RESOURCES FOR CONNECTING LEVELS OF ANALYSIS

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Abstract. CSCL research typically involves processes at the individual, small-group and community units of analysis. However, CSCL analyses generally each focus on only one of these units, even in multi-method approaches. Moreover, there is little data-based analysis of how the three levels are connected, although it is clear that such connections are crucially important to understanding learning in CSCL contexts. This paper proposes that the levels of individual learning, group cognition and community knowledge building may be connected by emergent *interactional resources*, which can mediate between the levels. A theory of the connection of the levels is sketched. Then examples of such connections by interactional resources are presented from logs of several CSCL investigations. Finally, a curriculum for gradually providing math teachers and math students with a complex of resources relevant to dynamic geometry is described as an example of how to support the connection of small-group interaction with individual understanding and with cultural practices in a CSCL adaptation of geometry.

I. THE PROBLEM OF CONNECTING LEVELS

Learning, cognition and knowledge building can be analyzed at multiple units of analysis. For instance, analyses of CSCL are often conducted on one of three levels: individual learning, small-group cognition or community knowledge building. This tri-partite distinction is grounded in the nature of CSCL. With its focus on collaborative learning, CSCL naturally emphasizes providing support for dyads and small groups working together. In practice, CSCL small-group activities are often orchestrated within a physical or virtual classroom context by providing some initial time for individual activities (such as background reading or homework practice) followed by the small-group work and then culminating in whole-class sharing of group findings. Thus, the typical classroom practices tend to create three distinguishable levels of activity. Often, the teacher sees the group work as a warm-up or stimulation and preparation for the whole-class discussion, facilitated directly by the teacher. Conversely, the importance of testing individual performance and valuing individual learning posits the group work as a training ground for the individual participants, who are then assessed on their own, outside of the collaborative context. In both of these ways, group cognition is treated as secondary to either individual or community goals. By contrast, the role of intersubjective learning is foundational in Vygotsky (1930/1978), the seminal theoretical source for CSCL. Regardless of which is taken as primary, the three levels are actualized in CSCL practice, and the matter of their relative roles and connections becomes subsequently problematic (Dillenbourg et al., 1996; Rogoff, 1995; Stahl, 2006).

While these different units, levels, dimensions or planes are intimately intertwined, research efforts generally focus on only one of them, and current analytic methodologies are designed for only one (Stahl, 2013a; Suthers et al., 2013a). Furthermore—and most importantly for this paper—there is little theoretical understanding of how the different levels are connected. To the extent that CSCL researchers discuss the connections among levels, they often rely upon commonsensical notions of socialization and enculturation, popularizations of traditional social science. There are no explicit empirical analyses of the connections, and it is even hard to imagine where one would find data that would lend itself to conducting such analyses (Stahl et al., 2012).

The individual unit of analysis is the traditional default in the learning sciences and in cognitive psychology. It is supported by widespread training of researchers in the methods of education and psychology. In the era of cognitive science, analysis made heavy usage of mental models and representations (Gardner, 1985). With the "turn to practice" (Lave & Wenger, 1991; Schatzki, Knorr Cetina & Savigny, 2001), the focus shifted to communities-of-practice. Group cognition lies in the less-well-charted middle ground (Stahl, 2006). It involves the semantics, syntactics and pragmatics of natural language, gestures, inscriptions, etc. These meaning-making processes involve inputs from individuals, based on their interpretation of the on-going context (Stahl, 2006, esp. Ch. 16). They also take into account the larger social/historical/cultural/linguistic context, which they can reproduce and modify (Stahl, 2013a).

This paper will argue that the connections between the individual, group and community planes take place through the mediation of *interactional resources* (Section II)[1]. To provide specificity and to ground the presentation in empirical data, the paper considers the resources that appear in recorded examples of mathematical work in computer-supported face-to-face and online collaborative-learning scenarios (Section III). Applying this problematic to the learning of mathematics, the paper adopts a discourse-centered view of mathematical understanding as the ability to engage in significant mathematical discussion (Sfard, 2008; Stahl, 2008). Here, "discourse" includes gesture, inscription, drawing, computer representation and symbol, as well as speech and text; these multiple modes are often closely interwoven in effective interaction (Çakir & Stahl, 2012; Çakir, Zemel & Stahl, 2009).

Computer technologies play a central role in mediating the multi-level, intertwined problem-solving, learning and knowledge-building processes that take place in CSCL settings. From a CSCL perspective, innovative technologies should be carefully designed to support this mediation. This involves considering within the socio-technical design process of collaboration environments how to prepare groups, individuals and communities to take advantage of the designed functionality and to promote mathematical thinking at all levels. This paper reports on the design of a curriculum in dynamic geometry to support group cognition, individual learning and community practices in a coordinated way, based on how interactional resources are visibly used in analyzed excerpts of pilot case studies of the use of dynamic-geometry software (Section IV). The curriculum addresses both communication issues—such as effective collaboration practices—and mathematical issues—such as focusing on dependencies among math objects—as well as technological issues of software usage.

II. THE THEORY OF CONNECTING LEVELS

The idea of viewing interactional resources as central to mathematical discourse around dynamic geometry was proposed by Öner (2013). Her paper cited a number of distinctions drawn in the CSCL literature for contrasting social/collaborative/relational resources with content-related resources:

- Text chat versus shared-whiteboard graphics (Çakir, Zemel & Stahl, 2009);
- Building a joint problem space (JPS) versus solving a problem (Roschelle & Teasley, 1995);
- A relational space versus a content space (Barron, 2000);
- Temporal dimensions of the JPS versus diachronic content (Sarmiento & Stahl, 2008);
- Project discourse versus mathematical discourse (Evans et al., 2011);
- Spatio-graphical observation (SG) versus technical reflection (T) (Laborde, 2004).

Öner then generated some data to explore the interaction of the contrasting dimensions by having two people work together face-to-face in front of a shared computer on a particular dynamic-geometry problem whose solution required a mix of spatio-graphical observation and technical reflection involving mathematical theory—a mix of SG and T resources, to use the distinction she adopted from Laborde.

[1] While the problem of connecting levels has recently been raised within the CSCL community—e.g., in the workshop at ICLS 2012 (Stahl et al., 2012) and in an editorial in ijCSCL (Stahl, 2012b)—this paper goes beyond those efforts to articulate a central role for interactional resources and to review supporting analysis of empirical CSCL data.

Inspired by Öner's experiment, Stahl (2013b) presented the same dynamic-geometry problem to two groups of people collaborating online in a CSCL system. We will review the sorts of resources that occur in the data generated in Öner's experiment and in Stahl's after first considering the idea of resources as connections between levels.

In Figure 1, we see highway ramps or bridges used as resources for connecting road levels or landmasses. While we are interested in linguistic interactional resources in this paper, it may be helpful to first consider the more intuitive case of a physical resource. A ramp or bridge often creates a possibility that did not otherwise exist for going from one level to another at a given point. To go from a local road to a limited-access superhighway, one must first find an available on-ramp. To cross a river from one side to the other, one may need a bridge. This is the individual driver's view. From a different vantage point—the perspective of the resource itself—the creation of a ramp or the building of a bridge "affords" connecting the levels (Bonderup Dohn, 2009).

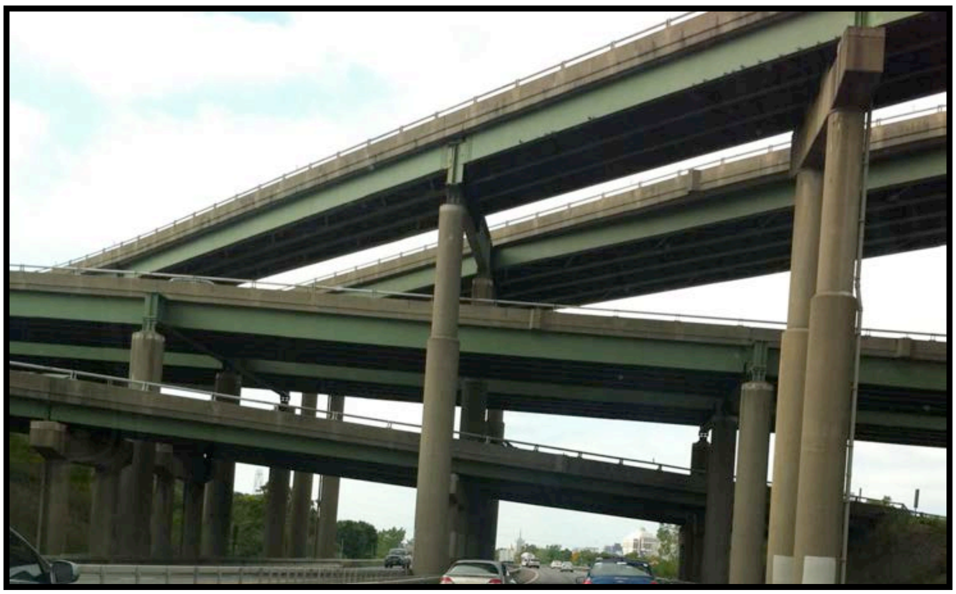

Figure 1. Connecting ramps for the I-90 bridge across the Hudson River. Photo: G. Stahl, Albany, NY, 2012.

By "affords," we do not simply mean that the connecting is a happy characteristic or accidental attribute of the bridge, but that the bridge, by its very nature and design, "opens up" a connection, which connects the banks of the river it spans. This view of artifacts was largely introduced in the philosophy of Heidegger; it later became influential in CSCL through various theories derived from his approach. In his early work, Heidegger (1927/1996) analyzed how the meaning of a tool was determined by the utility of the tool to the human user, within the network of meaning associated with that person's life and world. In his later writings, Heidegger (1935/2003) shifted perspective to focus on things like bridges (see Figure 2), paintings, sculptures, pitchers and temples in terms of how they themselves opened up new worlds, in which people could then dwell. In considering the intersubjective world in which collaboration takes place on multiple connected levels, we might say that the work of resources like bridges is to contribute the spanning of shores within the way that the world through which we travel together is opened up as a shared landscape of resources for discourse and action.

This transformation of perspective away from a human-centric or individual-mind-centered approach became characteristic for pioneering theories in the second half of the 20th Century, including various theories of situated and distributed cognition. It is a shift away from the individualistic, psychological view to a concern with how language, tools and other resources of our social life work. It is a post-cognitive move since it rejects the central role of mental models, representations and computations. The things themselves have effective affordances; it is not just a matter of how humans manipulate models in which the things are re-presented to the mind.

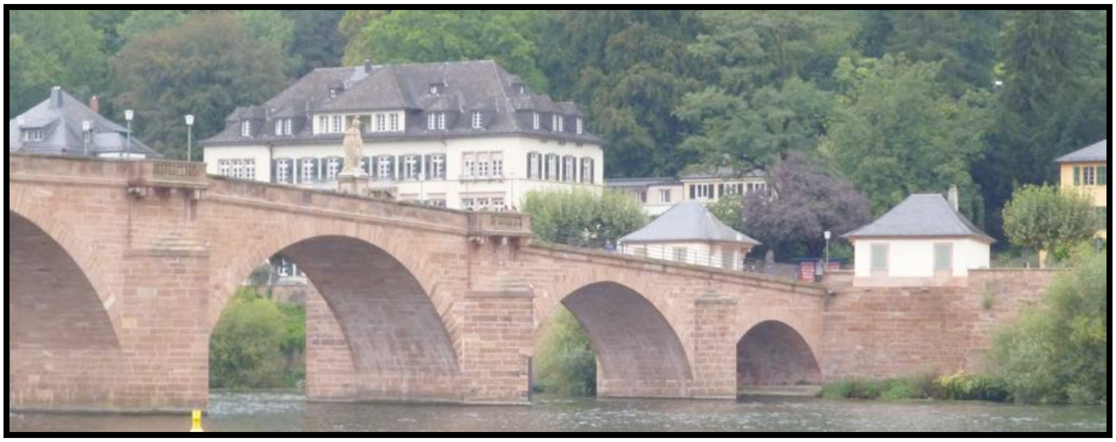

Figure 2. The bridge across the Neckar River, discussed by Heidegger (1950/1967). Photo: G. Stahl, Heidelberg, Germany, 2012.

The analytic focus and even the locus of agency are shifted from the individual mind to tools, artifacts, instruments, discourse and inscriptions. In phenomenology, Husserl (1929/1960) called for a return to "the things themselves" (*die Sache selbst*) and Heidegger (1950/1967) analyzed "the thing" (*das Ding*) separate from our representation of it. In ethnomethodology, Garfinkel and Sacks (1970) followed Wittgenstein's (1953) linguistic turn to focus on the language games of words and the use of conversational resources (Koschmann, Stahl & Zemel, 2004). In distributed cognition, Hutchins (1996) analyzed the encapsulation of historical cognition in cultural artifacts. In actor-network theory, Latour (1990; 1992; 2007) uncovered the agency of various kinds of objects in how they move across levels in enacting social transformations. Recently, Rabardel (Rabardel & Beguin, 2005; Rabardel & Bourmaud, 2003) analyzed the genesis of socio-technical instruments, which only gradually become useful as they are adapted and enacted in practice.

Our proposal in this paper to use the term "resources" is intended to carry forward into the 21st Century these groundbreaking approaches into the study of how the various planes of human interaction are connected. The phrase "interactional resource" is proposed as an inclusive expression for all the kinds of things that can be brought into discourse. Vygotsky (1930/1978) used the term "artifact" to refer to both tools and language as mediators of human cognition; we prefer to use the broader term "resource" as it has more recently been used in sociocultural analysis (Ackerman et al., 2008; Arvaja, 2012; Cekaite, 2009; Furberg, Kluge & Ludvigsen, 2013; Karlsson, 2010; Linell, 2001; Medina, Suthers & Vatrapu, 2009; Suchman, 1987) for entities referenced in discourse. Like artifacts, resources are often identifiable units of the physical world (including speech and gesture) that are involved in meaning-making practices—spanning the classical mind/body divide.

A central research issue for CSCL is how collaborative knowledge building takes place. The main problem seems to be to understand the role of both individual cognition and societal institutions in the small-group meaning-making processes. At ICLS 2000, Stahl (2000) presented a diagram that was intended more to raise this question than to answer it. This may have been the first explicitly theoretical paper in that conference series. It has always been a popular publication—probably because readers took the diagram as a model of what takes place and then tried to design CSCL systems that supported each element in the model. Figure 3 shows an updated version of that diagram. Note how "cultural artifact" serves to connect the three planes of meaning-making processes. The diagram was based on an eclectic combination of major theories influential in CSCL. Some of those approaches—collaborative knowledge building, group and personal perspectives, mediation by artifacts and interaction analysis—were further described in Stahl's (2002a) paper at CSCL 2002, a conference that explicitly sought to increase the theoretical discourse in the CSCL community (Stahl, 2002b). Work on theory then led to a need for new case studies to explore collaborative knowledge building.

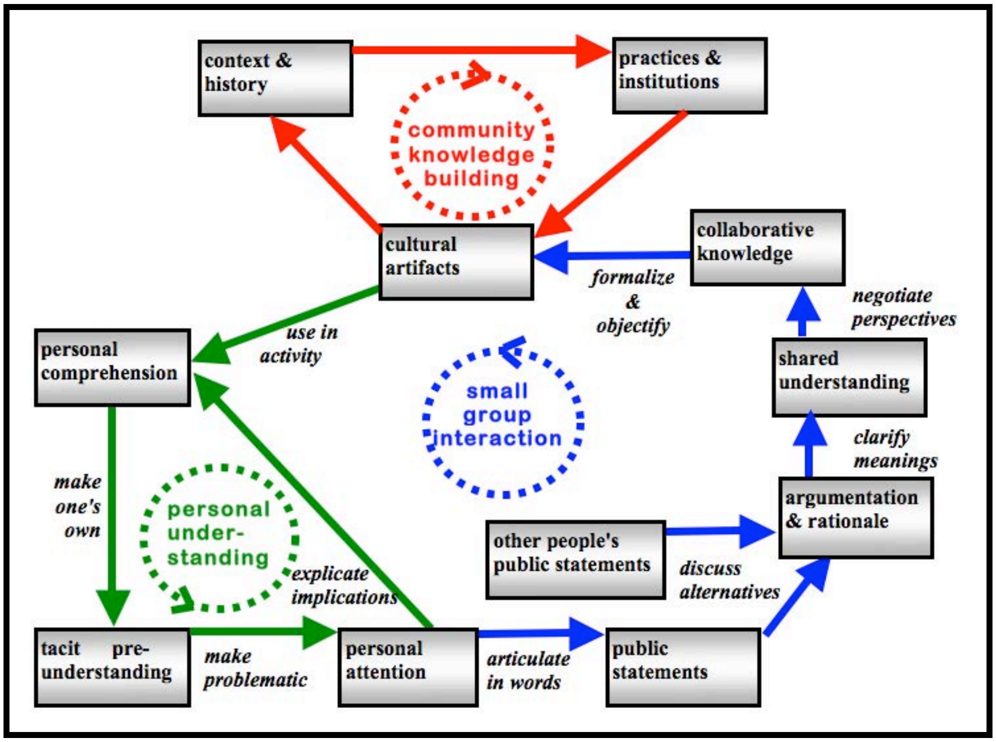

Figure 3. A model of collaborative knowledge building. Adapted from (Stahl, 2000).

In recent years, the Virtual Math Teams (VMT) Project (Stahl, 2009) has conducted case studies of small-group problem solving. In doing so, it has tried to focus exclusively on the small-group unit of analysis. This approach is based upon three observations:

1. That most CSCL studies in the past have focused either on the individual (cognitive) plane or on the community (practices) plane. For instance, they code utterances of individuals (Strijbos & Stahl, 2007) and reduce interaction to contributions of individuals or else they view interaction as participation in community processes and institutions. In terms of strict methodology, the small-group unit of analysis has been under-researched in CSCL.
2. That the small-group unit is fundamental to learning. As Vygotsky (1930/1978) said, one learns most human skills in social interaction first, only then being able to develop those skills individually. Furthermore, processes of meaning making and knowledge building are more visible in small-group interaction than in individual cogitation, making them easier to study.
3. That the multiple levels are so complexly intertwined that it is hard to imagine studying them all together without first understanding much of what takes place at each level, temporarily taken on its own.[2]

Based on the studies of virtual math teams (Stahl, 2009), the connections of resources from the small-group plane to the individual and community planes was then pictured as shown in Figure 4. The interaction of students in a typical CSCL setting is most appropriately analyzed at the small-group unit of analysis as a sequential progression (Schegloff, 2007; Suthers, 2007). The collaborative knowledge-building activity that takes place there is mediated by a variety of interactional resources (indicated in the figure by arrows).

[2] Different scientific approaches have accordingly focused on different units of analysis (Stahl, 2013a): cognitive science on the individual; ethnomethodology on the small-group interaction; quantitative social science on the community. These incommensurate methodological commitments rendered it almost impossible to theorize the connections between the levels (but see Suthers et al., 2013b).

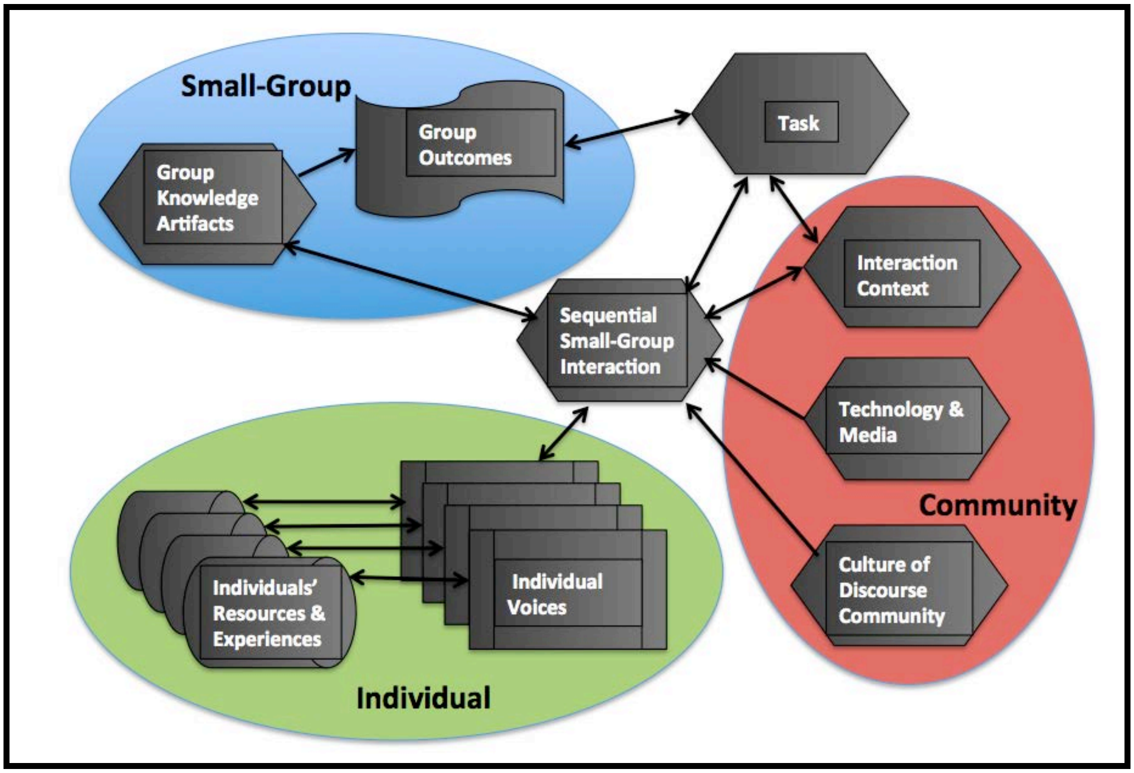

Figure 4. A diagram of sources of interactional resources and the connections they mediate. From (Stahl, 2013a).

Figures 3 and 4 are not meant to reify different levels or processes as necessarily having some kind of independent existence outside of our analyses, but to suggest some constraints between different phenomena to hypothesize and flows of influence to measure. The distinctions and resource-mediated connections represented by boxes and arrows in the chart are intended to operationalize an infinitely complex and subtle matter of collaborative knowledge building for purposes of concrete analytic work by CSCL researchers.

Some researchers, such as ethnomethodologists, argue against distinguishing levels. However, the view of levels of analysis in this paper may actually be consistent with ethnomethodology. For instance, in their introduction to ethnomethodologically inspired Conversation Analysis (CA), Goodwin and Heritage (1990, p. 283)—two of the writers most explicit about the theory underlying ethnomethodological studies—open with the following claim. "Social interaction is the primordial means through which the business of the social world is transacted, the identities of its participants are affirmed or denied, and its cultures are transmitted, renewed and modified." This statement implicitly distinguishes social interaction, individual identities and community cultures—asserting the tight connections between them, and a priority to the first of these. Social interaction typically takes place in dyads and small groups, so interaction analysis can be considered to be conducted at the small-group unit of analysis. Although CA, as a branch of sociology, refers to community-level social practices and linguistic resources, its case-study analyses usually involve interactions in dyads or small groups. CSCL researchers focus on small groups, but also want to analyze the levels of the individual and of the culture as such—e.g., the individual identities and learning changes, or the cultural practices and institutional forces. In this paper, we propose that interactional resources are centrally involved in mediating these connections within CSCL settings. The resources that CSCL must analyze are different from those of interest to CA, and the approach to interaction analysis is different. CA studies the interactional structure of informal conversation (e.g., adjacency pair typology and turn-taking rules) rather than the building of knowledge in online chat of school mathematics, has a different conception of resources for interaction as community practices ("member methods"), and is interested in the co-construction of social order rather than of domain knowledge. However, analysis of the ways in which interactional resources bridge from group phenomena to individual and community phenomena should be of similar concern to CA and CSCL.

CSCL sequential small-group discourse brings in—through indexical references, as described below—resources from the individual, small-group and community planes and involves them in procedures of shared meaning making. This interaction requires co-attention to the resources and thereby shares them among the participants. The process results in generating new or modified resources, which may then be retained at the various planes. The resources that are brought in and those that are modified or generated often take the form of designed physical artifacts and adopted elements of language. In other words, "small groups are the engines of knowledge building. The knowing that groups build up in manifold forms is what becomes internalized by their members as individual learning and externalized in their communities as certifiable knowledge" (Stahl, 2006, p. 16).

The question of how the local interactional resources that mediate sequential small-group interaction are related to large-scale socio-cultural context as well as to individual learning is an empirical question in each case. There are likely many ways these connections across levels take place and it may well be that they often involve mechanisms that are not apparent to participants. In this paper, we explore one way that such connections can occur: through mediation by interactional resources.

Sawyer (2005, p. 210f) argues that we can conceptualize the general level-bridging processes as forms of "collaborative emergence"—involving both ephemeral emergents and stable emergents: "During conversational encounters, interactional frames emerge, and these are collective social facts that can be characterized independently of individuals' interpretations of them. Once a frame has emerged, it constrains the possibilities for action." Sawyer's theory of ephemeral and stable emergents suggests a relationship among different kinds of resources along the lines pictured in Figure 5.

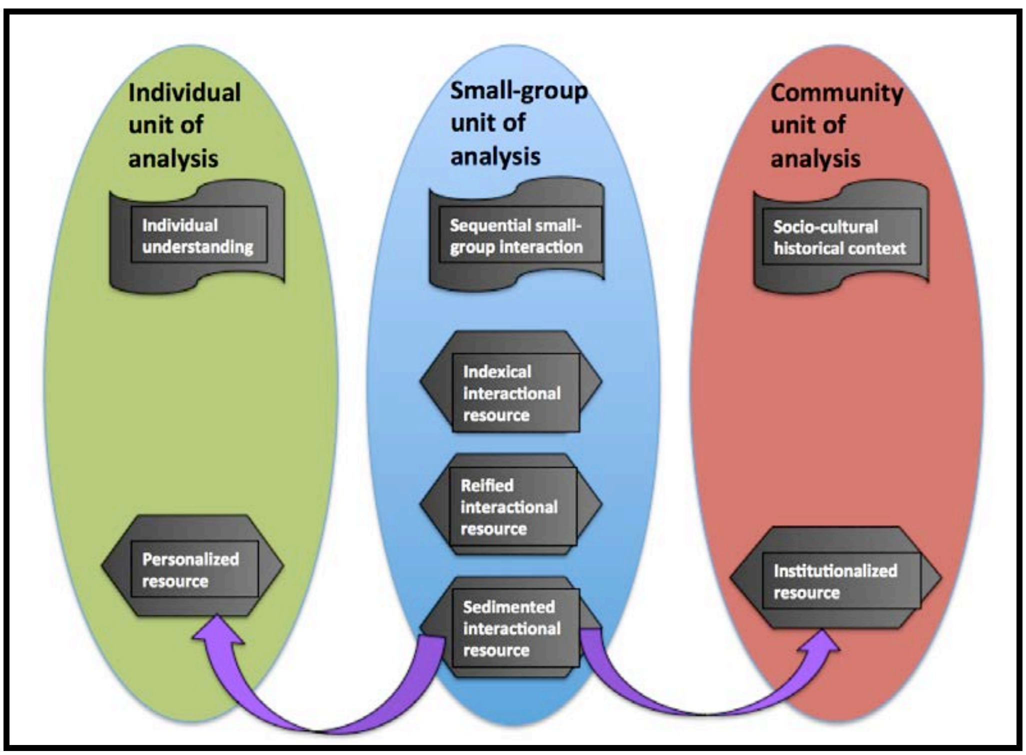

Figure 5. A diagram of emergent interactional resources bridging levels of analysis.

While Sawyer's analysis addresses a broad "sociology of social emergence," we have confined and adapted it to the concerns of CSCL. What is most relevant in his theory is the view of emergence arising out of the subtle complexities of language usage and small-group interaction—rather than from the law of large numbers, the interaction of simple rules or the chaotic behavior of non-linear relationships. The vast variety of interactional emergents form an intermediate level of analysis between the level of individuals and the level of community structures, providing a dynamic and processual understanding of social structures and infrastructures.

In this diagram, interaction is taken as being based on an "indexical ground of deictic reference" (Hanks, 1992). This means that the "common ground" (Clark & Brennan, 1991)—which forms a foundation for mutual understanding of what each other says in conversation—consists of a shared system of *indexical-reference resources*, such as deictic pronouns, which are used to point to unstated topics or resources. Interactional resources, which can be indexically referenced in the interaction, may undergo a process like Rabardel's (Rabardel & Beguin, 2005; Rabardel & Bourmaud, 2003) instrumental genesis: they may initially be constituted as an object of repeated discussion—an interaction frame (Goffman, 1974)—which we might call a *reified resource*, something capable of being picked out as having at least an "ephemeral-emergent" existence. Over time, continued usage can result in a *sedimented resource*, something whose existence has settled into a longer-term "stable-emergent" form. A sedimented resource is then susceptible to being taken up by a larger community as an *institutionalized resource* within a structured network of such resources, as in Latour's social-actor networks (Latour, 2007), contributing to the socio-cultural-historical context surrounding the interaction: not only referencing it, but partially reproducing it. On the other hand, interactional resources at various degrees of reification can also be taken up into the individual understanding of community members as *personalized resources*, integrated more or less into the intra-personal perspective of one or more group members. The personalization of previously inter-personal resources by individuals renders them into resources that can be referenced in activities of individual understanding—corresponding to processes of micro-genesis in Vygotskian internalization.

The various components of this view of interactional resources have been hinted at in previous theoretical contributions grounded in empirical examples. The term "reification" goes back to Hegel's philosophy of mediation (Hegel, 1807/1967). It has been applied to the formation of mathematical concepts by Sfard (Sfard, 2000; 2008; Sfard & Linchevski, 1994). Husserl (1936/1989) argued that the ideas of the early geometers became "sedimented" in the cultural heritage of the field of geometry. Livingston (1999) differentiated discovering a mathematical proof from presenting a proof; a transformational process takes place, in which the byways of exploration and possibly even the key insights are suppressed in favor of conforming to the institutionalized template of formal deductive reasoning. Netz (1999) (see also the review by Latour, 2008) documented the important role of a controlled (restricted) vocabulary to the development, dissemination and learning of geometry in ancient Greece. Analogously, Lemke (1993) argued that learning the vocabulary of a scientific domain such as school physics is inseparable from learning the science. Vygotsky (1930/1978, esp. pp. 56f) noted that the micro-genetic processes of personalizing a group practice into part of one's individual understanding—which he conceptually collected under the title "internalization"—are lengthy, complex, non-transparent and little understood. These seminal writings name the processes of reification, sedimentation, institutionalization and personalization of interactional resources; their empirical investigation remains as a major challenge for future CSCL research.

Analyses of CSCL research show that few studies have bridged multiple levels of analysis (Arnseth & Ludvigsen, 2006; Jeong & Hmelo-Silver, 2010). Yet, the desired CSCL research agenda (Krange & Ludvigsen, 2008; Stahl, Koschmann & Suthers, 2006; Suthers, 2006) calls for a study of representational artifacts and other resources that traverse between individual, small-group and community processes to mediate meaning making. The preceding sketch of emergent forms of evolving resources could be taken as a refinement of the research agenda for the field of CSCL: a hypothesis about how levels in the analysis of learning are connected; and an agenda for exploration. The following sections begin that undertaking. They present examples of interactional resources in small-group discussions of dynamic-mathematics problem solving and then describe some resources that are being prepared to help students engage in collaborative dynamic-mathematics problem solving.

III. The Analysis of Connecting Levels

A. *Table of Rockets*

An early attempt within CSCL to present an extended argument for the centrality of the small-group unit of analysis appeared in (Stahl, 2006), with a preliminary version in (Stahl, 2004). These lengthy discussions were grounded in a half-minute interaction among four students working with a computer simulation of model rockets. The excerpt involved the students coming to understand how to interpret a textual resource: a table of rocket components arranged to facilitate comparisons among differently configured rockets. At first, none of the students could see the designed affordance of the table, but after the half minute, they could all see the shared artifact as a resource for their scientific discourse. The interaction analysis of this excerpt showed how aspects of the table artifact were brought in as resources for the group discourse; as were shared and repeated words like "*same*" and "*different*." The words of the dominant student, Chuck, were reused in the interaction by others in order to orient Chuck to a new, shared understanding of the co-attended-to table. The resource that emerged for the group's subsequent practice was a sophisticated understanding of the organization of the table (Stahl, 2006, Ch. 12 & 13). This locally achieved understanding was congruent to a standard scientific understanding, which the instructor had assumed in designing the table and offering it as a resource for the group task. Here we can see the use of interactional resources connecting ideas from novice-individual and scientific-community planes in the small-group discourse, which led to a significant advance in the group's meaning-making ability. The table of rocket components was one such resource, which went through a process of instrumental genesis for the students, situated in their scientific problem-solving activity. The use of terms like "*same*" and "*different*"—introduced into the discourse by one member and then picked up by others as indices pointing to the problematic issue—became reified resources for the group. Interestingly, sameness and difference are fundamental concepts for mathematical discourse (aka thinking) according to Sfard (2008) (see also Stahl, 2008).

B. *Face-to-face Construction of Triangles*

In the experiment reported by Öner (2013), two graduate students work on a dynamic-geometry task, using a shared computer running Geometer's Sketchpad software. The task was specifically selected because it required a combination of exploring a figure to discover its dependencies and then duplicating the figure using those dependencies. Thus, it involved a combination of spatio-graphical (SG) observation and theoretical (T) mathematical construction. Figure 6 shows the group duplicating (on the left) the given figure (on the right), consisting of one equilateral triangle inscribed within another equilateral triangle.

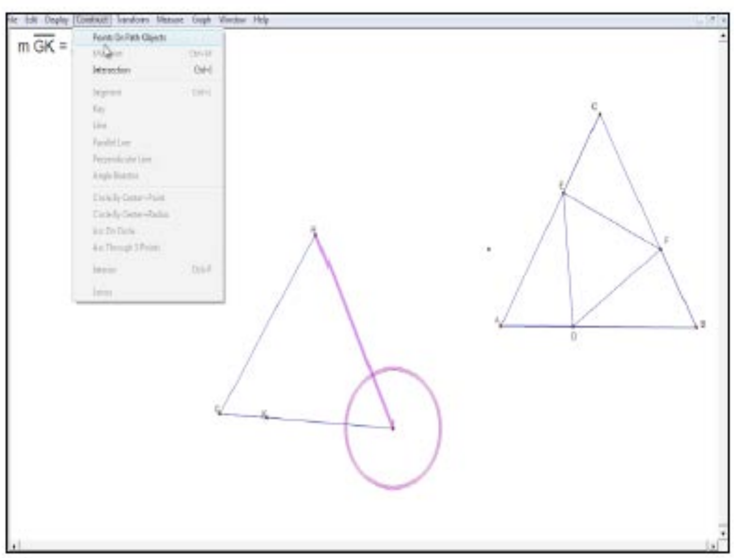

Figure 6. A dyad's attempt to duplicate a figure in Geometer's Sketchpad. From (Öner, 2013).

At the start of the group's work on this task, one of the students, Ayla, says, "*are t:these equal (.) these distances.*" The group then points to and measures the short segments along the outer triangle up to the interior triangle, which look about the same length. They confirm that EC=AD=FB, even when the figure is dragged and these lengths change in absolute size. The similar appearance of the three segments in the graphical view provides a perceptual resource, which Ayla brings into the discourse and points to both with her statement and with her finger on the computer screen, establishing co-attention to this interactional resource.

Later, at the crucial point in the construction at which a second vertex of the inner triangle is to be specified, the earlier finding about the original figure is recalled as a resource for duplicating it. As the other student, Mete, goes to position the second vertex on segment AB, Ayla points to segment EF and wonders quietly as if to herself, "*hm the distance does not have to be always equal.*" Then she says aloud, "*does it? (.) look EC and AD and FB (1.2) are always equal in length*," while pointing at the three segments on the screen. Mete immediately responds, "*ha then we'll do the thing we'll measure that gap*," and begins to do the corresponding construction. This is an instance of group memory (Sarmiento & Stahl, 2007), in which the group references a previous finding—an ephemeral reified resource—and re-situates it in the current interactional context, enacting it as a shared resource for the current work.

The subsequent 30 speech turns of the dyad are concerned with figuring out how to use the software tools to construct their equivalents of EC, AD and FB to be equal lengths. Geometer's Sketchpad provides a tool to do this simply in a couple of ways. However, the resourcefulness of the tool has to be reconstructed by the group interaction to be a usable and effective resource for the group effort. The reconstruction effort itself takes advantage of various interactional resources, such as the letters labeling (indexing for reference in discourse) the triangle vertices, which the group discusses in order to simplify the work of relating corresponding points between their duplicated figure and the original.

Examples of interactional resources from the analysis by Öner (2013) include those classified as theoretical (T), such as the geometry problem, the software tools or the relevant concepts, definitions, axioms and theorems of geometry. There are also spatial-graphical (SG) resources, including various visual properties of the figure like segment lengths, triangular shapes and point labels.

C. *Online Construction of Triangles*

Öner's experiment with reproducing the inscribed equilateral triangles was replicated within the VMT Project—for a somewhat more detailed analysis with chat logs see (Stahl, 2013b). Two groups of three adults—Team A and B—each spent about a half an hour in the online VMT collaboration environment including multi-user GeoGebra. The software supported text chat with graphical referencing and dynamic-geometry construction, providing a contrast to the face-to-face speech and finger pointing in the Öner scenario. The task was identical to Öner's, so many of the resources for group work were identical: concepts and theorems of geometry (to the extent that the groups had working knowledge of them) and the visual properties of the figure (as it was dragged in both of the dynamic-geometry software displays).

Although Team A in the VMT experiment focused on observing the spatio-graphical behavior of the points under dynamic dragging, it took them a long time to make Ayla's key observation. Finally, Jan typed: "*So I think F is CD units away from B on BC. Its not constructed as an equilateral triangle, it happens to be an equilaterl triangle because of the construction*." Here, the SG observation leads immediately to a T statement about the construction of the internal triangle, namely that it is not constructed by making its sides or angles equal, but rather their equality is a consequence of imposing a different dependency involving distances of the vertices of the interior triangle from those of the exterior triangle. Visual resources are turned into resources for construction and reflection. Note that the SG-visual resources that enter the group discourse are based on the perceptual abilities of the individual group members, while the T-geometric conceptualizations that enter the small-group interaction are taken from community scientific knowledge. That is, the small-group interaction brought together resources from the individual and the community levels of analysis.

Team B took even longer to arrive at the key observation for constructing the inscribed triangles. Its members pursued multiple strategies, such as using geometric theorems about centers of triangles and correspondences of similar triangles. Finally Lauren typed, "*I abandoned the center, and worked with the lengths of the sides*." Then she "*used the compass tool to measure the distance from D to C*" and constructed the circles around points A and B as seen in Figure 7, each with radius equal to CE to locate the vertices for constructing the interior triangle.

The use of social conventions and other relationship-building resources in addition to the content-oriented phases of chats seem to play an important role in problem-solving interactions. As Mercer and Sams (2006, p. 517) put it, "while working in classroom groups, children use talk to do much more than engage in curriculum tasks: they form relationships, develop social identities, and pursue 'off-task' activities which may be more important to them than the tasks in which they officially engaged—and as Wegerif (2005) has argued, may be essential to the process of establishing good relationships so that effective 'on-task' activities result." This may be even more pronounced in online interactions, which lack some of the social resources provided by physical presence.

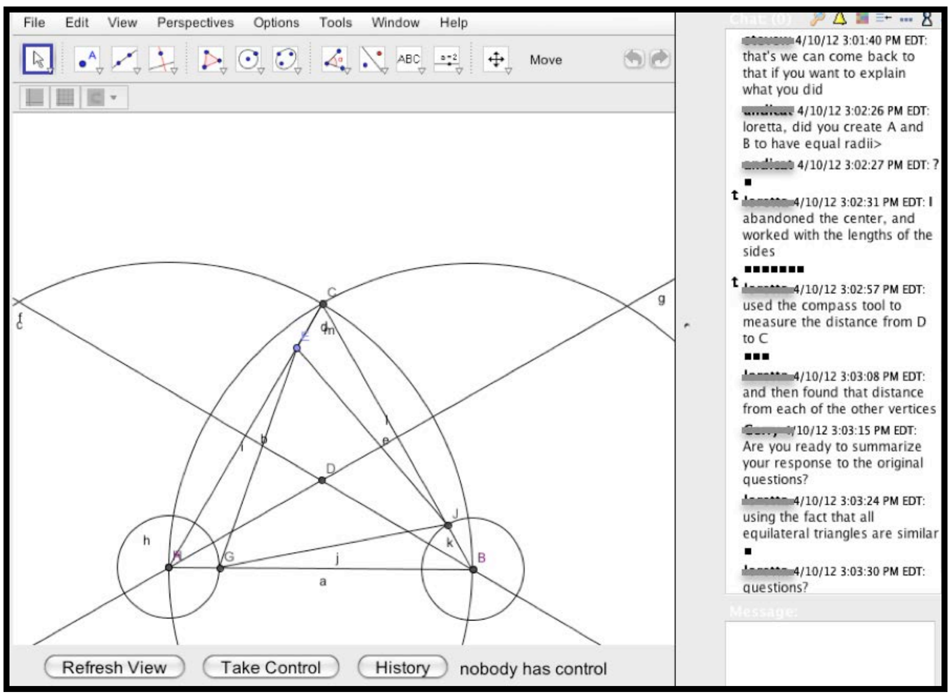

Figure 7. Finding the center and constructing equal line segments in VMT with GeoGebra. From (Stahl, 2013b).

Groups in CSCL contexts can be seen to be making considerable use of diverse resources to accomplish their interactional work. Often, they bring in resources from their individual backgrounds or from a community plane (the classroom, the history of mathematics, the subculture of social texting, the practices common in society, the conventions of ordinary language). Frequently, they build local resources within the group, available for repeated use and for personalization as resources for the individuals or for institutionalization into resources for the larger community. The resources must be shared—attended to by the group and similarly understood—for them to be effectively used. This may be achieved through pointing, questioning, explaining, drawing or illustrating (Stahl et al., 2011). In a problem-solving session, one of the first resources co-constructed by the group might be a formulation of the question that they will pursue, based perhaps on an assigned task, which they must understand and articulate collaboratively (Zemel & Koschmann, 2013). The formulation of the question may involve the generation of ephemeral reified resources, such as shared terms to reference the question within the group discourse. Open-ended mathematical tasks can involve the formulation of mathematically interesting problems as an on-going part of the group work. For such so-called "wicked" problems, the interpretational resources and math understanding generated by solving the problem may be necessary for knowing how to even state the problem effectively (Rittel & Webber, 1984).

The use of resources can be accumulated in the sequentiality of interaction to produce larger group-cognitive accomplishments such as mathematical problem solving (Stahl, 2011). Across a somewhat larger time scale, resources can build on one another, much as Euclid's proofs built upon previous proofs. Groups can use their earlier formulations of interactional resources to construct higher-level resources and to refine previous understandings, just as scientific knowledge advances by accumulation and revision of institutional resources (Kuhn, 1972).

CSCL research can connect the levels in its research data by identifying the resources that are being brought into collaborative interactions and by tracking how they are constituted, understood and applied in the meaning-making process. CSCL studies can contribute to our understanding of collaborative meaning-making processes by providing detailed analyses of the ways in which group discourses involve resources interactionally and how the resources are shared, interpreted, refined and preserved.

IV. THE PEDAGOGY OF CONNECTING LEVELS

If resources play such an important role in collaborative learning, then how can CSCL designers support the use of resources? Can research into the role of resources in learning at different levels of analysis and across these levels guide pedagogical design? Clearly, it would be useful to make sure that students have access to relevant resources and that they understand how to use them. In situations where teachers play a central role in guiding the collaborative learning, it would similarly be important to ensure that the teachers have access to relevant resources and that they understand how to facilitate student use of them. Early attempts to support resources for teachers and students were proposed in (Stahl, Sumner & Owen, 1995) and (Stahl, Sumner & Repenning, 1995).

In the Virtual Math Teams (VMT) Project, we have learned through pilot trials of the VMT-with-GeoGebra environment that this relatively complex system requires careful preparation and training for teachers, students, online groups and classes to use effectively without encountering frustration. In response to this, we have drafted a set of dynamic-geometry curricular activities, interspersed with tutorial tours of the technology features (Stahl, 2012a). These materials are designed for use both by teachers in professional-development contexts and by student teams in online-classroom or after-school settings.

We believe that computer-supported dynamic geometry can revitalize the study of classical Euclidean geometry—particularly through a translation into a collaborative-learning format (Stahl, 2013b). The socio-technical design of the VMT-supported CSCL approach to collaborative dynamic-mathematics is intended to have small groups of learners build ephemeral-reified interactional resources through online discourse and joint geometric construction. This is done by providing computer support for the discourse and the construction, but also delivering just-in-time instructions at the individual level and scaffolded practice at the group level. Most of the mathematical activities are based on postulates of Euclidean geometry (Euclid, 300 BCE/2002); as groups of students explore these activities, they not only enact these institutional resources, but they gain hands-on understanding of the meaning and insight sedimented in this cultural heritage.

The VMT curriculum activities have also been designed to promote effective collaborative-learning interaction practices, particularly as they can occur in significant mathematical discourse about geometry. We do this by providing a carefully structured set of scripted activities for use by teachers and students. These include the following interactional resources:

1. Resources for engaging in significant mathematical *discourse*; to collaborate on and discuss mathematical activities in supportive small online groups. This includes suggested uses of linguistic and interactional resources for coordinating collaboration, as well as tutorials in using the communication tools of the VMT software.
2. Resources to collaboratively *explore* mathematical phenomena and dependencies; to make mathematical phenomena visual in multiple representations; and to vary their parameters. This includes scaffolded exercises

in noticing visual characteristics of dynamic-geometric figures being dragged and in wondering in chat postings about their dependencies.

3. Resources for *constructing* mathematical diagrams—understanding and exploring their structural dependencies. This consists primarily of a semester-long sequence of construction activities, initially with step-by-step instructions and tutorials about GeoGebra tools.
4. Resources to notice, wonder about and form conjectures about mathematical relationships; to justify, explain and *prove* mathematical findings. This involves discussion prompts and situated examples of explanations or proofs.
5. Resources to understand core concepts, relationships, theorems and constructions of basic high-school *geometry*. The included materials and activities cover central conceptual and procedural resources from Euclid's first book of propositions and from the Common Core standards for beginning geometry (CCSSI, 2011).

The presentation of resources is organized developmentally, so that understanding of the resources presented first can be used to build understanding of resources presented subsequently. Concomitant with this is a progressive shift from scaffolded explanation of basic resources (like software tools) to open-ended inquiry of more complex resources (like mathematically interesting micro-worlds).

1. The *discourse* begins with having students greet each other online and then negotiate about who will do what, when in the online environment. Students are next asked to comment on their noticings and wonderings. Later, they are to make conjectures. Finally, they are expected to explain things to each other, make sure that everyone understands, and produce presentations of group findings. Linguistic, conceptual and procedural resources developed in collaborative work eventually contribute to resources for individual thinking.

2. The *exploration* begins with being introduced to software widgets and tools. It goes on to increasingly complicated geometric drawings. Then, students are expected to construct geometric objects themselves and in small groups. Finally, they are given open-ended scenarios and encouraged to figure out how to bring resources to bear on unknown mathematical territory.

3. *Construction* skills gradually grow from dragging pre-configured dynamic applets, to constructing with step-by-step instructions, to figuring out how to construct objects with specific dependencies, to defining their own custom construction tools, to constructing objects of their own design in open-ended micro-worlds. The skill level in using such resources progresses from novice to a reasonable command of GeoGebra's geometry tools. A transition to GeoGebra's algebra connection (analytic geometry) is provided at the end, opening up GeoGebra's multiple representations and resources of geometric diagrams, analytic-geometry graphs, spreadsheet data, 3-D transformations and a computer-algebra system.

4. *Proof* in geometry is introduced slowly, with an initial focus on noticing and wondering. This is followed by formulation of text-chat-based explanations and use of multi-media resources to document findings. The explanations gradually entail increased levels of justification, finally approaching formal proofs, without ever reaching the completely formalized version of routinized two-column proof. The use of resources for analysis, explanation and argumentation are practiced in prompted exercises.

5. The *geometry* content starts by covering many of the classic activities in Book I of Euclid's *Elements* (300 BCE/2002), but implemented in the computer-supported collaborative-learning medium of multi-user dynamic geometry. It incorporates the beginning standards for high school geometry in the reformed *Common Core Standards* (CCSSI, 2011), including resources related to congruence, symmetry and rigid transformations. The fundamental features of triangles are examined first, and then students are encouraged to explore similar resources for analyzing quadrilaterals. For instance, students are involved in designing hierarchies of kinds of triangles or quadrilaterals based on the resources of alternative representations and dependencies of congruence, symmetry and rigid transformations. Finally, a sampling of creative objects, micro-worlds and challenge problems are offered for student-centered exploration.

There is a theoretical basis for gradually increasing skill levels in terms of both geometric understanding and deductive proof. The van Hiele theory (see deVilliers, 2003, p. 11) specifies several levels in the development of students' individual understanding of geometry. The implication of van Hiele's theory is that students who are at a given level cannot properly grasp ideas presented at a higher level until they work up to that higher level. Thus, a developmental series of activities pegged to the increasing sequence of levels is necessary to effectively present the various resources of geometry, such as, eventually, the formal, institutionalized structure of deductive proof. Failure to lead students through this developmental process is likely to reinforce student feelings of inadequacy and consequent negative attitudes toward geometry. The VMT collaborative curriculum aims to facilitate a progression of reified resources for discussing math in groups corresponding to the van Hiele levels for individual understanding.

In his book, deVilliers (2003) suggests that students be introduced to proof by gradually going through a sequence of levels of successively more advanced roles of proof through a series of well-designed activities. In particular, the use of a dynamic-geometry environment can aid in moving students from the early stages of these sequences (recognition and communication) to the advanced levels (deduction and systematization). The use of dragging geometric objects to explore, analyze and support explanation can provide helpful resources for the developmental process. The design and construction of geometric objects with dependencies to help discover, order and verify relationships can further the process. The construction can initially be highly scaffolded by providing helpful resources; then students can be guided to reflect upon and discuss the constructed dependencies; finally, they can practice constructing objects with gradually reduced scaffolding. This can bring students to a stage where they are ready for using resources related to deduction and systematization. Furthermore, by working through the various resources for different varieties of proof, math teachers and students are exposed to a richer conception of proof, in line with contemporary theories of proof, such as those by Lakatos (1976) and Livingston (1999).

A particularly important resource for understanding and working in dynamic geometry is the concept of *dependency*. GeoGebra allows one to construct systems of inter-dependent geometric objects. The dependencies built into dynamic-geometry constructions are intimately related to proofs illustrated by those constructions. Often, to understand a dependency and to be able to implement it in a construction is tantamount to being able to articulate a proof and to explore its validity dynamically (Stahl, 2013b). Students have to learn how to think in terms of these dependencies. They can learn through use of resources like visualizations, manipulations, constructions and verbal articulations. These can all be modeled by examples, and these resources can be provided gradually. Small-group discussion while working together on geometric activities can articulate the importance of dependencies, as seen in the case studies of the previous section.

The VMT Project is now drafting and piloting versions of curricular activities designed to develop significant mathematical discourse focused on dependencies among geometric objects (Stahl, 2012a). Concomitantly, it is implementing software support for teachers and students to explore the dependencies and assembling materials for professional development to prepare teachers to enact this curriculum with their students (Stahl & Powell, 2012). The set of activities is designed to provide the most important basic geometry resources to math teachers and students, taking them from a possibly novice level to a more skilled level, at which they will have a sufficient portfolio of resources for engaging in significant mathematical discourse without continuing scaffolding. The resources of classical Euclidean geometry were decisive in the historical development of rational thinking by literate individuals and of scientific culture in the modern world (Netz, 1999; Stahl, 2013b). We hope to adapt these resources to the CSCL context, where they may enter into collaborative online interactions and thereby influence individual understanding and classroom practices.

In on-going experiments within the VMT Project and elsewhere, our colleagues and we will be logging the use of resources by teachers and students in order to analyze how resources connect levels of learning in a CSCL setting. We will track individual and group performance in significant mathematical discourse as resources and practices from community levels are taken up in sequential small-group interaction. Perhaps we will witness the formation of local practices and group interactional resources, which can influence individual and community levels over time. In these ways, we will study resources for connecting levels of learning in CSCL.

More generally, through analysis of the nature and work of interactional resources in case studies of a broad variety of CSCL interactions, the CSCL research community can expect to reach a better understanding of the nature of different levels of analysis in CSCL research and how the levels may be connected in terms of their mediation by diverse resources. Gradually, we will discover how resources are enacted, understood, shared, designed, adapted and preserved—and how they mediate connections among levels of learning through social interaction.

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