in place of shuttered classrooms. Unfortunately, this rarely took advantage of recent research in the learning sciences or in computer-supported collaborative learning like VMT, instead using business software, social media apps and non-collaborative pedagogy.

To suggest how to fill the glaring educational gap during the pandemic, the final version of the curriculum for the VMT Project was made publicly available on the GeoGebra website and as a free e-

book: *Dynamic Geometry Game for Pods* (Stahl, 2020). It includes a sequence of 50 dynamic-geometry challenges at increasing levels of expertise. Each level is demanding enough to benefit from collaboration, as most students would likely become stuck without partners to figure out what was required.

The game provides specific scaffolding for the adoption of the various kinds of group (or individual) practices that facilitate constructing, dragging, discussing, analyzing and adapting effective dependencies in digital geometry – as discussed above. The challenges include standard geometry problems, open-ended tasks where a group has to define the problem and approach as well as evaluate their answer. Some challenges define open themes for inquiry (Dewey, 1938/1991; Papert, 1980). There are also suggestions of math domains to explore – e.g., sequences of transformations or taxicab geometry (Krause, 1986). These provide various ways to investigate mathematical dependencies.

For students who do not have access to VMT or working relations with appropriate pod-mates, options are outlined for individual study, for home schooling and for online pick-up teams. In addition, an associated article delineates a proposal for blended learning (Stahl, 2021a). It proposes integrating individual, small-group and classroom activities around the game challenges.

The *Game for Pods* and the VMT Project leading up to it provide a concrete, detailed, tested example of an interactive learning approach to fostering an understanding of dependencies in dynamic geometry. The underlying research involved a multidisciplinary consideration of interrelations among various cognitive units, technological media, mathematical systems, semantic structures, interpersonal interactions and social practices. This can provide a model for learning and teaching mathematics in the Anthropocene. The new epoch presents multiple challenges to mathematics education. As we have already seen with the impact of the Pandemic on schooling and the influence of climate denial on public acceptance of science, the need for and the urgency of appropriate innovations are rising rapidly.

The VMT research project was a design-based research approach to exploring CSCL in realistic educational settings. Its research questions included:

- Can mathematics be learned collaboratively?
- Can CSCL technology, curriculum, pedagogy be developed to support online collaborative learning of school mathematics?
- Specifically, can CSCL technology, curriculum, pedagogy be developed to support online collaborative learning of digital geometry?
- Can the understanding of dependency be effectively taken as a core concept for mastering digital geometry?
- Can analysis of data from case studies show how the collaborative learning of mathematics and of dependency relations take place?
- Can this analysis help to refine a theory of collaborative knowledge building or group cognition?

These questions were each largely answered in the affirmative by the VMT Project. Its analyses and conclusions are presented in the series of books discussed above reporting on various stages of the Project. Thousands of published pages of empirical analysis of VMT case studies have documented in detail how CSCL can support group cognition. These analyses were largely conducted collaboratively during years of weekly data sessions by a group of math educators, CSCL researchers and social scientists at the Math Forum. Several international CSCL researchers also participated. In particular, a longitudinal study over eight sessions shows how a small group gradually developed shared understandings of dependency.

Now it is time to see if CSCL can be effectively used to help prepare students with the analytic skills necessary for understanding the world they face – the era of climate change, the age of the Anthropocene. Accordingly, this paper suggests the following hypotheses for future research in interactive and intelligent learning:

- Understanding the current world requires an ability to deal with multidisciplinary complex systems, in which natural and social phenomena interact on many scales.
- A key concept for developing appropriate understandings in contemporary physical, social and environmental sciences is that of “dependency” or “interdependency.”
- Digital geometry can provide a good model or foundation for understanding, designing and manipulating dependencies.
- The concept of dependency in digital geometry can be adapted to help understand dependencies in the multidisciplinary sciences of the Anthropocene.
- Interactive and intelligent technology, curriculum and pedagogy can be developed to support online collaborative learning of many foundational relationships in contemporary science. The VMT Project can be taken as a model for the design-based research to accomplish this.

CSCL has the potential to provide unique and effective approaches to the challenge of preparing students for knowledgeable life in the Anthropocene. With its dual focus on interactive (collaborative) learning and on intelligent (computer) support, CSCL unites social and technological educational design concerns. Using a design-based research approach, it can develop pedagogical units that have been iteratively tested in realistic educational conditions. It can thereby become a multi-faceted science of learning appropriate to the intricate nature of the Anthropocene. It should apply this potential to prepare students to understand the dependency relationships definitive of our new age of climate change. This should become a focus of future exploration and research in interactive and intelligent learning.

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