A Multi-Dimensional Coding Scheme for Mathematical Collaboration

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This coding scheme in Table 1 was developed in the summer of 2004 as part of the Virtual Math Teams (VMT) Project. Stahl was the PI and Strijbos was a visiting researcher. The scheme was tested by Steve Weimar (Director of the Math Forum) and three PhD students who were Research Assistants on the VMT Project: Johann Sarmiento, Murat Cakir and Ramon Toledo.

The results of this scheme were reported in (Stahl, 2009), especially (Strijbos, 2009; Zemel, Xhafa & Çakir, 2009).

Table 1. Math collaboration dimensions coding scheme (version 15)_8-6-04

Dimension	Subcategory	Code	Description	Examples [Any text between brackets refers to a previous utterance as potentially expressed by another group member to clarify the example]
Conversation				
	State	S	An utterance in which a statement of any kind is made that is NOT a response to a previous utterance of any kind NOR are they as specific as an offer, request, regulate or repair typing.	"I think I got it" "I don't have enough paper"
	Offer	O	An utterance in which new conversational content that has not been discussed before is introduced in the conversation and this content is focused on the problem solving process.	"I think we should focus on the altitude"
	Request	Rq	An utterance in which a request is made to another group member of any kind. This can be a request for explanation, information, confirmation, disconfirmation. Usually it includes a question mark. If an utterance is framed as a question, but a more specific responding conversational category applies to the content – often the content is a critique or regulate – the utterance is NOT coded as a request, but as critique or regulate. An utterance that consists only of a question mark is still coded as a 'request' (? is a chat convention).	"Why do we need the height?" "So how is it goin?" "if it's equilateral it's a 45-45-90 triangle?" "Is it not?" "[Chat convention]?"
	Regulate	Rg	An utterance that is focused on regulating the process by expressing information about an individual activity that is being or is going to be performed (e.g. creating a drawing, work with pencil paper), time and/or turn-taking. These can also be utterances that refer to actions that have to be taken according to the script by the participants or utterances that aim to get the attention of other participants. Including any utterance that confirms or disconfirms activity that has been – or is to be peformed – by any group member.	"*scribbeling*", "searching google", "let me draw a pic", "wait a minute", "I got left behind", "IM it [the picture] to Powwow chief", "the guy said something about a link ro sending", "but look", "[You drew the triangle] No, I didn't" "Is there anyone out there" "don't desert me when I need you"
	Repair typing	Rt	An utterance in which a typing error in a previous utterance is corrected.	"[We need to use proportions] *proportions*" "[I got 214.708] 124.708"
	Respond	Rp	An utterance that is a response to an utterance by another group member, but it is NOT as specific as the 'responding' conversational acts listed below that respond to utterances that contain content regarding the problem solving process.	"I have no idea" "sorry" "it's ok"

	Follow	F	An utterance in response to an utterance by another group member, it specifically aims to indicate that one is confirming, disconfirming, giving approval, signaling that one is paying attention to the problem solving – without explicitly agreeing or disagreeing – and/or indicating that one is following/not following someone's idea or problem solving activity and/or following/not following an utterance that contained a (dis)agreement about the content of the problem solving. If the utterance that they are responding to does NOT contain content regarding the problem solving process, it is coded as a respond.	"ok" "right" "I see" "I'm not following" "I understand" "We could do that"
	Elaborate	El	An utterance that clearly builds on a previous utterance by the same person and the content is focused on the problem solving. The utterances may be separated by utterance(s) from another group member. The utterances add new content to the conversation building on the content in the utterance that preceded – this can be in the form of a 'reasoning' such as 'If X is A then Y is B" and "This is value 4, so X this will equal Y", "Before we can compute X we must know Y". It can also be in the form of subsequent modifications to a mathematical expression. Compared to an extend, an elaborate adds something new to the conversation, whereas an extension is part of split utterance. In addition, an elaboration CAN receive codes in other dimensions.	"[equilateral means all sides are equal] therefore all angles are equal too" "[oooooh we can squareroot both sides I think (offer)] 3<2n + 3n + 2 (elaborate)" followed by "3<5n +2 (elaborate)"
	Extend	Et	An utterance that clearly extends a previous utterance by the same person. The utterances may be separated by utterance(s) from another group member, but is part of an utterance that has been split into smaller utterances. An extend is a part (fragment or clause) of a compound utterance – but it does NOT receive a conversational code of itself. In the case of an extend the conversational code is determined by looking at the whole compound utterance – but the code is assigned to the first part of the compound utterance. Parts of a compound utterance can NOT be coded in any of the other dimensions. If this is the case, it is likely that this part of the compound utterance is an elaborate instead of an extend.	See example 5 in section on extend: "and I know (offer)" "that it is (extend)" "the sides (extend)" "are between (extend)" "21 (extend)" "an 21.5 (extend)"
	Setup	Se	An utterance that is similar to an extend but instead of following a previous utterance it precedes the utterance with which it forms a compound utterance. The same rules stated for extend apply for a setup.	"for the 9 [you did it wrong]"
	Agree	A	An utterance in which someone explicitly agrees with the content of a previous utterance that contains some content regarding the problem solving process, usually followed by additional information and sometimes a supportive remark. 'Yes' as a response to an explain is coded as an 'agree'.	"Yes, but …" "I concur" "Yeah, that's true"
	Disagree	D	An utterance in which someone explicitly disagrees with the content of a previous utterance that contains some content regarding the problem solving process and/or any utterance that starts with – or only contains – 'No' (but an utterance does not have to contain a no to be assigned this code). Often the use of 'no' is followed by additional information and sometimes a refuting remark. In the case of a double negation the utterance is NOT coded as 'disagree', but either as follow or agree depending on how explicit the content of the utterance is.	"No, because …" "You did it wrong" "[the 12 triangle is 124.68 cubits] hey its 124.708"
	Critique	C	An utterance in which a critique is provided to an utterance by another group member that is focused on the content – and/or process – of problem solving. In contrast to disagree, this type of utterance questions the content of the preceding utterance or provides an alternative without expressing any definite judgement.	"You sure about that, cause ..." "I don't think so" "I don't think there is a formula, though" "okay, sorry, you should have said two"

				"are you allowed to that"
	Explain	E	An utterance in which an explanation is provided to any utterance by another member demanding an explanation that focuses on the content of the problem solving. In most cases an 'explain' is preceded by a 'request' and the preceding utterance contains content regarding the problem solving process. An explain can be a reply to a critique that is phrased as a question.	"[Why use Pythagorean theorem?] cause both halves are right triangles" "[can sum 1 explain 4 me?? PLEASE?] we just multiplied them out"
	No Code	Nc	An utterance that can NOT be assigned any conversational codes that are listed above NOR any other code in any of the other dimensions. If the Internet connection (or software) of the facilitator has broken down, lines that seem to be responding but it cannot be judged for a conversational code are also assigned 'No code' and no threading is assigned either. A no code is also assigned to entry and exit lines, and scripted facilitation.	"o" "or something" "[my iq is in the negatives] [this is off topic] just a little"
Social ref.			***Description***	***Examples***
	Identity self	Is	An utterance that reflects one's own abilities vis-a-vie the problem at hand. These could be about the strengths or weakness of the self and are often made in the context of some expected norm.	"I have an idea" "I prevail" "My IQ is now like in the negatives" "Now I'm up to speed"
	Identity other	Io	An utterance that reflects another group member's abilities vis-a-vie the problem at hand. These could be about the strengths or weakness of the other (in relation to the self) and are often made in the context of some expected norm.	"You are really lost with this problem" "[Who are you referring to] me or X"
	Interest	I	An utterance with a reference to the problem at hand and it demonstrates investment of effort in working on the problem and/or a general expression not specifically focused on the problem at hand that reflects general interest in math. Questions asking for interest are NOT coded as interest; only the actual expression of interest or an answer asking for interest is coded as interest.	"Interesting problem" "this is easy" "I like doing geometry" "this is kinda cool"
	Risk-taking (Expressing uncertainty)	Ri	An utterance in which uncertainty or a lack of knowledge is expressed without concern for being wrong or losing face – this includes utterances that express that a group member (self or other) has realized being wrong or having made a mistake. The difference with 'identity self' is that the utterance is not focused on their (lack of) *ability* to solve the problem.	"[Is there a formula for 60/60/60?] I have no idea" "[the square root of that is 7.794] ooooopppss …"
	Resource	Rs	An utterance in which one is drawing on other sources (e.g., family members, friends, the web, related math principles) to help solve the current problem. This includes utterances in which a link to website – other than a geometric drawing – is posted by a group member.	"My dad is doing the problem, but he does want to give me any hints" http://somewehere.on.the/web "search google"
	Norms	N	An utterance that refers to general expectations about how to interact such as greeting behavior and netiquette.	"SHOUTING is not always appropriate"
	Home	H	An utterance that refers to the [constraints and advantages of the] home environment.	"My mother is calling me"
	School	S	An utterance that refers to the [constraints and advantages of the] school environment. This includes references to a (school) library.	"We have not worked on the problem in class yet"
	Collaboration group	Cg	Any utterance with a reference to the group (i.e. 'we', 'all group members', or 'everybody', 'who'). This refers to the collaboration in a broad sense, for example an activity that has been done, or is assumed to be done or will be done by the group.	"We have made some progress so far" "Let's get going" "Lets not do that"
	Collaboration individual	Ci	Any utterance with a reference to the self or another member (i.e. 'screenname', 'he', 'she', 'I', 'you'). This refers to the collaboration in a broad sense, for example	"I was doing the problem by my self" "Now I'm following"

			an activity that has been done, or is assumed to be done or will be done by the self or another group member.	"I see"
	Sustain social climate	Ss	An utterance in which an expression is made that reflects on - or aims - to sustain the social climate; or an utterance that is a response to such an utterance.	"lol", "jk", "np", "sorry", "haha", "not funny", "*goes off to sulk*", " You have an interesting profile", "thanks", "I'm doing a spoof", "Super", "it's ok", "got lost for a second", "good", "good idea"
	Greet	Gr	An utterance in which a group member greets the other group members – including a greeting by the facilitator. This in includes saying goodbye.	"What's up", "Hi", "bye", "c ya later"
Problem S.			***Description***	***Examples***
	Orientation	O	An utterance in which the original problem – or part of the original problem is restated, parsed in sub problems, reinterpreted or analyzed – without formulating a particular goal or strategy. Referential words can signal the code. In one of the examples on the right, the word 'it' refers to the problem statement.	"We are trying to find a triangle whose area is the sum of the areas of those two triangles" "It never said what the order of the lengths are"
	Strategy *(Long term strategy)*	S	An utterance in which a specific problem solving strategy is directly suggested or proposed to be followed through by the group. Including representing the problem or a path to follow. Including mentioning a path to a sub goal and/or a strategy that has been used or was applied.	"Let's calculate the area of the triangles with know side length of 9 and 12" "draw the altitude" "you have to make it like a parallelogram"
	Tactic *(Short term strategy)*	T	An utterance in which a partial component – tactic – of a specific problem solving strategy is suggested to be followed through or referred to with or without revealing the whole strategy and/or any preceding utterances in which the other parts were already suggested. This includes implicit partial strategies to be followed through by the group, including utterances that intend to find out whether a group member should reveal the whole strategy.	"do we know what these equations all equal" "do they by chance equal zero" "[what is the order of the lengths] but how are we going to find the correct one?" "and you can plug it into the quadratic formula" "[what does that do] we could find a range"
	Perform	P	An utterance in which a chosen strategy is – and associated mathematical actions are – performed or executed. This includes modification of an expression or equation to a different form. If an utterance contains a procedure that is performed – but has already been stated in the conversation – it is NOT coded as a 'perform' but as a 'Restate'. An activity – or use of an active verb – can signal a 'perform' (e.g. made, found). Whenever a perform is coded a mathematical move has to be assigned.	"20.25 + X = 81" "I am calculating the height" "(n^2 +4+4n)<9<(n^2+5n) is possible" "I made (n^2+5n)+(n^2+4n+4)>9" "9<2n2 + 9n + 4"
	Check *(check in terms of actions; error focused)*	Ch	An utterance in which a problem solving strategy and/or a mathematical operation that was performed by another group member is checked or evaluated (usually preceded by a mathematical operation that has been performed). This includes utterances that either confirm or disconfirm the mathematical operation performed and/or the result, BUT there has to be clear connection in the problem solving threading that the utterance to receive the code 'check' targets that utterance. If there is any doubt, the code is NOT assigned. An utterance is a 'Check' when the check is focused on 'actions' or 'manipulations' performed by another student. Whenever a check is coded a mathematical move has to be assigned.	"You did it wrong" "it's 8.352" "right" "exactly" "where did you get 9<2n2 + 9n + 4" "3<5n + 2 doesn't work"
	Restate	Re	An utterance in which a single problem solving activity or outcome of that activity is restated – including utterances that aim to (re)establish what they agreed upon. When an utterance is coded as a 'Restate' no mathematical move code is given, but a 'problem solving thread' is assigned to link back to the utterance in which the content was stated previously in the conversation.	"We know X is Y cubits" "We agree it is 10.392" "[we just said that (N+2)^2 is n^2 +4 + 4n] we just multiplied them out" "(n+2)^2 is in the form of (a+b)^2"
	Summarize	Su	An utterance in which the content of a previous problem solving sequence or strategy is summarized and presented as a condensed interpretation of what	"[(n+2^2] + [n^2 +4n +4] we just said that (n+2)^2 is n^2+4n+4"

			transpired over a long time period. In a summary the content of multiple utterances is summarized, repeated and/or related in a single utterance. When an utterance is coded as a 'Summary' no mathematical move code is given, but a 'problem solving thread' is assigned to link back to the utterances in which the summarized content was stated previously in the conversation.	
	Reflect *(check in terms of reasoning)*	Rf	An utterance in which is reflected on the chosen strategy or previous utterances in the conversation and/or outcomes of mathematical actions; i.e. surpassing a summary of the problem solving process so far. This includes utterances in which a member attempts to refocus the problem solving – in terms of shifting from a sub strategy to the main strategy of problem solving. Reflect includes utterances in which a member reflects on or checks his/her own or someone else's problem solving input. An utterance in which a check is performed that focuses on 'reasoning' is also coded as 'Reflect'.	"I wonder if we really needed the areas" "Okay guys, I don't think there is a formula to find the height" "okay, it's TWO 30/60/90 triangles" "are you allowed to use that "greater than sign" as an equal sign" "are you allowed to do that" "but if it's to great it won't work"
	Result	R	An utterance in which the results of a 'perform' of a solution step is stated. This includes any result from transforming or manipulation a geometric or algebraic expression. A result is the final form and/or outcome of an expression or perform.	"for the 9 triangle it is about 7.79" "for the 12 it's 10.39", "7.794" "it's 8.352" "[(n +2)^2] so I got n^2 + 4n + 4" "well I said earlier that I just used trial and error and factored it out using the number I had picked and I found that it had to be less than 4"
Math move			***Description*** ***A move is defined as the active introduction of information or manipulation of formulas and/or values. In other words, there is something that is actively created and/or added to the problem solving conversation.*** Outcomes are coded as part of the specific math move; if only a number is stated use the previous 10 lines to determine the code.	***Examples***
General				
	Counting	Co	An utterance in which counting is performed.	"One, two, three … five units"
	Numeric computation	Nc	An utterance in which numeric computation is performed or an utterance that contains a numeric approach to an algebraic move (such as permutations or factoring).	"10.39 * 12" "and that times the base (12)" "I say there are six possible orders of length"
	Geometric expression or link to drawing	Geo	An utterance in which the problem and/or the solution is geometrically expressed or approached using geometrical notation and/or in words, including utterances that refer to a drawing made by the students – this includes spatial references to a drawing or a visual object. This includes the active application or manipulation of formula's, values and symbolic expression. In other words, something new is created – which includes utterances in the form of reasoning, giving explanations and critiquing.	"We know that length of AB = 9." "You can see my drawing at http://somewhere.on.the/globe" "[After seeing drawing] okay, it's TWO 30/60/90 triangles"
	Algebraic expression	Alg	An utterance in which the problem and/or solution is algebraically expressed or approached with the use of algebraic notation – such as using variables like 'X' or notation like 'r^2' – and/or in words. Including active application or manipulation of algebraic formula's, values and symbolic expression. In other words, something	"20.25 + X = 81" "then subtract 9 form both sides"

			new is created – which includes utterances in the form of reasoning, giving explanations and critiquing. Including arithmetic in an algebraic sense such as subtracting form both sides (transformation) of an equation.	
Specific	*Import new math info*	I	An utterance that introduces a new mathematical concept, fact or information in the discussion from either individual knowledge or archive – with or without a reference or relevance to the problem at hand – that could be used by the group in the problem solving process but it's NOT linked to the problem solving process. The concept, fact or information in the utterance was not provided in the original problem description. Asking for a formula is NOT considered as a mathematical move ("Is there a formula for a 60/60/60 triangle?"). This includes a critique – phrased as question – that introduces new mathematical information.	"We should multiply the height and the edge length to get the area" "Area of triangle is ½ *b*h", "proportions?" "Pi*r^2" "The Pythagorean theorem can be used on all kinds of triangles"
	Import & apply new math info	Ia	An utterance that introduces a new mathematical concept, fact or information in the discussion imported from either individual knowledge or archive AND it is actively linked to the problem solving process. The concept, fact or information in the utterance was not provided in the original problem description. This includes a critique – phrased as question – that introduces new mathematical information.	"Do you think we could solve for n using the quadratic formula" "Splitting the triangle gives two right triangles, so we can use the Pythagorean Theorem" "like the theorem"
	Similar problem (instantiation)	Sp	An utterance in which the current problem is matched against a similar problem with a known solution or in a know form or theorem.	"I've reduced it to a problem I've solved before" "((N+2)^2 is in the form of (a+b)^2"
	Simplified case	Sc	An utterance in which the current problem is transformed in a simplified version of the problem.	**NO EXAMPLE YET**
	Infer pattern	Ip	An utterance in which a pattern or relationship is identified, described or defined by comparing different triangles or the different computational outputs. This includes the use of randomly picked (guessed) numbers to check for a pattern.	"You think it's the same for all triangles"
	Sub problem	Su	An utterance in which the problem is split in sub problems to be worked out and results combined – or whose possible solution or outcome is prerequisite to other operations that must be performed.	"We need the area before we can compute the height" "then find the area of each triangle" "has anyone come up with an equation or expression to solve for n"
	Test for boundaries	T	An utterance in which different test cases are tried by putting in numbers in the algebraic expression to find boundaries/ extremes: small numbers, with zero, with big numbers, negatives. A test and the way it is performed are both deliberate decisions. This includes the use of randomly picked (guessed) numbers to rule out possibilities.	"We could try to use different values"
	Estimate	E	An utterance in which the answer is estimated [based on a previously stated math move]. A degree of uncertainty is expressed, but the answer is partly supported by some sort of reasoning – or activities – in preceding utterances in the problem solving conversation.	"[It's equilateral so] I think the length is 7"
	Trial and error	Te	An utterance in which the answer is guessed by plugging different values into a formula, but there is no support from any reasoning – or activities – in preceding utterances in the problem solving conversation. This includes the use of randomly picked (guessed) numbers to find a starting point for solving the problem, as well as stating the solution without any reasoning or explanation.	"I guess it is 15" "this is easy, it's 15" "well I said earlier that I just used trial and error and factored it out using the number I had picked and I found that it had to be less than 4"
	Conduct unit analysis	Cu	An utterance in which the units are compared that a group member expects to use in the answer with the units in the given information and where s/he looks for conversion factors involving these units.	"Maybe we should look at the relationship between edge length and area of equilateral triangles"
	Work	Wb	An utterance in which a predicted solution is stated [and usually followed by	"The area of the third triangle equals the other two,

	backwards		actions needed to get there].	so we need to compute their area's first"
	Combinatoric	Cb	An utterance that contains reasoning through permutations and combination, such as factorial expressions.	**NO EXAMPLE YET**
M. accuracy			***Description***	***Examples***
	Accurate	Ma	A math move utterance whose content or action is accurate.	"Pythagorean theorem is for right triangles only"
	Inaccurate	Mi	A math move utterance whose content or action is inaccurate.	"Circumference equals Pi times r squared"
M. progress			***Description***	***Examples***
	Toward solution	Pt	A math move utterance that makes the group progress toward the solution.	Depends on problem solution
	Away from solution	Pa	A math move utterance that makes the group progress away from the solution.	Depends on problem solution
Sys. support			***Description***	
	Entry	E	An automated chat line indicating a person entered the chat	"Anonymous" has entered the chat room
	Exit	Ex	An automated chat line indicating a person left the chat	"JayDoubleU" has left the chat room
	Technical problem	Tp	An utterance in which a technical problem with computer technology is expressed and/ or a solution to a technical problem with computer technology is provided	"My explorer did not open"
	Scripted facilitation	Sf	An utterance posted by the facilitator that is a part of the scripted facilitation guidelines.	"Here's the URL for the problem you will be working on …"
	Unscripted facilitation	Uf	An utterance posted by the facilitator that is NOT a part of the scripted facilitation guidelines,	"We have a new participant who wants to join. Do you mind?"
	Drawing facilitation	Df	An utterance posted by the facilitator that specifically refers to the (process of) posting of a drawing.	"If you have a picture you can send it to …" "To see the picture go <link> somewhere <link/>
	Contact facilitator	Cf	An utterance by a group member directed at the facilitator. This can be a question that is asked to the facilitator or response to a question by the facilitator.	"the dood is back!"
Conversation Thread			***Description***	
	Reply to U_i	U_i	An utterance that is a reply to a previous utterance. Different single utterances can be reply to the same utterance. When assigning the conversational thread code usually no more than 20 preceding lines need to be considered. Extensions are connected to the first part of the extended utterance. An utterance that has a setup is linked to the setup (no forward linking).	Indicate the line number that the utterance directly responds to U21, U 33.
Problem S. Thread			***Description***	
	Connect U_i	U_i	An utterance that connects to an earlier utterance seen from a problem solving perspective. Restatements are linked to the original statement, explanations are linked to the utterance requesting the explanation and elaborations are linked to the statement that they elaborate upon, a critique is linked to the utterance that it critiques and a summary is linked to the statements that are summarized (which is an exception of multiple links).	Indicate the line number that the utterance directly responds to U21, U 33.