Co-constructed Narratives in Online, Collaborative Mathematics Problem-Solving

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Abstract. Our approach to the study of learning of mathematical problem-solving extends the notion of narrative learning environments to include the dynamics of collaborative dialogs and related emergent narratives. This perspective favours the conception of the dialogical aspects of interaction as shared achievements of co-participants and as central meaning-making procedures, based on our qualitative analysis of transcripts from online collaborative math problem-solving interactions. From these observations we attempt to establish a link between narrative learning environments and dialogical perspectives and explore relevant implications for the design of the Virtual Math Teams collaborative learning environment.

Introduction

Research in the field of Narrative Learning Environments (NLEs) is concerned with questions such as how to characterize the contribution of narratives and narration to learning, and how to use knowledge of narratives to design learning environments. As part of the Virtual Math Teams research project (mathforum.org/wiki/VMT/), we have investigated talk-in interaction within the context of online collaborative mathematical problem solving and have found similarities between the narrative approach and a dialogical perspective on sense-making and interaction. Therefore, we propose to extend the idea of NLEs to encompass collaborative learning environments which, in addition to using narrative structures, also offer the possibility of joint participation and interaction with a diverse set of linguistic and extra-linguistic objects (e.g. mathematical objects and their derivative properties).

1. Narrative Learning Environments (NLE)

Research and development on NLEs explores intelligent learning environments where "narrative is approached and applied" to support learning and the construction of meaning [1]. As such, NLEs build and extend the long held interest in AI for the structuring power that narratives and narration exert on cognition (e.g. [2], [3]). A narrative learning environment is expected to promote three main kinds of activities for learners: ***co-construction*** (the ability to participate in the construction of a narrative), ***exploration*** (engagement in active exploration of the learning tasks, following a narrative approach and trying to understand and reason about an environment and its elements), and ***reflection*** (consequent analysis of what happened within the learning session). To date, research on NLEs has concentrated on the analysis and use of narrative elements such as virtual storytelling, interactive drama, and participatory narratives, mostly within the context of literacy development and language learning (e.g. [4]) and the exploration of points of intersection between AI, educational technologies and narratology. Generally, this approach treats narrative as an object and a fixed structure of interaction.

2. The Dialogical Perspective on Learning

The *dialogical* perspective pursues meaning-making as an interactional achievement of co-participants, rather than as a fixed property of linguistic objects. Theorists of the dialogical aspect of language and meaning (e.g. Bakhtin [5,6,7], Harré [8], Sacks [9], Schegloff [10]) point to the features of talk as action, and of shared action in itself, as core processes of human meaning-making. These socially shared procedures might point to general sense-making strategies with applicability to particular domains (e.g. fictional storytelling, or math problem solving).

As Wegerif stresses [11], the dialogical perspective on learning attempts to access the creative space of "the interanimation of more than one perspective" that emerges in the dynamics of interactive narratives and collaborative meaning-making. What is common to both narration and collaborative dialogues is *the discourse*; the emergent coherence of the sequencing, projection and referencing of utterances generated within meaning making shared with others and with meaningful artefacts [14]. As such, narration and dialogues as interactive events open up opportunities for participants to engage in *co-construction* of possible worlds, to *explore* them in dialogue, and to *reflect* together on the experience. Participation and engagement are then central to the learning processes conceived as a socio-cultural practice [12], speech and interaction being extremely important mediators in this process. Furthermore, as Vygotsky states in his concept of the Zone of Proximal Development [13], children's potential learning abilities are especially accessible within their interactions with others, a fact that adds practical and theoretical support to the use of collaborative learning.

Participatory or interactive narratives offer opportunities for co-construction of meaning precisely based on the dialogic principle of interactivity resulting on an intermix of classical narrative structures and other frameworks of shared participation, a point we seek to illustrate within the domain of collaborative mathematical problem solving. In summary, we propose to connect narrative learning environments and collaborative learning environments by virtue of their common concern for the role of discourse and interaction in learning and its potential support via designed artefacts.

3. Collaborative Math Problem-solving: Co-construction, exploration and reflection

The Virtual Math Teams (VMT) research program investigates the innovative use of online collaborative environments to support effective K-12 mathematics learning as part of the research and development activities of the Math Forum (mathforum.org) at Drexel University. VMT extends the Math Forum's "Problem of the Week (PoW)" service by bringing together groups of 3 to 5 students in grades 6^{th} to 11^{th} to collaborate online in discussing and solving non-routine mathematical problems. Currently, participants interact using a computer-supported collaborative learning environment which combines quasi-synchronous text-based communication (e.g. chat) and a shared whiteboard among other interaction tools. At the core of VMT research is the premise that primarily, group knowledge arises in discourse and is preserved in linguistic artifacts whose meaning is co-constructed within group processes ([15]). Key issues addressed by the VMT include the design challenge of structuring the online collaborative experience in a meaningful and engaging way, and the methodological challenge of finding appropriate methodological approaches to study the forms of collaboration and reasoning that take place.

3.1. Data sources and Methodology

As part of the initial exploratory phase of research, the VMT offered more than 20, one to one and a half hour online sessions in which small groups of students used AOL Instant Messenger© technologies to interact and collaboratively attempt to solve a mathematical problem provided. Through these events we have collected a corpus of chat transcripts that constitute our main source of data. The VMT implements a multidisciplinary approach to the analysis of these transcripts, which integrates quantitative modelling of students' interactions as well as ethnographic and conversation analytical studies of collaborative problem solving. A coding scheme has been developed for the quantitative analysis of the sequential organization of postings recorded in a chat log. This coding scheme includes nine content and threading dimensions (e.g. conversation, problem-solving content and threads) of each chat line (see [16] and [17] for further discussion). The analysis presented here represents an example of the complementary ethnographic analysis of these same data.

Several researchers have explored the interdependencies between discourse, narratives, and mathematics in general (Cocking & Chipman [18]) as well as the role of narratives in mathematics learning (Burton, [19],[20]). Our qualitative analysis of collaborative mathematical problem-solving, based on the conversation analysis (e.g. [9],[10]), seeks to understand the methods that co-participants use to organize their shared interactions. The object of inquiry in conversation analysis (CA) is not exclusively conversation as a linguistic entity, but rather talk and social interaction. The interest of CA is "with the local production of [social] order and with 'members' methods' for doing so" ([21], p.19). Using the methods of CA, our analysis of transcripts of online collaborative problem-solving revealed, in particular instances, narrative elements—e.g. the emergence of a narrator and a narratee as well as structured sequences of events, that participants oriented to in their collaborative production of problem solutions.

3.2. Emergent Narrative Elements from Shared Participation.

The following analysis illustrates the ideas proposed by using data from one of the online transcripts of a VMT collaborative problem-solving session. The session presented here has three main participants, SKI, YAG and GOH. "*Press for Time*" is the problem assigned for the session:

> *The Rational Reader, a popular daily newspaper, has to be printed by 5 a.m. so that it can be distributed. Late one night, a major story broke and the front page had to be rewritten, which delayed the start of the printing process until 3 a.m. To try to get the printing done on time, the Reader used both their new printing press and their old one. The new press is three times as fast as the old one, and with both of them running, the printing was finished exactly on time. How long does it take to print a normal edition of the paper using only the new press?*

From the transcript we can infer that, at least two of the participants (SKI and YAG) had worked on the problem prior to their joint participation in the online collaborative session and, as a result, the group members orient themselves to an "expository" mode of interaction in which reports of "ways" to solve the problem are offered in the form of story-like narrations. The process of narrating, the constituting of narrator and narratee voices as well as the resulting narrative, however, are to be considered as an interactional achievement of all the participants. On the other hand, an interactive narrative within the speech genre of mathematics problem solving (in the Bakhtinian sense [7]), has specific characteristics that govern the space of possible transformations of the different "events" of the narrative being produced. The following excerpts allow us to illustrate these ideas:

1. 7:26:10 SKI i started and solved	57. 7:29:38 GOH how come 1/x and 1/y

		with a system	
2.	7:26:12	SKI	of equations
3.	7:26:14	YAG	let SKI explain...
4.	7:26:24	SKI	lets just say x is the time for the old machine and y is for the new
5.	7:26:29	GOH	ok
6.	7:26:35	SKI	our first equation is like this
7.	7:26:41	SKI	if we atke the recip of x
8.	7:26:45	YAG	*choughSHOWOFFchough*
9.	7:26:55	YAG	:P
10.	7:26:57	YAG	:-D
11.	7:26:59	SKI	thats how much of the job the old one does in one hour
12.	7:27:02	YAG	yep
13.	7:27:12	SKI	and the reciprocal of y is how much of the job the new one does in one hour
14.	7:27:16	YAG	recip [of] y is the new one
15.	7:27:24	SKI	ok
16.	7:27:29	SKI	recip=reciprocal
17.	7:27:33	SKI	anyways
18.	7:27:38	YAG	and, recip y+ recip x = 1/2
19.	7:27:43	SKI	we add 1/x and 1/y
20.	7:27:48	SKI	ya
21.	7:27:50	SKI	what YAG said
22.	7:27:53	SKI	1/2
23.	7:27:56	YAG	in hours and fraction of work
24.	7:28:04	YAG	needed to be done
25.	7:28:05	SKI	cuz they together get half the job done in one hour
26.	7:28:09	YAG	:P
27.	7:28:13	SKI	are u getting our first equation?
...			

		added equal 1/2?	
58.	7:29:42	SKI	ok
59.	7:29:47	YAG	ummm
60.	7:29:50	YAG	pure luck!
61.	7:29:51	SKI	1/x is how much the old one does in one hour
62.	7:29:57	GOH	right.
63.	7:29:58	SKI	how much of the job it does in an hour
64.	7:30:01	YAG	(frac of job done)
65.	7:30:03	SKI	1/y is for the new machine
66.	7:30:08	GOH	right
67.	7:30:11	SKI	add those up
68.	7:30:18	YAG	and since they do it together at 3-5
69.	7:30:20	SKI	thats how much of the job they do together in one hour
70.	7:30:22	YAG	it took 2 hrs
71.	7:30:25	SKI	ya
72.	7:30:29	SKI	listen to [YAG]
...			
84.	7:31:06	SKI	the whole job took 2 hours
85.	7:31:14	YAG	with both machines
86.	7:31:19	SKI	so in one hour they did 1/2 of the job
87.	7:31:34	YAG	and in the 2nd hour they did the other half
88.	7:31:54	GOH	Okay, I got it. 1/2 is how much of the job they do together in one hour
89.	7:31:58	SKI	rite
90.	7:32:00	YAG	yepyepyep
91.	7:32:06	SKI	u know what x and y represent rite?
...			

As can be seen in these excerpts, even in this "expository" orientation, co-participants take active roles in co-constructing the explanation. Even though SKI initiates his story-like report with the form of a first person narrative ("*i started and solved with a system of equations"),* the shared narrative space of this interaction is constituted with YAG and GOH's uptake of SKI's narrator voice (lines 3 and 5) and their subsequent participation. SKI's narration seems to shift to the first person plural ("*our first equation is like this"*) and subsequently we can observe how SKI and YAG share the narrator role by completing each other postings or interjecting new ones (e.g. lines 23 and 25). SKI and YAG have, at this point, constituted themselves as a recognizable collectivity (Lerner [22]) oriented towards the task of producing an intelligible narrative explanation for GOH (e.g. line 27).

On the other hand, by virtue of the interactional nature of the conversation being produced, GOH is by no means restricted to a passive audience role. One of the interesting peculiarities of our attempt to intersect the framework of narratology and the domain of collaborative mathematical problem-solving, results in a unique instantiation of the idea of "possible worlds." The complex world of linguistic and mathematical objects which SKI, YAG and GOH both access and co-construct (e.g. the proposition "*The new press is three times as fast as the old one*" included in the problem statement, and SKI's posting "*the reciprocal of y is how much of the job the new one does in one hour*), their individual perspectives, and the transformations that they exert on such objects (e.g. SKI use of "cuz" - *because* - on line 25) are governed not by strict logical laws (as is sometimes assumed in narrative semantics) but by the local sense-making procedures of the co-participants and their orientation to joint-activity. For, instance, SKI in line 27 asks GOH for an assessment of her state of participation, and

GOH eventually (line 57) requests that the co-constructed narrative be reoriented towards a further sense-making on the mathematical and narrative objects so far established (e.g. 1/x, "*the old one*," "*how much of the job they do together in one hour," etc.*).

In addition to the co-construction of the narrative explanation in itself, the dialogical orientation opens the space for the exploration of possibilities of the local world of mathematical objects and, what is perhaps even more interesting as far as learning is concerned, to anticipate the intelligibility of the co-constructed narrative. In line 91, SKI's question to GOH seems to represent, both an orientation towards a prerequisite for the intelligibility of the mathematical narrative being produced, as well as an anticipation of a potential problem of understanding. It is in these instances of dialogical interaction where we are able to observe the power of what Feurenstein [23], elaborating on Vygotsky, has characterized as "mediated learning experiences:" interactions through which co-participants place themselves between each other and the world, and co-construct the meaning of their joint activity (i.e. verbal or otherwise). In mediation, stimuli and responses are selected, changed, amplified and interpreted in complex ways that represent a "type of organization (which) is basic to all higher psychological processes" ([13], p. 40). Needless to say this role is also shared among co-participants.

Although we have referred to this context as collaborative problem solving, it might appear that the work being done is closer to an "explanation" than to co-construction of knowledge. Yet, the participants, perhaps influenced by the very nature of dialogic interactions, make such explanations interactive and participatory for all members of the group. The outcome of this approach is that there is a constant interchange between first person singular and third person plural narration, and a consequent change in agency and authorship embedded within certain mathematical objects: "my way" (e.g *"I started and solved with a system of equations")* contrasted to "your way" (e.g. "*YAG its kinda hard to understand ur way*"), and sometimes becoming "our way" (e.g. *"so 8 hours is 480 minute[s], divide by 3, to get 160 minutes our answer!!!!"*). Of central interest to our analysis are the methods used by co-participants to orient themselves to certain forms of participation that guide them in their collaborative sense making. The use of the "expository" mode of interaction here differs slightly from Mercer's [24] conception of the three kinds of inter-subjective talk: *disputational*, *cumulative*, and *exploratory*. In Mercer's framework, *disputational* talk is characterized by the speakers being concerned with defending their own selves, at the possible expense of any attempt at a solution . In *cumulative* talk, each speaker seeks to support the other's self but fails to explore facts and solutions. *Exploratory* talk, according to Mercer occurs when speakers "engage critically but constructively with each other's ideas" (p.98). For a more complete analysis of the two main "participation frameworks" identified in VMT research see [16]. Although one could argue that the structure of the task itself (a word or "story" problem) might contribute to the emergence of narrative elements in the dialogical interactions among participants, similar phenomena has been observed on geometry and other non-word problems.

4. Implications for design, future research.

The analysis presented in the previous section illustrates how certain narrative structures may emerge from the dialogical interactions and the ways participants orient themselves to their shared sense-making during mathematical problem-solving. Although we have presented a single in-depth case, we seek to identify a diverse array of patterns of participation, through discourse and conversation analysis in parallel with statistical natural language processing techniques (e.g. [25], [17]), with the goal of informing the design of the appropriate learning supports for online, collaborative math problem-solving. Engagement, participation, and ultimately, learning might be emergent aspects of distributed activity systems that offer rich

opportunities for the learners to construct meaning through language and interaction in true dialogical contexts. Further research and development is necessary to integrate, in the design of future learning environment, theories of sense-making that account for the narrative and dialogical aspects of individual, small-group and community interactions. Additional text processing is envisioned, such as automated narrative summarization and intelligent indexing with the specific intent of facilitating the re-usability of collaborative problem-solving dialogs for specific learning purposes, including the potential support for an online community of math problem-solvers represented as a "narrative of dialogues".

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