

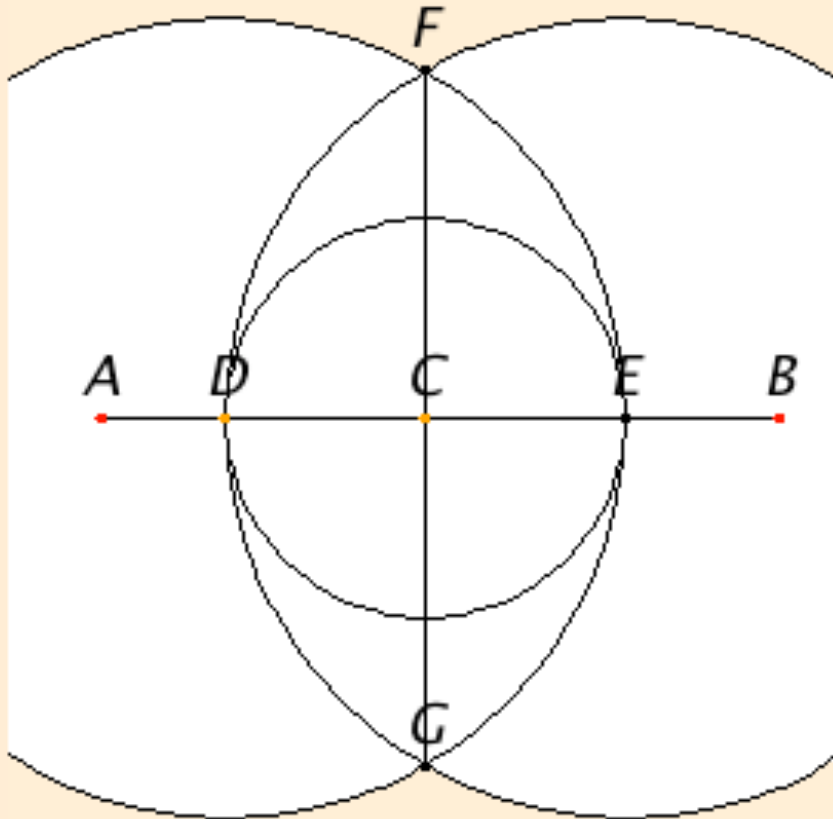
Distinguished Lecture  
School of Communication and Information  
Rutgers University--New Brunswick  
Friday, February 24, 2012

# Translating Euclid into CSCL

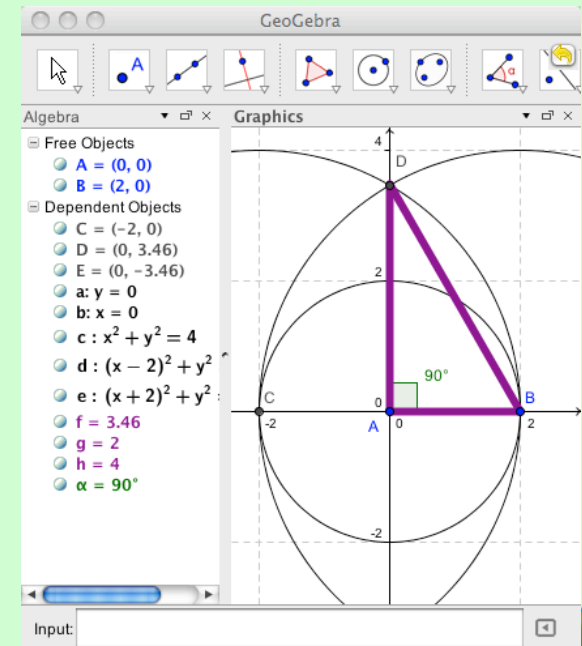
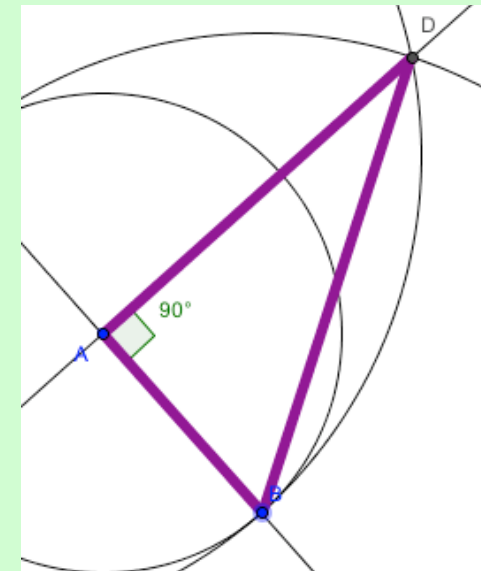
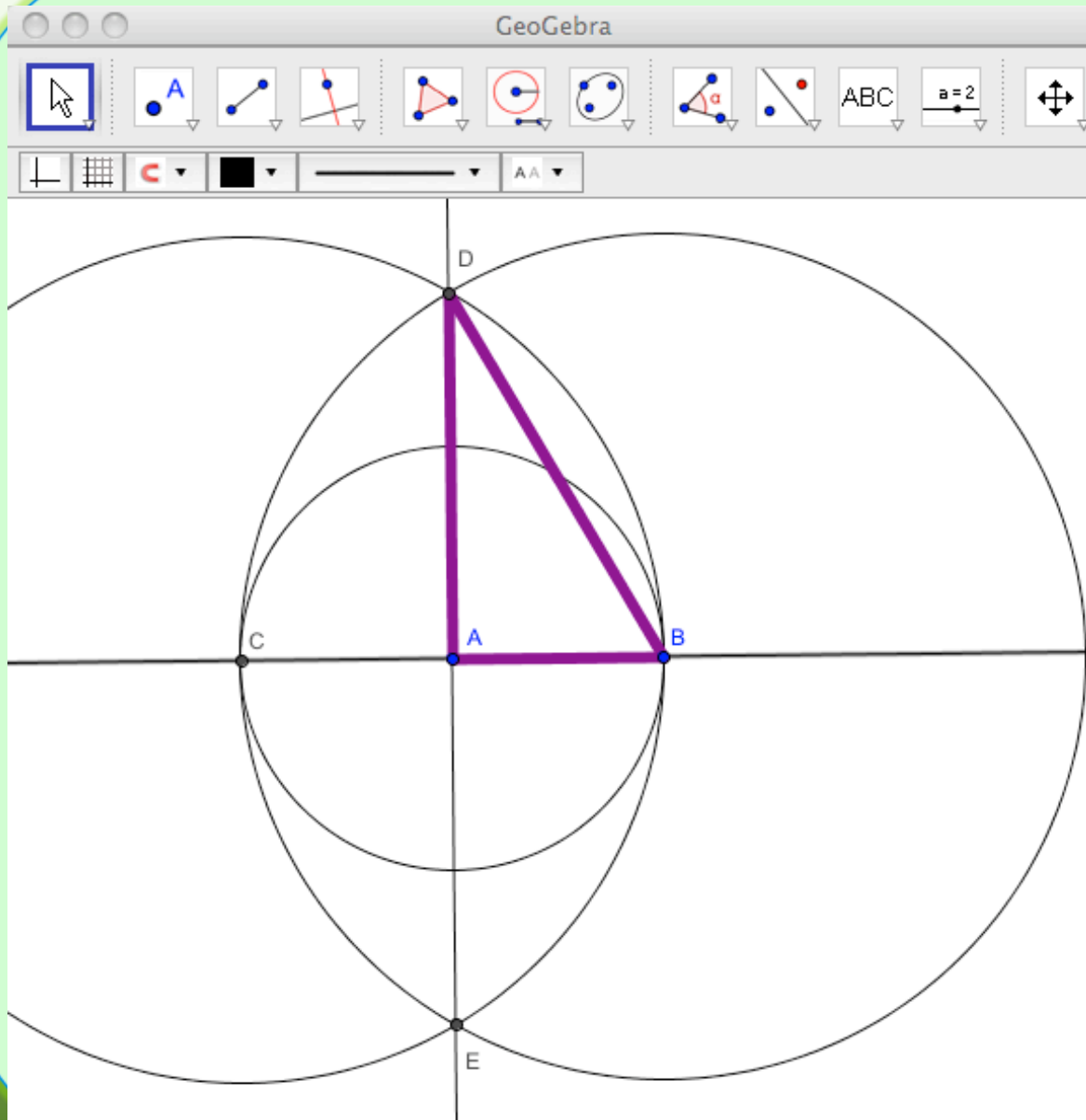
Issues in Design-Based Research on  
Computer-Supported Collaborative Learning  
of Significant Mathematical Discourse

Gerry Stahl

# Euclid constructing a right angle on a clay tablet



# Dynamic Geometry in GeoGebra



# Accountable Talk – (idealized) log of discourse with agent about construction

**Sam:** I drew a circle around A to find a point the same distance as B

**Agent:** Chris, can you check that Max is following Sam?

**Chris:** Hey Max, can you explain what Sam just did?

**Max:** Yeah, I think he found a point C such that  $AC = AB$

**Chris:** But how do you know the lengths are exactly the same?

**Max:** Because all radii of a circle are equal

**Sam:** Now I am drawing circles around C and B

# Exploration & Discourse: VMT-with-GeoGebra construction & chat

TabbedChat: vmt (CID:1261090089485)

File Edit Chat GG File GG Edit GG View GG Options GG Tools GG Window GG Help

Material: Whiteboard GeoGebra Summary

Algebra View Graphics View

Free Objects

- A(-0.4, 9.58)
- B(2.38, 1.76)
- C(9.24, 0.66)
- D(8.22, 6.4)

Dependent Objects

- E(3.91, 7.99)
- F(0.99, 5.67)
- G(5.81, 1.21)
- H(8.73, 3.53)
- a = 8.3
- b = 6.95
- c = 5.83
- d = 9.19
- e = 3.73
- f = 6.57
- g = 3.73
- h = 6.57
- i = 13.13
- poly1 = 48.41
- poly2 = 24.21

Auxiliary Objects

Chat (0)

Current users: Gerry vmt

EFHG, here

vmt 6:03:26 PM EST: interesting ... I dragged corner A around and watched how the areas of poly 1 and e changed

Gerry 6:04:05 PM EST: yeah, i c

Gerry 6:04:30 PM EST: poly2 seems to always be about half of poly1

vmt 6:05:14 PM EST: I bet that is always true because it is built from the midpoints of poly1

vmt 6:08:11 PM EST: Look! I connected A and C -- that forms two sets of similar triangles. I bet that if we made triangles DEH and DAC that DEH would be 1/4 the area of DAC because its b and h are 1/2

Gerry 6:10:00 PM EST: Cool! That proves it. If we draw BD, we will have 4 triangles each with a quarter the area of half the quadrilateral! Very elegant

Message: Nice work, partner! Thanks for explaining it to me.

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# 1. Translating Euclid

**Classic Education –constructing a right angle on a clay tablet**

**Dynamic Geometry – constructing using GeoGebra tools**

**Accountable Talk –discourse with agent about constructing**

**Exploration & Discourse – constructing, visualizing, exploring in VMT-with-GeoGebra**

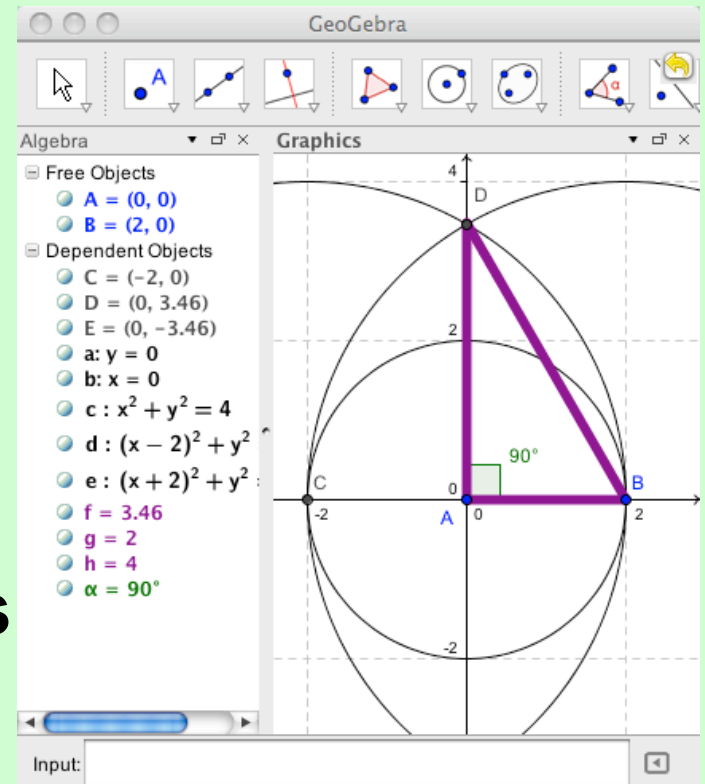
# **The VMT Project Today: Intro**

- 1. The Promise of CSCL: From clay tablets to iPads**
- 2. The Domain of Geometry Education**
- 3. The Role of Discourse**
- 4. The Approach of Design-Based Research**
- 5. The Enactment of Technology**
- 6. Design Issues**
- 7. The Use of Conversation Analysis**
- 8. The Philosophic Issues**

# 2. Math Education with Dynamic Geometry

**Guiding development of skills & tacit understanding**

- **Visualization**
- **Exploration**
- **Dependencies**
- **Multiple Representations**
- **Concrete & Abstract**
- **Math Talk**





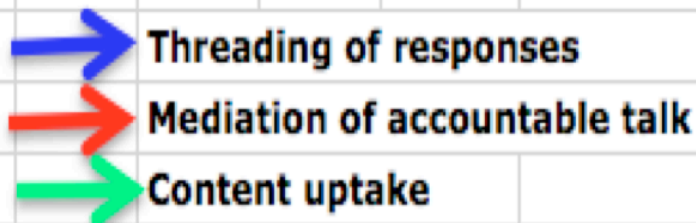
## 3. Discourse

- **Socialization in math community**
- **Language of math & practices**
- **Collaborative learning thru discourse**
- **Accountability to each other, to math domain, to math community standards**
- **Conceptualization & articulation**

# Accountable Talk: Model or Software Agent

**Ideal Agent-Mediated Accountable Talk Response Structure**

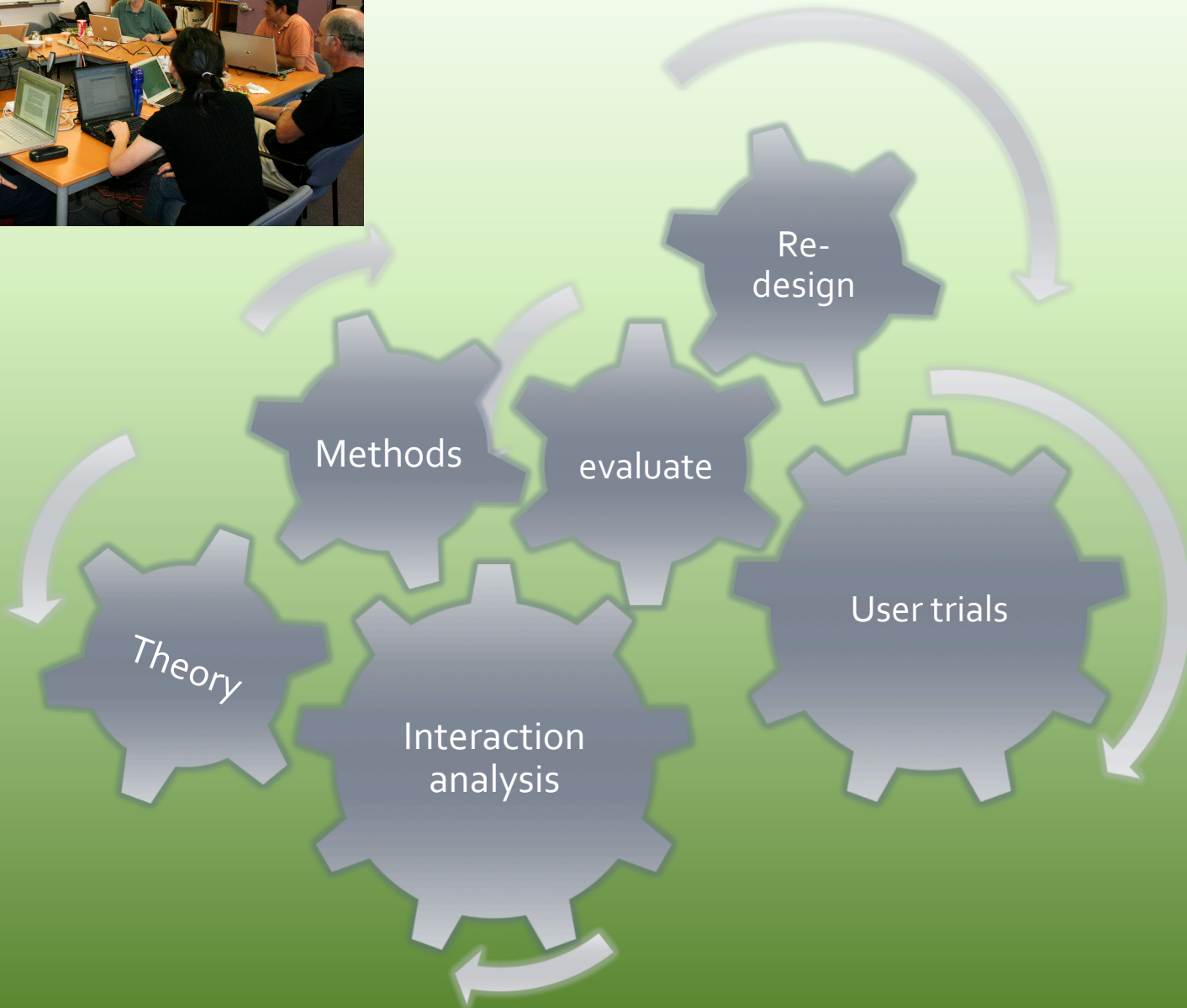
Agt	S1	S2	S3	
	1			1 S1 states something
2				2 Agent responds to S1 by prompting S2
		3		3 S2 responds to the agent, asks S3 to restate S1' content
			4	4 S3 restates S1's statement -- as mediated by the agent
	5			5 S1 accepts S3's restatement
			6	6 S3 builds on S1's restatement
7				7 Agent prompts S2 to add to S3 and S1
			8	8 S2 adds to S3 and S1's statements -- as mediated by the agent



## 4. Design-Based Research

- **Innovation – nothing fully known or fixed**
- **Technology, Practices of usage, User needs, user skills, hypotheses, central concepts, assessment methods**
- **Start simple; explore in iterative cycles of trial, evaluation, re-design**
- **Evaluation of technology & scaffolds: observe use and enactment by non-designers**

# Design-Based Research



## 5. “Virtual Math Teams” Computer Support

- **An integrated socio-technical environment for individual, group and community cognition**
- **Lobby: find rooms, social networking, teacher administrative tools, activity overview tools**
- **Chat room: group chat, Web browser, Wiki editor, Topic viewer, Help manual**
- **GeoGebra: group and individual construction areas**

# The VMT Lobby



## Virtual Math Teams 3.0-Dev.03

Welcome Professor

- [New to VMT?](#)
- [List of All Rooms](#)
- [My Profile](#)
- [My Teammates](#)
- [My Rooms](#)
- [Messages](#)
- [Manage Activities](#)

- [VMT Help Pages](#)
- [VMT Sandbox Room](#)
- [VMT Lounge Room](#)
- [VMT Wiki Pages](#)
- [VMT Replayer 2.2](#)
- [VMT Replayer 3 Dev 2 \(12/22 - 1/30\)](#)
- [VMT Replayer 3 - current](#)
- [Logout](#)

View Chat Rooms as

Math Subject Tree    Tabular List

Filter Chat Rooms By...

Project:     Last Activity:

---

▼ **Geometry** (2 Topics)

- ▶ **Activity1** 🏆 (9 Rooms, 0 Active )
- ▼ **Activity2** 🏆 (9 Rooms, 0 Active )
  - ▶ **Lucky Number 7\_1** 🏆
  - ▼ **Team Bee\_1** 🏆

Username	# of Messages	Last Active
at373	10	Feb 1, 2012 19:50
cbartizek	25	Feb 1, 2012 19:56
charlie_mcmichael	141	Feb 1, 2012 19:50
gerry	12	Jan 31, 2012 23:19
professor	8	Feb 1, 2012 19:57

# VMT chat room with multiple tabs

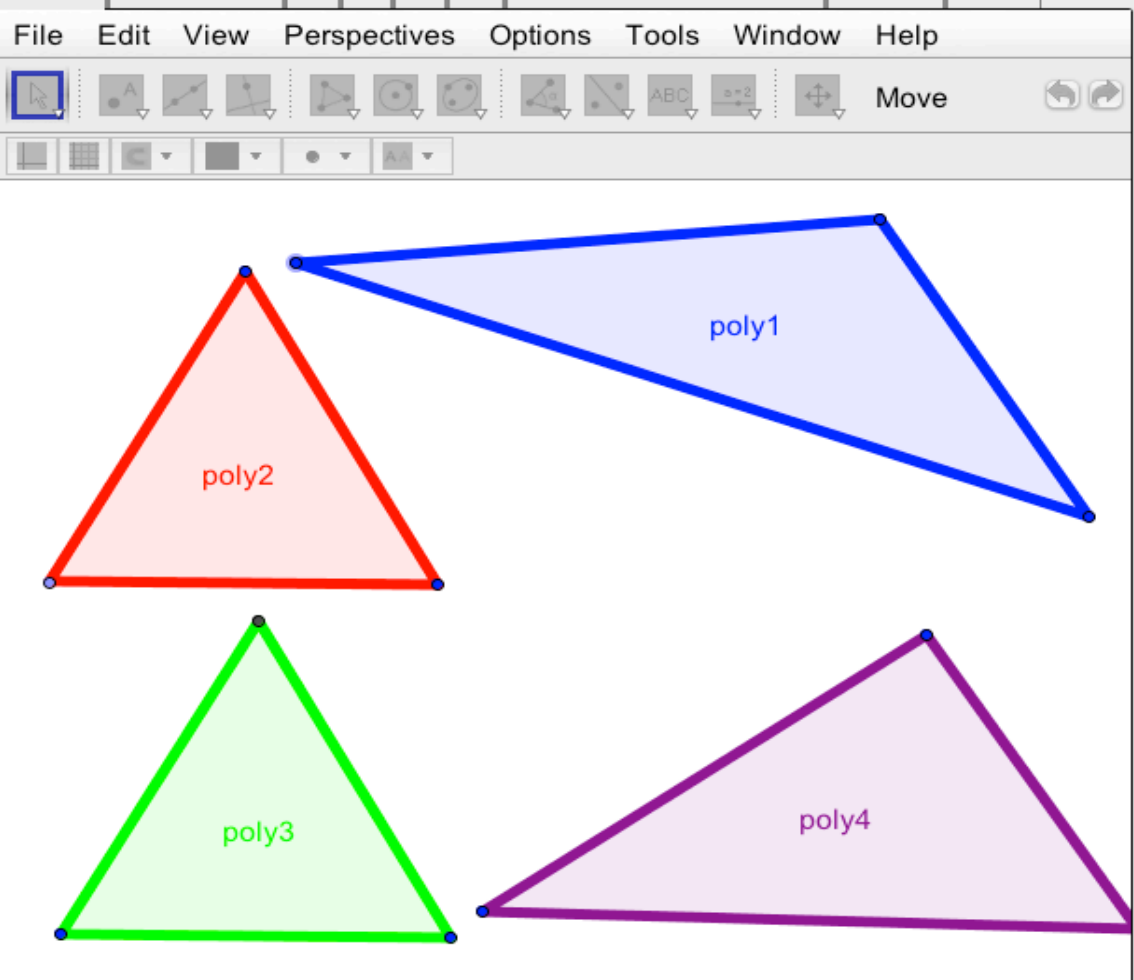
TabbedChat: Professor (CID:1328585967654)

File Edit Chat GeoGebra

Material: +

GeoGebra Summary A B C D Shared Whiteboard Topic Wiki

File Edit View Perspectives Options Tools Window Help



poly2

poly1

poly3

poly4

Refresh View Take Control nobody has control

Current users:  
Professor

Chat: (1)

- Professor leaves the room 2/6/12 11:03:57 PM EST
- Professor joins the room 2/6/12 11:24:03 PM EST
- Professor leaves the room 2/6/12 11:25:12 PM EST
- Professor joins the room 9:14:50 PM EST

Professor 9:18:40 PM EST: I moved the blue triangle

Professor 9:18:52 PM EST: So it seems to be generic

Professor 9:19:00 PM EST: or scalene

Professor 9:19:18 PM EST: I think there is an equilateral

Professor 9:19:27 PM EST: and a right triangle

Professor 9:19:38 PM EST: and probably an isosceles

Professor 9:20:03 PM EST: Does everyone else agree with my conclusion?

Message:  
How do you think these were constructed?

- **Summary whiteboard:** textbox with mathML, referencing, history
- **Wiki page** for activity of group – for Activity – for group – for community
- **Logs** and Replayer
- **Integrated curriculum** of activities
- **Professional development and training** in individual intellectual, group collaborative and community accountable practices – primarily in terms of significant mathematical discourse.



# Probability

Here are a set of challenges related to probability problems. **You can contribute** by adding your ideas about applying a strategy to a problem (adding content to a P#S# page), proposing a new strategy (adding a new column) or adding a new challenge (row).

Probability Strategies & Problems	S1. Drawing balls from a jar	S2. Solve Complementary Problem	S3. Enumerate & Organize your cases	S4. Use a Tree Diagram	S5. New Strategy
P1. <a href="#">The sock drawer</a>	P1S1	P1S2	P1S3	P1S4	P1S5
P2. <a href="#">Box with three cards</a>	P2S1	P2S2	P2S3	P2S4	P2S5
P3. <a href="#">Seating arrangements</a>	P3S1	P3S2	P3S3	P3S4	P3S5
P4. <a href="#">Baseball World Series</a>	(P4-S1 Example)	(P4-S2 Example)	(P4-S3 Example)	(P4-S4 Example)	P4S5
P5. <a href="#">Duck hunters</a>	P5S1	P5S2	P5S3	P5S4	P5S5
P6. <a href="#">Clock hands</a>	P6S1	P6S2	P6S3	P6S4	P6S5
P7. <a href="#">Length of Random Chords</a>	P7S1	P7S2	P7S3	P7S4	P7S5
P8. <a href="#">New Problem</a>	P8S1	P8S2	P8S3	P8S4	P8S5

If you need them, here are some [resources for probability](#)

Categories: [ProblemSolving](#) | [VMT](#)



### navigation

- [VMT Lobby](#)
- [Wiki Main Page](#)
- [Recent changes](#)
- [Help](#)

### search




### toolbox

- [What links here](#)
- [Related changes](#)
- [Upload file](#)
- [Special pages](#)
- [Printable version](#)
- [Permanent link](#)

# The VMT Re-Player

Material: GeoGebra

Current users: tutorA

Chat (0)

Free Objects

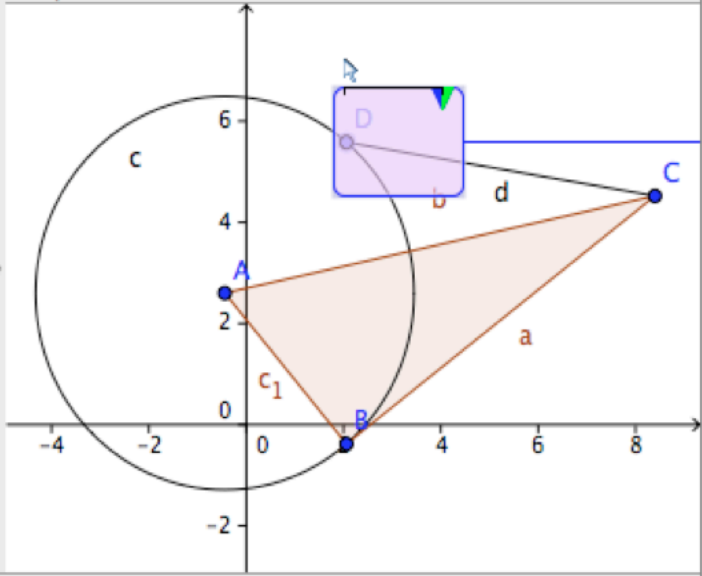
- A(-0.44, 2.6)
- B(2.06, -0.38)
- C(8.4, 4.52)
- D(2.06, 5.58)

Dependent Objects

- a = 8.01
- b = 9.05
- c:  $(x + 0.44)^2 + (y - 2.6)^2$
- c<sub>1</sub> = 3.89
- d = 6.43
- poly1 = 15.57

Auxiliary Objects

Algebra View Graphics View



Message: Here is point D on the circle and on line segment CD. Try to drag this point and watch the circle

Speed: 1

Time to previous: -0:22 (Message by: tutorA)  
Current action at: 4:39:56 PM (Awareness info)  
Time to next: 0:00 (Message by: tutorA)

## 6. Current “Hot” Design Issues

- a) **Instruction vs. exploration**: explaining interface vs. notice and wonder
- b) **Synchronous**/collaborative chat vs. **asynchronous**/individual read and try vs. class replay and reflection
- c) **Custom perspective** for all, community, class, group, individual?
- d) **Custom tool** for all, community, class, group, individual?
- e) **Agent** intervention: scripting vs. help agent
- f) **Curriculum** design criteria
- g) Design-based **research agenda**

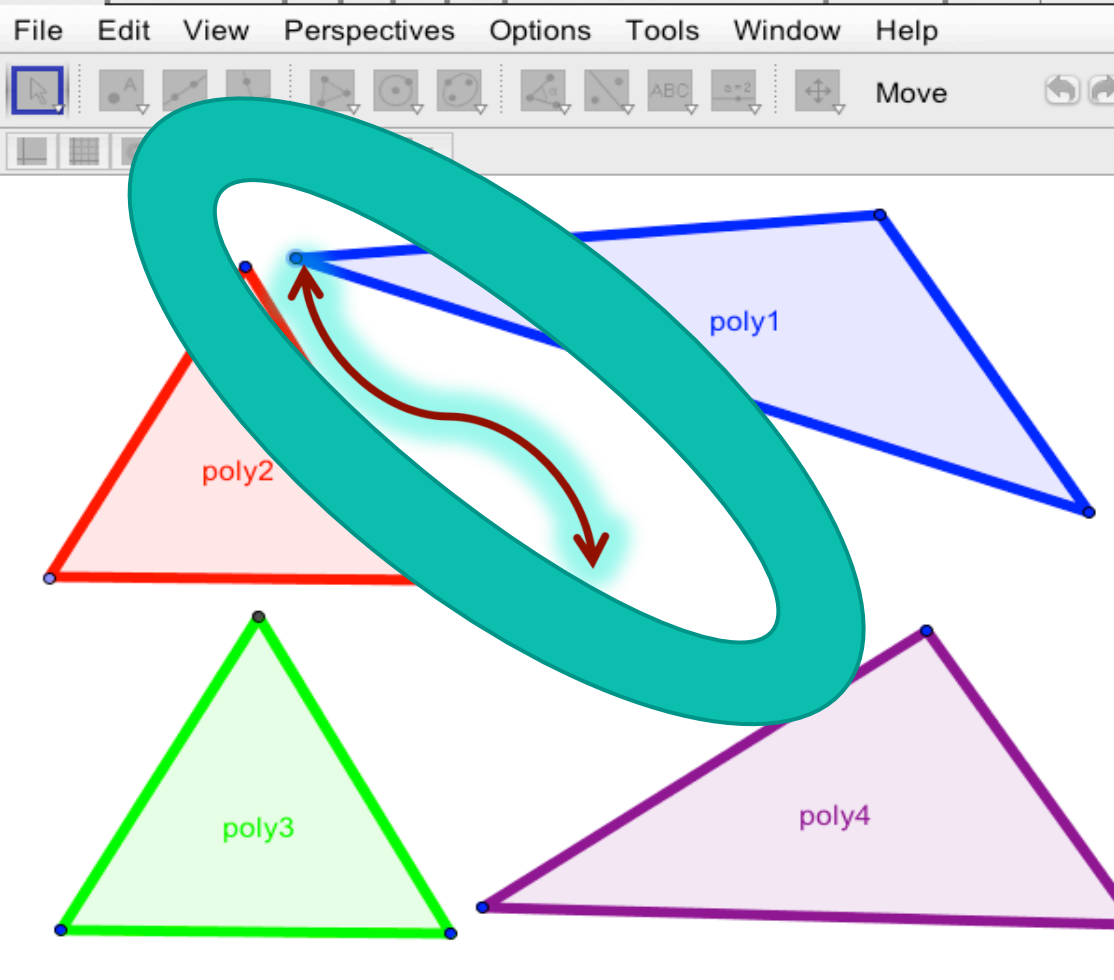
# More exploration, less Instruction

TabbedChat: Professor (CID:1328585967654)

File Edit Chat GeoGebra

Material: GeoGebra Summary A B C D Shared Whiteboard Topic Wiki

File Edit View Perspectives Options Tools Window Help



poly1

poly2

poly3

poly4

Refresh View Take Control nobody has control

Current users: Professor

Chat (1)

- Professor leaves the room 2/6/12 11:03:57 PM EST
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Professor 9:20:03 PM EST: Does everyone else agree with my conclusion?

Message: How do you think these were constructed?

# Shared & personal spaces; synch/asynch

The screenshot shows a web browser window titled "TabbedChat: Professor (CID:1328585967654)". The browser's address bar and menu bar are visible. A teal oval highlights the browser's tab bar, which contains several tabs: "GeoGebra", "Summary", "A", "B", "C", "D", "Shared Whiteboard", "Topic", and "Wiki".

The main content area is a GeoGebra workspace. It features a toolbar with various geometric construction tools. Below the toolbar, four polygons are displayed on a grid background:

- poly2**: A red triangle.
- poly3**: A green triangle.
- poly4**: A purple triangle.
- poly1**: A blue quadrilateral.

At the bottom of the workspace, there are buttons for "Refresh View" and "Take Control", along with the text "nobody has control".

On the right side of the browser window, there is a chat window titled "Chat (1)". It shows a list of messages from "Professor" with timestamps:

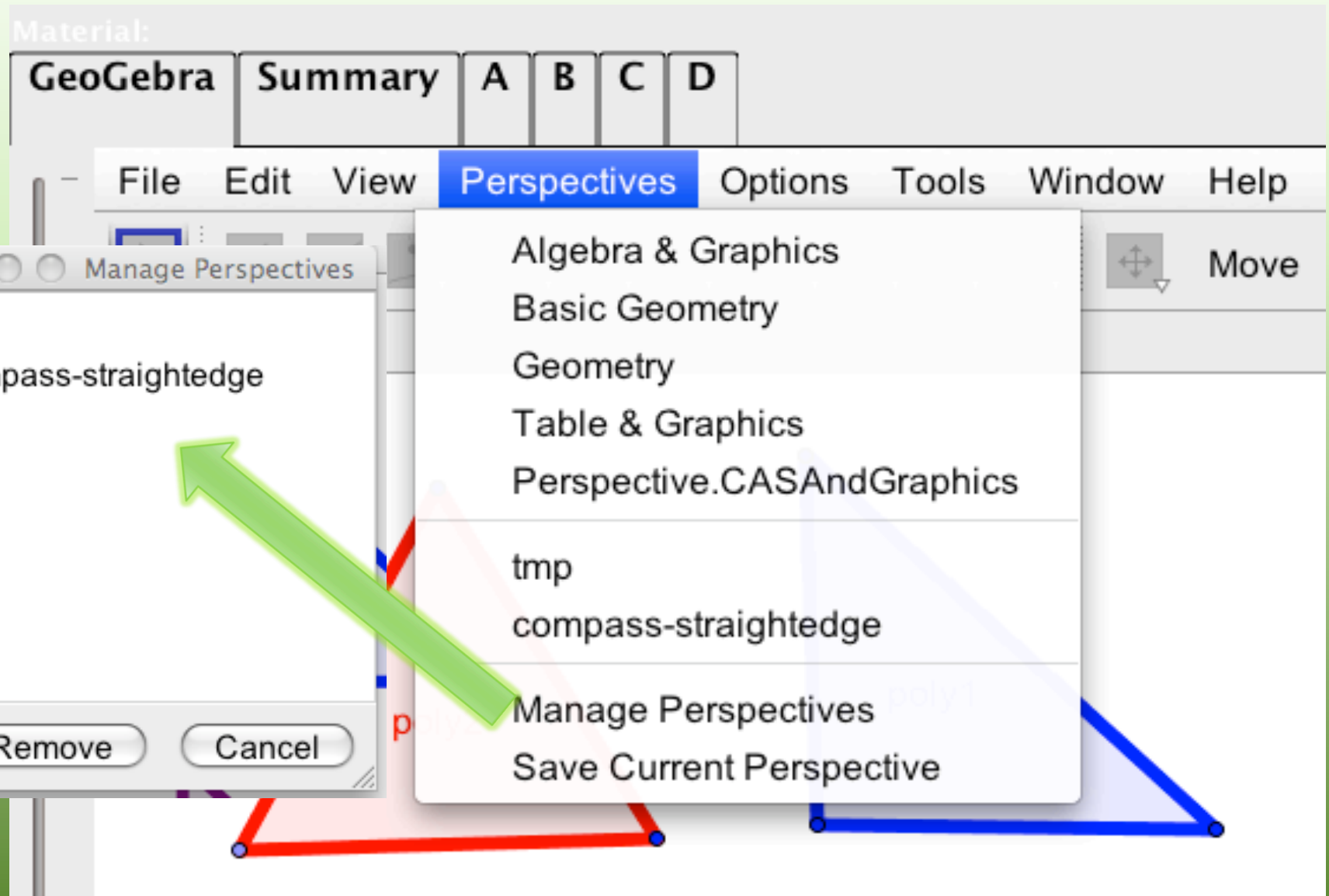
- Professor leaves the room 2/6/12 11:03:57 PM EST
- Professor joins the room 2/6/12 11:24:03 PM EST
- Professor leaves the room 2/6/12 11:25:12 PM EST
- Professor joins the room 9:14:50 PM EST

Below these messages, there is a separator line of red dashes. The chat continues with the following messages:

- Professor 9:18:40 PM EST: I moved the blue triangle
- Professor 9:18:52 PM EST: So it seems to be generic
- Professor 9:19:00 PM EST: or scalene
- Professor 9:19:18 PM EST: I think there is an equilateral
- Professor 9:19:27 PM EST: and a right triangle
- Professor 9:19:38 PM EST: and probably an isosceles
- Professor 9:20:03 PM EST: Does everyone else agree with my conclusion?

At the bottom of the chat window, there is a "Message:" input field containing the text "How do you think these were constructed?".

# Custom perspectives tool



# Custom right triangle tool

Free Objects

- A = (0, 0)
- B = (2, 0)

Dependent Objects

- C = (-2, 0)
- D = (0, 3.4)
- E = (0, -3.4)
- a:  $y = 0$
- b:  $x = 0$
- c:  $x^2 + y^2$
- d:  $(x - 2)^2 + y^2$
- e:  $(x + 2)^2 + y^2$
- f = 3.46
- g = 2
- h = 4
- $\alpha = 90^\circ$

Create New Tool

Output Objects | Input Objects | Name & Icon

Select objects in construction or choose from list

Segment f: Segment [D, A]

Segment g: Segment [A, B]

Segment h: Segment [D, B]

< Back | Next > | Cancel

# Accountable Talk – (idealized) log of discourse with agent about construction

*Sam:* I drew a circle around A to find a point the same distance as B

*Agent:* Chris, can you check that Max is following Sam?

*Chris:* Hey Max, can you check that Sam just did?

*Max:* Yeah, I think he found a point C such that  $AC = AB$

*Chris:* But how do you know the lengths are exactly the same?

*Max:* Because all radii of a circle are equal

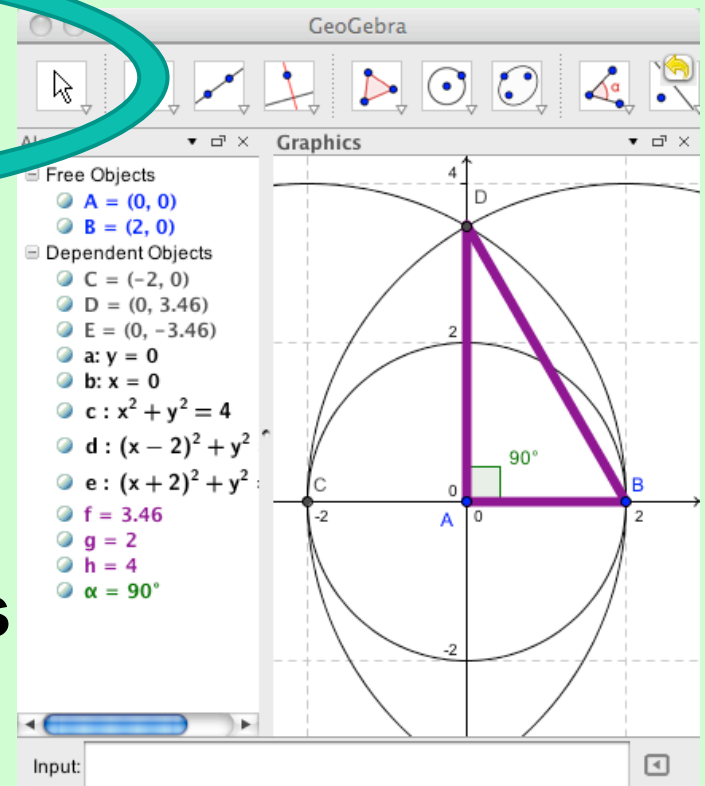
*Sam:* Now I am drawing circles around C and B



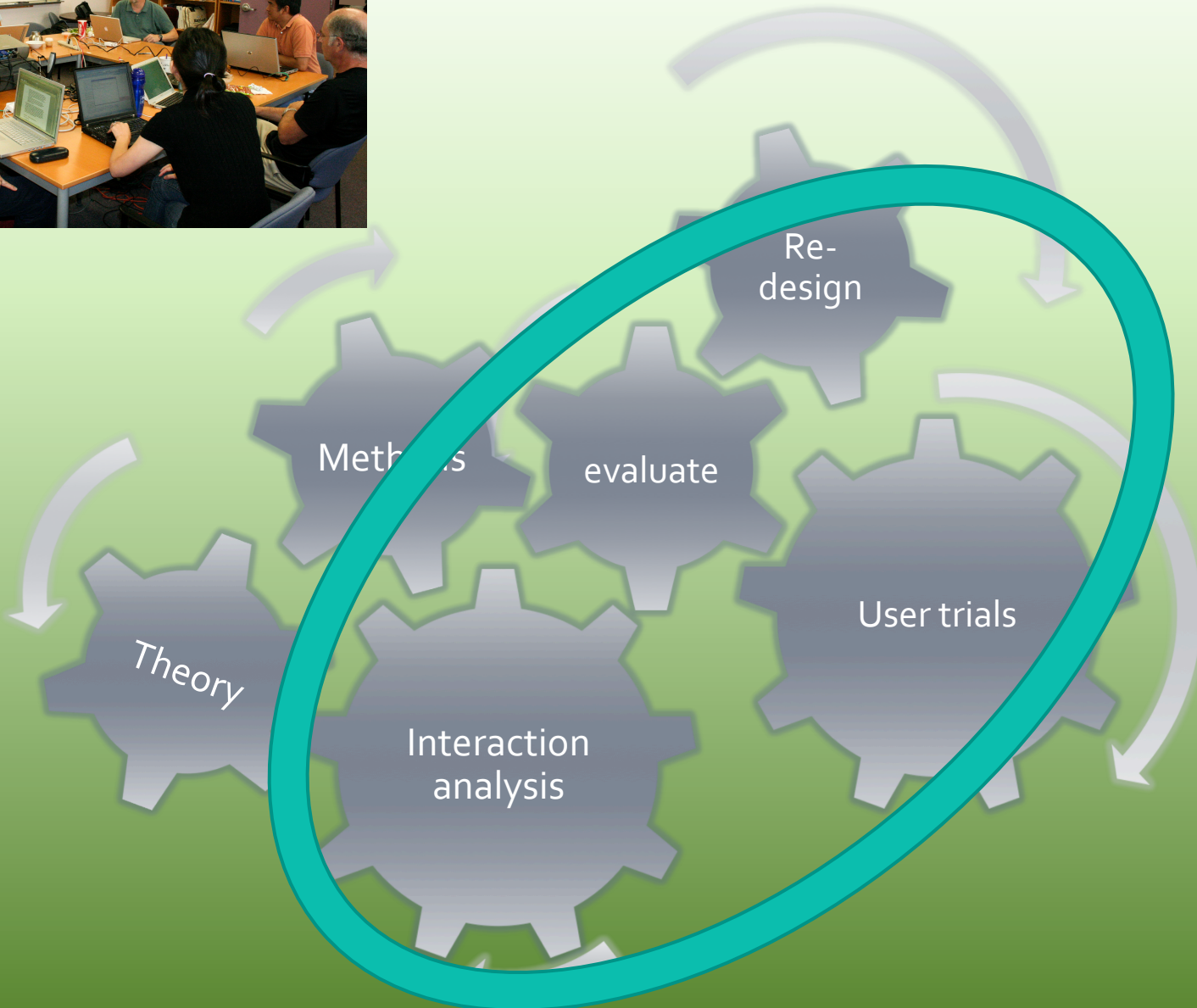
# Math Education Design Criteria

## Guiding development of skills & tacit understanding

- Visualization
- Exploration
- Dependencies
- Multiple Representations
- Concrete & Abstract
- Math Talk

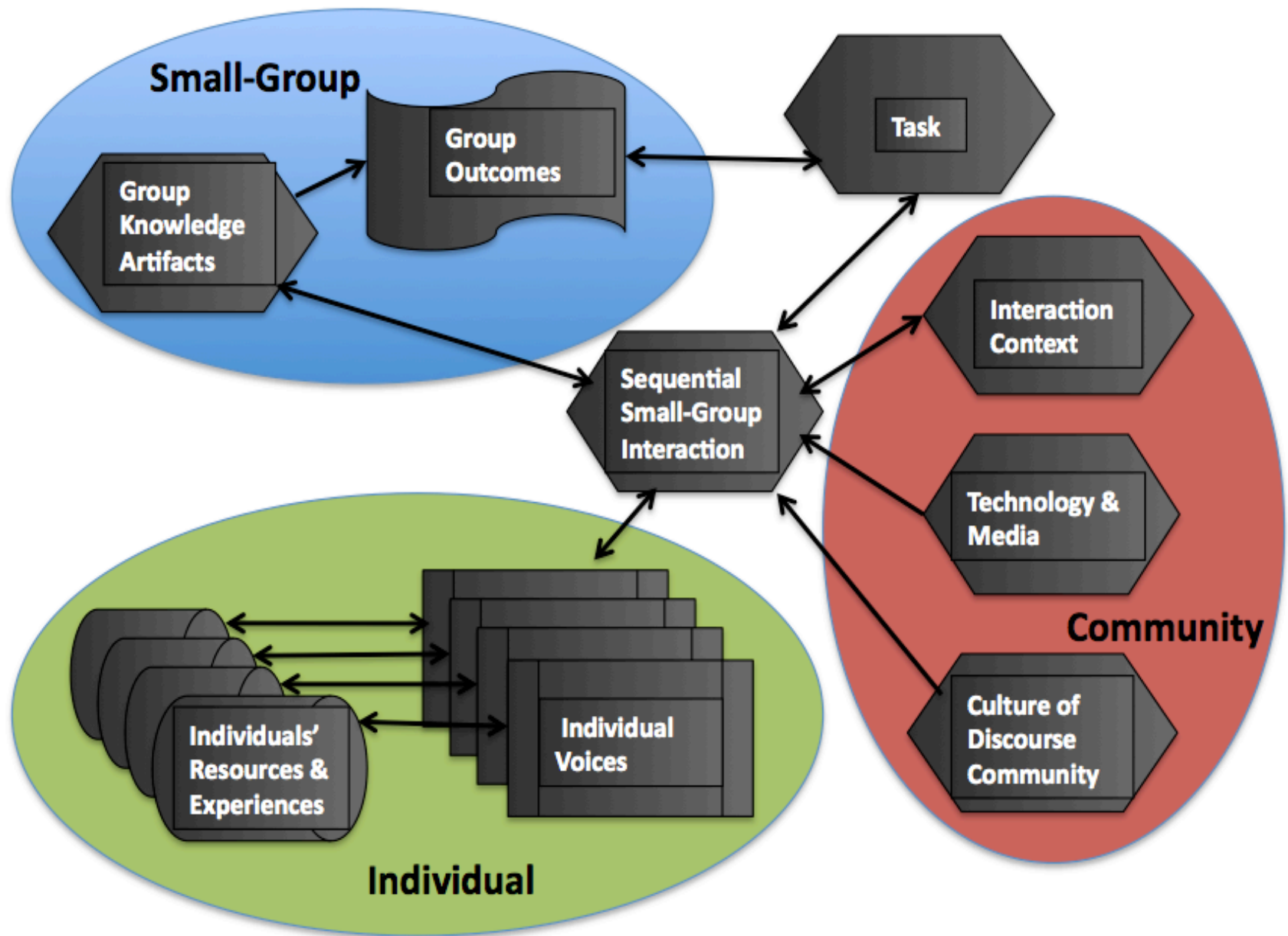


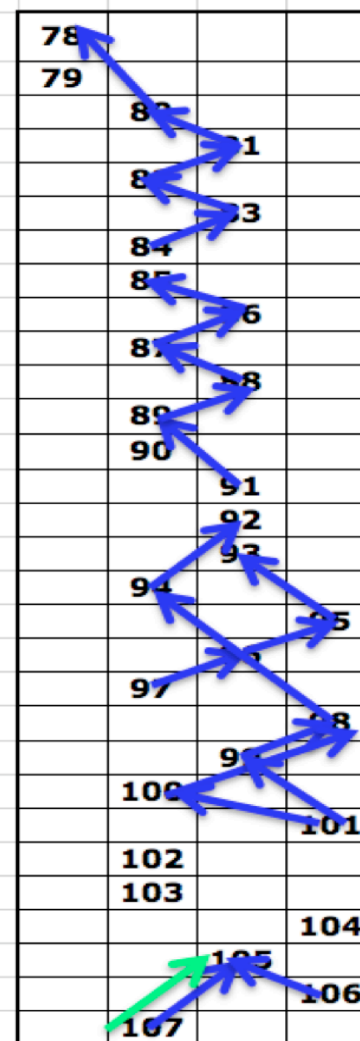
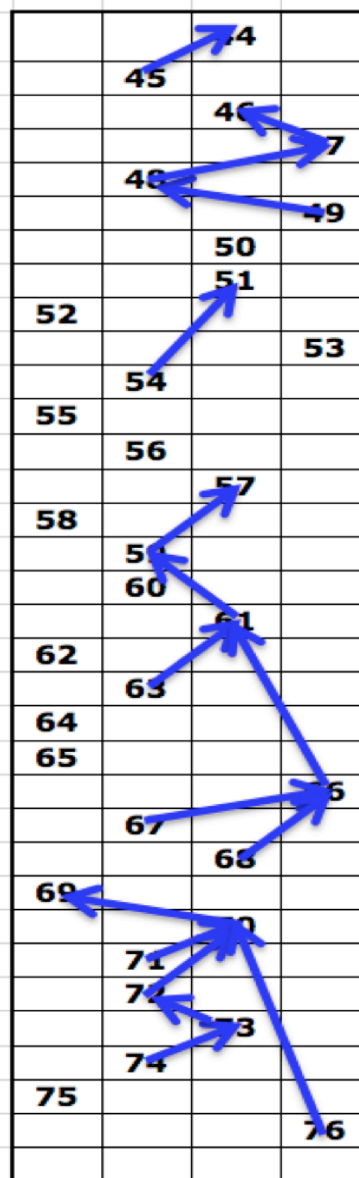
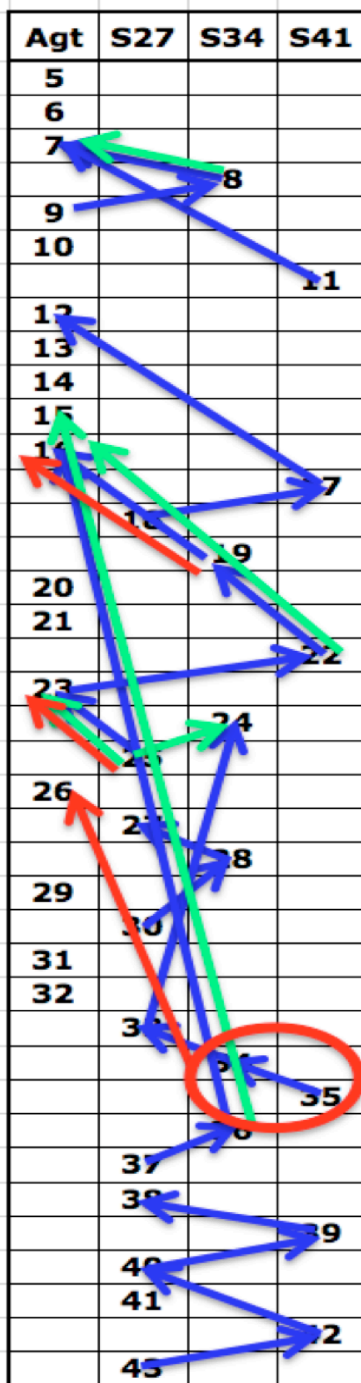
# Design-Based Research






## **7. Conversation Analysis: for Formative Evaluation**

- **Careful micro-analysis of discourse of users engaged in computer-mediated tasks**
- **From complete chat logs of interaction**
- **And replaying of technology usage**
- **Analysis of group interaction and problem-solving processes**
- **Based on visible evidence, not inferences about mental events or reliance on self reporting**





-  Threading of responses
-  Mediation of accountable talk
-  Content uptake

# Group cognition in math: The longer-sequence structure

- Log 1. Open the topic**
- Log 2. Decide to start**
- Log 3. Pick an approach**
- Log 4. Identify the pattern**
- Log 5. Seek the equation**
- Log 6. Negotiate the solution**
- Log 7. Check cases**
- Log 8. Confirm the solution**
- Log 9. Present a formal solution**
- Log 10. Close the topic**

## 8. Philosophic Issues

- **How can dynamic-math support enhance geometric cognition by individuals, small groups and classrooms?**
- **How can it integrate:**
  - **Rational planning of geometry, dependencies, software functions, scripts, curriculum, software agents**
  - **Normative social practices of collaborative learning, accountable talk, human motivation, undistorted communication**

- **Face-to-face with computer simulation:  
Seeing an artifact as meaningfully structured**



Stahl, 2006, *Group Cognition*, p. 242

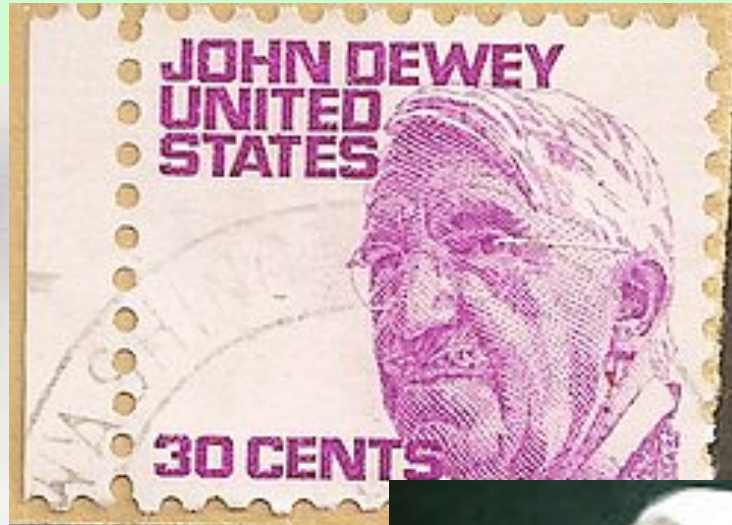


- **Geometry is the paradigm of purposive-rational calculation. The structure of the classic proof is:**
  - **Statement of conjecture to be proven**
  - **Construction of deduction**
  - **Restatement of that which was to be proven**
- **This is the general form of planning:**
  - **Goal, implementation, reflection checking on completion**
- **This is also the form of computer programming:**
  - **Requirements, programming, testing**
- **Like computer programming, *dynamic-geometry* construction is a matter of assembling a (ideally optimal) set of constraints among *dependent* objects to achieve the desired realization.**

**Group geometric cognition can integrate purposive-rational thinking and social-normative interaction. According to Habermas, there is a duality of the logic of efficient work and the logic of social interaction. By integrating geometric construction with collaborative discourse, we combine these.**

**The proper balance is the precondition of democracy.**

# Vygotsky, Dewey, Habermas



- **Classical geometry epitomized a literate world-view, dominated by factual propositions and fixed truths. This logical, deductive, abstract view began with the early geometricians, was extended by Socrates, Plato and Aristotle, was codified by Euclid and was rigidified by Roman thought.**
- **Dynamic geometry can translate fixed views of formal truths into exploratory, temporal, human-constructed systems of visible dependencies. Truths are seen to be socially constructed, understandable and modifiable. Proofs are not eternal and God-given; rather, students can construct systems of dependencies with designed consequences.**

# Conclusion

- **Euclidean geometry pointed the way for rational thinking in literate Western society, culminating in CS and AI**
- **A CSCCL approach to dynamic geometry learning can integrate such rational thinking with interpersonal understanding and can show the socially constructed and negotiable nature of rationality**
- **But successful implementation and deployment require meeting a complex and ill-defined set of diverse challenges**

*For Further Information:*

- **“*Group Cognition*” (2006, MIT Press)**
- **“*Studying Virtual Math Teams*” (2009, Springer)**
- ***Gerry Stahl’s e-Library* (collections of papers free for iPad, Kindle, PDF or low-cost print-on-demand): [GerryStahl.net/elibrary](http://GerryStahl.net/elibrary)**
  
- **These slides: [GerryStahl.net/pub/rutgers2012.ppt.pdf](http://GerryStahl.net/pub/rutgers2012.ppt.pdf)**

