# Group Cognition in Online Collaborative Math Problem Solving

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*This is a case study of online collaboration on an algebra problem. It adapts the methodology of conversation analysis to quasi-synchronous, text-based chat room technology. The analysis is conducted within the context of a design-based research effort, so a primary goal is to identify technological barriers caused by standard chat technology with an eye to designing a more appropriate and supportive online collaborative learning environment.*

*"Group cognition" is a theoretical framework in which cognitive processes are identified as resulting from the dynamic interaction of multiple personal interpretive perspectives within contexts of group discourse and collaboration. The analysis is conducted within a theoretical framework that focuses attention on the small group unit of analysis as the site of problem-solving agency, rather than on cognitive processes of the individual participants. The analysis results in the identification of interactive methods of "doing mathematics" as a group. This, in turn, reflects back on the theoretical framework and refines the notion of group cognition.*

*The analysis aims to motivate the following theoretical, methodological and design-based claims:*

- *The discourse displays elements of mathematical understanding, problem-solving strategies and logical rationality by the group that parallel those of individual students.*
- *Interaction among the student participants can be conceptualized as an instance of "group cognition."*
- *Excerpts of online collaborative math problem solving can productively be analyzed at the small group unit of analysis.*
- *The methodology of conversation analysis can effectively be adapted to interpret text-based online interaction.*
- *Group cognition displays the potential to achieve more than the individual participants seem capable of accomplishing on their own, but also displays interactional problems that prevent the group from achieving its full problem-solving potential.*
- *Conclusions can be drawn from such an analysis that are relevant to the design of improved computer-supported collaborative learning environments.*

## Introduction

A micro-analysis is conducted of a three-and-a-half minute long excerpt from an online interaction. The interaction took place among three students participating in the Virtual Math Teams service of the Math Forum. Conversation analysis of the interaction highlights various methods that the group of students engages in—both mathematical and social interaction methods. It identifies interactions through which the group members constitute the group as a problem-solving agent, define individual roles within the group, establish the style of communication, define their individual and group identities, articulate the problem, suggest math strategies, make mathematical proposals and negotiate math knowledge. Close analysis of the dialogical work done by sequences of chat postings reveals complex social and mathematical moves that can be characterized as instances of group cognition.

In the chat log, members' postings constitute the group discourse as such and orient to it as salient. For instance, individuals make proposals to be shared by the group. These proposals are often explicitly presented as the individual's personal opinion, but acceptance by another group member makes them part of the shared flow of considerations and sets them up for being built upon by anyone in the group. Subsequent postings reference them, identifying them as integral to the group discourse.

## Doing mathematics together online

Computers offer many opportunities for innovation in education. One of the major avenues is by supporting the building of collaborative knowledge (Stahl, 2006). For instance, it is now possible for students around the world to work together on challenging math problems. Through online discussion, they can share problem-solving experiences and gain fluency in communicating mathematically.

In a research project at the Math Forum @ Drexel (http://mathforum.org), we have begun to invite middle school students to participate in online chats about interesting problems in beginning algebra and geometry. The following problem, discussed in the example in this paper, is typical:

> If two equilateral triangles have edge-lengths of 9 cubits and 12 cubits, what is the edge-length of the equilateral triangle whose area is equal to the sum of the areas of the other two?

We rely on a variety of methods from the learning sciences to guide our research and to analyze the results of our trials. In particular, we use *conversation analysis* (Pomerantz & Fehr, 1991; Psathas, 1995; Sacks, Schegloff, & Jefferson, 1974; Sacks, 1992; ten Have, 1999) to interpret the interactions that take place in the student chats. In this paper, we adapt the findings of conversation analysis to math chats and develop a specific form of adjacency pairs that seem to be important for math chats. Before presenting this, it may be useful to describe briefly how the notion of adjacency pairs differs from naïve conceptions of conversation.

There is a widespread common-sense or folk-theory (Bereiter, 2002; Dennett, 1991) view of conversation as the exchange of propositions (Shannon & Weaver, 1949). This view was refined and formalized by logicians and cognitive scientists as involving verbal expression in meaningful statements by individuals, based on their internal mental representations. Speech served to transfer meanings from the mind of a speaker to the mind of a listener, who then interpreted the expressed message. Following Wittgenstein (1953) in critiquing this view, speech act theory (Austin, 1952; Searle, 1969) argued that the utterances spoken by individuals were ways of acting in the world, and were meaningful in terms of what they accomplished through their use and effects. Of course, the expression, transmission and interpretation of meaning by individuals can be problematic, and people frequently have to do some interactional work in order to re-establish a shared understanding. The construction of common ground has been seen as the attempt to coordinate agreement between individual understandings (Clark & Brennan, 1991).

Conversation analysis takes a different view of conversation. It looks at how interactional mechanisms, like the use of *adjacency pairs* (Duranti, 1998; Schegloff, 1991), co-construct inter-subjectivity. Adjacency pairs are common sequences of utterances by different people—such as mutual greetings or question/answer interchanges—that form a meaningful speech act spanning multiple utterances that cannot be attributed to an individual or to the expression of mental states. We are interested in what kinds of adjacency pairs are typical for math chats.

Online math chats differ from ordinary informal conversation in a number of ways. They are focused on the task of solving a specific problem and they take place within a somewhat formal institutional setting. They involve the *doing* of mathematics (Livingston, 1986). And, of course, they are computer-mediated rather than being face-to-face. The approach of conversation analysis is based on ethnomethodology (Garfinkel, 1967), which involves the study of the methods that people use to accomplish what they are doing. So, we are interested in working out the methods that are used by students in online math chats. In this paper we discuss a particular method of collaboration in math chats that we have elsewhere called *exploratory participation*: participants engage each other in the conjoint discovery and production of both the problem and possible solutions (Zemel, Xhafa, & Stahl, 2005).

The medium of online chat has its own peculiarities. Most importantly, it is a text-based medium, where interaction takes place by the sequential response of brief texts to each other (Livingston, 1995; Zemel, 2005). As a quasi-synchronous medium (Garcia & Jacobs, 1999), chat causes confusion because several people can be typing at once and their texts can appear in an order that obscures what they are responding to. Furthermore, under time pressure to submit their texts so that they will appear near what they are responding to, some chat participants break their messages into several short texts. Because of these peculiarities of chat, it is necessary for researchers to carefully reconstruct the intended threading of texts that respond to each other before attempting to interpret the flow of interaction (Cakir *et al.*, 2005; Strijbos & Stahl, 2005).

## Math proposal adjacency pairs

In order to begin to analyze the methods that students use in math chats, we take a close look at an excerpt from an actual chat. Figure 1 shows an excerpt from near the beginning of the log of one of our first online collaborative math problem-solving sessions. Three students—named Avr, Sup and Pin—have just entered the chat room, said hello to each other and read the problem involving three triangles.

The first thing to notice here is a pattern of proposals, discussions and acceptances similar to what takes place in face-to-face discourse. Proposals about steps in solving the math problem are made by Avr in lines 1, 3, 8, 17 and by Pin in lines 20, 27. These proposals are each affirmed by someone else in lines 2, 6, 10, 19, 22, 28, respectively.

To avoid chat confusion, note that line 21 responds to line 19, while line 22 responds to line 20. The timestamps show that lines 20 and 21 effectively overlapped each other chronologically: Avr was typing line 21 before she saw line 20. Similarly, lines 24 and the following were

1. Avr (8:21:46 PM): Okay, I think we should start with the formula for the area of a triangle
2. Sup (8:22:17 PM): ok
3. Avr (8:22:28 PM): A = 1/2bh
4. Avr (8:22:31 PM): I believe
5. pin (8:22:35 PM): yes
6. pin (8:22:37 PM): i concue
7. pin (8:22:39 PM): concur*
8. Avr (8:22:42 PM): then find the area of each triangle
9. Avr (8:22:54 PM): oh, wait
10. Sup (8:23:03 PM): the base and heigth are 9 and 12 right?
11. Avr (8:23:11 PM): no
12. Sup (8:23:16 PM): o
13. Avr (8:23:16 PM): that's two separate triangles
14. Sup (8:23:19 PM): ooo
15. Sup (8:23:20 PM): ok
16. Avr (8:23:21 PM): right
17. Avr (8:23:27 PM): i think we have to figure out the height by ourselves
18. Avr (8:23:29 PM): if possible
19. pin (8:24:05 PM): i know how
20. pin (8:24:09 PM): draw the altitude'
21. Avr (8:24:09 PM): how?
22. Avr (8:24:15 PM): right
23. Sup (8:24:19 PM): proportions?
24. Avr (8:24:19 PM): this is frustrating
25. Avr (8:24:22 PM): I don't have enough paper
26. pin (8:24:43 PM): i think i got it
27. pin (8:24:54 PM): its a 30/60/90 triangle
28. Avr (8:25:06 PM): I see
29. pin (8:25:12 PM): so whats the formula

Figure 1. Excerpt of 3½ minutes from a one-hour chat log. Three students chat about a geometry problem. Line numbers have been added and screen-names anonymized; otherwise the transcript is identical to what the participants saw on their screens.

responses to line 20, not line 23. We will correct for these confusions in Figure 2, which reproduces a key passage in this excerpt.

In Figure 1, we see several examples of a three step pattern:

1. A proposal is made by an individual for the group to work on: “I think we should ….”
2. An acceptance is made on behalf of the group: “Ok,” “right”
3. There is an elaboration of the proposal by members of the group. The proposed work is begun, often with a secondary proposal for the first sub-step.

This suggests that collaborative problem-solving of mathematics may often involve a particular form of adjacency pair. We will call this a *math proposal adjacency pair*.

Many adjacency pairs allow for insertion of other pairs between the two parts of the original pair, delaying completion of the pair. For instance, a question/answer pair may be interrupted by utterances seeking clarification of the question; the clarification interaction may itself consist of question/answer pairs, possibly with their own clarifications—this may continue recursively. With math proposal adjacency pairs, the subsidiary pairs seem to come after the completion of the original pair, in the form of secondary proposals, questions or explanations that start to do the work that was proposed in the original pair.

Proposals seem to lead to some kind of further mathematical work as a response to carrying out what was proposed. Often—as seen in the current example—that work consists of making further proposals. It is striking that the proposed work is not begun until there is agreement with the proposal. This may represent consent by the group as a whole to pursue the proposed line of work. Of course, it is not so clear in the current example, where there are only three participants and the interaction often seems to take place primarily between pairs of participants. As confirmed by other chat examples, however, the proposal generally seems to be addressed to the whole group and opens the floor for participants other than the proposer to respond. The use of "we" in "we should" or "we have to" (stated or implied) constitutes the multiple participants as a plural subject, an effective unified group (Lerner, 1993). Any one other than the proposer may respond on behalf of the group.

Moreover, there seems to be what in conversation analysis is called an interactional *preference* (Schegloff, Jefferson, & Sacks, 1977) for acceptance of the proposal. That is, if one accepts a proposal, it suffices to briefly indicate agreement: "ok." If one wants to reject a proposal, then one has to account for this response by giving reasons.

We would like to characterize in more detail the method of making math proposal adjacency pairs. Often, the nature of an interactional method is seen most clearly when it is breached (Garfinkel, 1967). Methods are generally taken for granted by people; they are not made visible or conducted consciously. It is only when there is a *breakdown* (Heidegger, 1927/1996) in the smooth, tacit performance of a method that people focus on its characteristics in order to overcome the breakdown. The normally transparent method becomes visible in its breach. We can interpret Sup's posting in line 23 as a *failed proposal*. Given the mathematics of the triangle problem, a proposal like Sup's related to proportionality might have been fruitful. However, in this chat, line 23 was effectively ignored by the group. While its character as a failed proposal did not become visible to the participants, it can become clear to us by comparing it to successful proposals in the same chat and by reflecting on its situation in the chat in order to ask why it was not a successful proposal.

## A failed proposal

Let us look at line 23 in its immediate interactional context in Figure 2. We can distinguish a number of ways in which it differed from successful math proposals that solicited responses and formed math proposal adjacency pairs.

17, 18. Avr (8:23: 29 PM): i think we have to figure out the height by ourselves … if possible
19. pin (8:24:05 PM): i know how
21. Avr (8:24:09 PM): how?
20. pin (8:24:09 PM): draw the altitude'
22. Avr (8:24:15 PM): right
24. Avr (8:24:19 PM): this is frustrating
23. Sup (8:24:19 PM): proportions?
24. Avr (8:24:22 PM): I don't have enough paper

Figure 2. Part of the chat log excerpt in figure 1, with order revised for threading.

(a) All the other proposals (1, 3, 8, 17, 20, 27) were stated in relatively complete sentences. Additionally, some of the proposals were introduced with a phrase to indicate that they were the speaker's proposal (1. "I think we should …," 17. "I think we have to …," 20. "i know how …" and 27. "i think i got it …"). The exceptions to these were simply continuations of previous proposals: line 3 provided the formula proposed in line 1 and line 8 proposed to "then" use that formula. Line 23, by contrast, provided a single word with a question mark. There was no syntactic context (other than the question mark) within the line for interpreting that word and there was no reference to semantic context outside of the line. Line 23 did not respond in any clear way to a previous line and did not provide any alternative reference to a context in the original problem statement or elsewhere. For instance, Sup could have said, "I think we should compute the proportion of the height to the base of those equilateral triangles."

(b) The timing of line 23 was particularly unfortunate. It exactly overlapped a line from Avr. Since Avr had been setting the pace for group problem solving during this part of the chat, the fact that she was involved in following a different line of inquiry spelled death for an alternative proposal at the time of line 23. Pin either seemed to be continuing on his own thread without acknowledging anyone else at this point, or else he was responding too late to previous postings. So a part of the problem for Sup was that there was little sense of a coherent group process—and what sense there was did not include him. If he was acting as part of the group process, for instance posing a question in reaction to Pin and in parallel to Avr, he was not doing a good job of it and so his contribution was ignored in the group process. It is true that a possible advantage of text-based interaction like chat over face-to-face interaction is that there may be a broader time window for responding to previous contributions. In face-to-face conversation, turn-taking rules may define appropriate turns for response that expire in a fraction of a second as the conversation moves on. In computer-based chat, the turn-taking sequence is more open. However, even here if one is responding to a posting that is several lines away, it is important to make explicit somehow to what one is responding. He could have said, "I know another way to find the height – using proportions." Sup's posting does not do anything like that; it relies purely upon sequential timing to establish its context, and that fails in this case.

(c) Sup's posting 23 came right after Pin's proposal 20: "draw the altitude." Avr had responded to this with 22 ("right") but Pin seems to have ignored that. Pin's proposal had opened up work to be done and both Avr and Pin responded after line 23 with contributions to this work. So Sup's proposal came in the middle of an ongoing line of work without relating to it. In conversational terms, he made a proposal when it was not time to make a proposal. It is like trying to take a conversational turn when there is not a pause that creates a turn-taking opportunity. Now, it is possible—especially in chat—to introduce a new proposal at any time. However, to do so effectively, one must make a special effort to bring the on-going work to a temporary halt and to present one's new proposal as an alternative. Simply saying "proportions?" will not do it. Sup could have said, "Instead of drawing the altitude, let's use propostions to find it."

(d) To get a response to a proposal, one must elicit at least an affirmation or recognition. Line 23 does not really solicit a response. For instance, Avr's question, 21: "how?" called for an answer—that was given by Pin in line 20, which actually appeared in the chat window just prior to the question and with the same time stamp. But Sup's posting does not call for a specific kind of answer. Even Sup's own previous proposal in line 10 ended with "right?"—requiring agreement or disagreement. Line 10 elicited a clear response from Avr, line 11("no") followed by an exchange explaining why Sup's proposal was not right.

(e) Other proposals in the excerpt are successful in contributing to the collaborative knowledge building or group problem solving in that they open up a realm of work to be done. One can look at Avr's successive proposals on lines 1, 3, 8 and 17 as laying out a work strategy. This elicits a response from Sup trying to find values to substitute into the formula and from Pin trying to draw a graphical construction that will provide the values for the formula. Sup's proposal in line 23, however, neither calls for a response nor opens up a line of work. There is no request for a reaction from the rest of the group and the proposal is simply ignored. Since no one responded to Sup, he could have continued by doing some work on the proposal himself. He could have come back and made the proposal more explicit, reformulated it more strongly, taken a first step in working on it, or posed a specific question related to it. But he did not—at least not until much later—and the matter was lost.

(f) Another serious hurdle for Sup was his status in the group at this time. In lines 10 through 16, Sup had made a contribution that was taken as an indication that he did not have a strong grasp of the math problem. He offered the lengths of the two given triangles as the base and height of a single triangle (line 10). Avr immediately and flatly stated that he was wrong (line 11) and then proceeded to explain why he was wrong (line 13). When he agreed (line 15), Avr summarily dismissed him (line 16) and went on to make a new proposal that implied his approach was all wrong (lines 17 and 18). Then Pin, who had stayed out of the interchange, re-entered, claiming to know how to implement Avr's alternative proposal (lines19 and 20) and Avr confirmed that (line 22). Sup's legitimacy as a source of useful proposals had been totally destroyed at precisely the point just before he made his ineffective proposal. Less than two minutes later, Sup tries again to make a contribution, but realizes himself that what he says is wrong. His faulty contributions confirm repeatedly that he is a drag on the group effort. He makes several more unhelpful comments later and then drops out of the discourse for most of the remaining chat.

The weaknesses of line 23 as a proposal suggest some characteristics for successful proposals: (a) a clear semantic and syntactic structure, (b) careful timing within the sequence of postings, (c) a firm interruption of any other flow of discussion, (d) the elicitation of a response, (e) the specification of work to be done and (f) a history of helpful contributions. In addition, there are other interaction characteristics and mathematical requirements. For instance, the level of mathematical background knowledge assumed in a proposal must be compatible with the expertise of the participants, and the computational methods must correspond with their training. Other characteristics will become visible in other examples of chats.

At this time, the notion of math proposal adjacency pairs is just a preliminary proposal based on a single chat log excerpt. It calls for extensive conversation analysis of a corpus of logs of collaborative online math problem solving to establish whether this is a fruitful way of interpreting the data. If it turns out to be a useful approach, then it will be important to determine what interactional methods of producing such proposals are effective (or not) in fostering successful knowledge building and group cognition. An understanding of these methods can guide the design of activity structures for collaborative math. As we are collecting a corpus of chat logs, we are evolving computer support through iterative trials and analyses.

## Designing computer support

If the failure of Sup's proposal about proportions is considered deleterious to the collaborative knowledge building around the triangles problem, then what are the implications of this for the design of educational computer-based environments? One response would be to help students like Sup formulate stronger proposals. Presumably, giving him positive experiences of interacting with students like Avr and Pin, who are more skilled in chat proposal making, would provide Sup with models and examples from which he can learn—assuming that he perseveres.

Another approach to the problem would be to build functionality into the software and structures into the activity that scaffold the ability of weak proposals to survive. As students like Sup experience success with their proposals, they may become more aware of what it takes to make a strong proposal. (Livingston, 1986)

Professional mathematicians rely heavily upon inscription: the use of specialized notation, the inclusion of explicit statements of all deductive steps and the format of the formal proof to support the discussion of math proposals—whether on an informal whiteboard, a university blackboard or in an academic journal. Everything that is to be indexed in the discussion is labeled unambiguously. To avoid ellipsis, theorems are stated explicitly, with all conditions and dependencies named. The projection of what is to be proven is encapsulated in the form of the proof, which starts with the givens and concludes with what is proven. Perhaps most importantly, proposals for how to proceed are listed in the proof itself as theorems, lemmas, etc.—organized sequentially.

One could imagine a chat system supplemented with a window containing an informal list of proposals analogous to the steps of a proof. After Sup's proposal, the list might look like Figure 3. When Sup made a proposal in the chat, he would enter a statement of it in the proof window in logical sequence. He could cross out his own proposal when he felt it had been convincingly argued against by the group.

- Given: 2 equilateral triangles of edge-length 9 cubits and 12 cubits

- formula for a triangle: A = 1/2bh
- Area of each triangle = ?
- ~~b, h = 9, 12~~
- draw the altitude
- use proportions for ratio of altitude to base

- Find: The edge-length of the equilateral triangle whose area is equal to the sum of the areas of the other two triangles

Figure 3. A list of proposals

The idea is that important proposals that were made would be retained in a visible way and be shared by the group. Of course, there are many design questions and options for doing something like this. Above all, would students understand this functionality and would they use it? The design indicated in Figure 3 is only meant to be suggestive.

Another useful tool for group mathematics would be a shared drawing area. In the chat environment used by Sup, Pin and Avr, there was no shared drawing, but a student could create a drawing and send it to the others. Pin did this twelve minutes after the part of the interaction shown in the excerpt. Before the drawing was shared, much time was lost due to confusion about references to triangles and vertices. For math problems involving geometric figures, it is clearly important to be able to share drawings easily and quickly. Again, there are many design issues, such as how to keep track of who drew what, who is allowed to erase, how to point to items in the drawing and how to capture a record of the graphical interactions in coordination with the text chatting.

## Conclusions

Some methods of contributing proposals are effective, others are not. We can identify several problems with a particular failed proposal in the excerpt: (a) it lacks semantic clarity and has a weak syntactic structure; (b) its timing in being followed immediately by a stronger proposal is unfortunate; (c) the proposer has a history of distracting from the flow of the group problem-solving rather than contributing to it; (d) the proposer lacks alignment with the group focus of discussion. Due to the quasi-synchronous nature of the chat medium, there is a competition to time and structure postings and social relationships so as to increase the likelihood of a posting being taken up into the group discourse; this particular posting faired poorly in this competition.

Methods for effectively doing math collaboratively integrate skills of text chatting (typing, abbreviating, posting quickly, referencing other postings), socializing (establishing roles, legitimacy, social relations), formulating proposals mathematically (proper level of abstraction, use of symbols, strategies, explanations) and interacting (making effective proposals, leading the group discussion, eliciting desired responses). Some of the math methods are simultaneously methods of interacting socially and constituting group identity (e.g., who can play what role, whose proposals will be taken seriously). Many common methods of doing math appear in the chat: selecting levels of abstraction, using formulas, substituting numeric values.

Online methods tend to parallel face-to-face methods, but technical mediation makes a difference, making some things easier (like working in parallel) and other things harder (reconciling computational differences, repairing losses from overlap). Whether face-to-face or online, the flow of the discourse from one proposal to the next, from method to method and from problem to strategy to sub-problem to solution defines the logic of mathematical group discourse as a specific form of group cognition.

The mathematical discourse of the chat log follows quite closely the sequential format of a proof. Student proposals are strategic steps in a proof that would, if successfully completed, derive an answer to the problem from its givens. The accountability of each step means that the group cannot continue after a proposal is made until that proposal is accepted. Permissible responses to a proposal are to accept it because one recognizes it as legitimate, to reject it, or to question it. If one rejects it, then the rejecter is accountable for providing a convincing reason for rejection.

Considered at the small group unit of analysis, the group of students accomplished cognitive achievements. The group underwent cognitive change and pursued a math problem. It reflected upon and selected problem-solving strategies. It built knowledge and fashioned symbolic artifacts, including formulae. It thought as a group. Viewing its behavior as a thinking group constituted by interacting interpretive individuals, the analysis makes visible how shared meaning was constructed. It analyzes the group discourse and looks at issues of sequentiality, accountability, sociality and shared meaning-making through the negotiation and acceptance of proposals. Many of the questions concerning mathematics education and thinking that arise for individual students present themselves at the group unit of analysis as well.

If sequentiality and accountability are the hallmarks of high-order rational thought, then group discourse meets the criteria for being considered an important form of cognition. Moreover, there is reason to hope that computer-supported collaboration can produce high-order cognitive achievements that rival and ultimately surpass those of individuals. This is not to deny the problems that can arise in groups, that may even hold back individual accomplishments. Nor is it to claim that current technologies provide the required forms of support. But it does point to a potential that transcends the limitations of the individual human mind and allows people to think together. The analysis summarized here suggests several software features that would help to avoid the kinds of problems that arose in the studied chat and could foster the greater potential of online collaborative math problem solving: a separate list of proposals— perhaps structured in a proof format—could allow the group to periodically review what proposals were made, responded to, accepted, built upon; a shared whiteboard would allow diagrams to be shared and annotated; a threading mechanism might reduce chat confusion caused by overlapping postings hiding sequential relationships and references.

The notion of group cognition developed here may be considered a strong form of distributed cognition. It goes considerably beyond Norman’s (1993) argument that the individual mind extends outside the head to artifacts in the world as forms of external memory, like a reminder string on the finger. It is similar to Hutchins’ (1996) example of the cognition that steers a large ship being distributed across people, artifacts and procedures. The case analyzed here shows creative high-order problem-solving and mathematical knowledge building taking place as group cognition.

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