Chapter 25

A Model for Analyzing Math Knowledge Building in VMT

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Abstract: This work describes a methodology for analyzing the social construction of mathematical knowledge within a chat environment like VMT. It proposes a model for representing the flow of discourse by linking contributions based on information uptake. A framework for analysis using the model is designed to represent: (1) the co-construction and manipulation of mathematical representations and artifacts such as symbols, concepts, math formulas and linguistic expressions; (2) segmentations that identify critical boundaries during chat interactions; (3) meaning-making paths intertwining through series of uptakes; (4) pivotal moments during interactions influencing the direction of the discourse and (5) elements of the model for educators to apply in understanding the learning of mathematics by groups. The long-term goal behind this research is to develop a structure for analyzing online collaborative math learning. More specifically, this methodology seeks to contribute to a holistic approach to understanding the process of meaning making embedded in interactions among chat postings. We discuss this methodology in the context of data collected in VMT from small groups of junior-college students solving mathematics problems using three different types of problem design.

Keywords: Meaning making, up-take, segmentation, pivotal contributions, Collaborative Interaction Model, problem design, individual uptake descriptor table

Participants in chat sessions in settings like the VMT environment learn as an indirect result of having to keep up their end of conversation. This process prompts learners to construct meaning, relate experiences and construct knowledge (Baker, Jensen & Kolb, 2002). Participants have to think of a response to what they have heard. Their reasoning process leading to their response requires analysis of what they have heard for an extraction of something meaningful, and then relating this meaning to resources from past experiences (Schank, 2002). Collaboration requires conversation, in which participants work in groups to socially negotiate a shared understanding of the approaches they use to accomplish tasks (Jonassen, Peck & Wilson, 1999).

Networked computers offer many opportunities to introduce conversation in an online environment in order to support the building of collaborative knowledge. People who are geographically apart can access chat software through a network of computers connected through a server to communicate and co-construct knowledge. In quasi-synchronous chat environments, the generation of communication occurs when textual and graphical inscriptions are interpreted by one or more participants, who subsequently construct new representations in the chat medium. This social construction process involves interpretation of another person's understanding and reflection upon this understanding in a cultural sense that is similar to the other's (Bruner, 1995). Here, the understanding is situated in the context of creation (Brown, Collins & Duguid, 1989) and externalized in the form of representations afforded by the chat environment. When the conversation content is seen rather than heard, the methods participants use to facilitate their conversation are clearly dependent on the medium in which interaction takes place. This context must be taken into account by researchers trying to interpret and understand the meaningful interaction among participants.

Our research explores patterns in chat transcripts to look for instances of intersubjective cognitive activity distributed across participants and their manipulations of representations. We interpret this activity from both the researcher's and participant's perspectives. We build on the work of social network analysis (Scott, 1991; Wasserman & Faust, 1992), information uptake (Suthers, 2006b), group cognition (Stahl, 2006) and interaction analysis (Jordan & Henderson, 1995) to propose a model for analyzing small groups of collaboration in quasi-synchronous chat environments like VMT.

Our work adopts the concept of information uptake (Suthers, 2006a; 2006b) to understand group cognition in small group problem solving (Stahl, 2006). We propose a *Collaborative Interaction Model* (CIM) to provide a structural view of the uptakes. By linking contributions together in a diagrammatic model, we provide a representation to support deeper analysis of the way an individual's contribution is influenced by the uptake or interpretation of another participant's contribution. Using this model, we identify the construct of a *pivotal contribution* as one that is central to the group's knowledge-building or problem-solving process, and the construct of a *stage transition* that shifts direction in the discourse. A sequence of postings forms the elemental cell of interactional meaning making. Subsequent sections will explain the development of the proposed model, using chat segments to examine how participants construct knowledge and mediate shared understanding in the VMT chat environment.

Organization of the Chapter

This chapter is organized with the following sections:

- A review of common methodologies to analyze online conversation.
- An overview of the VMT learning environment and of the context and background of the usage of the environment for collecting our data.
- Three types of mathematical problem designs that we deployed in the environment.
- Samples of transcripts using the problem designs, constructed from the replay of the chats.
- The proposed analysis model and the underlying assumptions for using the model.
- The process followed for constructing analyses using the model, and the key features of the model.
- Further implications and features of the model, as well as its broader applicability to students and educators.

Research Methods for Analyzing Online Conversations

Various studies have suggested methods to analyze online conversations (asynchronous and synchronous environments) from the perspective of the researcher. Garcia & Jacobs (1999) proposed using the methodology of *conversation analysis* (Goodwin & Heritage, 1990; Sacks, Schegloff & Jefferson, 1974) to study interactions taking place in online chat environments with video capture of participants' computer screens during chat sessions. They argued that for some research questions, the use of single-point logs to analyze interaction transcripts did not sufficiently capture external interaction processes such as the behaviors of participants when using the computer to transmit information (Rintel, Mulholland & Pittam, 2001). Their research was further developed by O'Neill & Martin (2003) through the illustration of how repairing problematic postings by participants could be easily managed and how the timing of chat postings may disrupt conversational coherence. The characteristic of a chat environment makes it challenging to identify appropriately the referential relationships among postings. Hence, it is important for researchers when doing analysis to take into account the disruptive nature of "quasi-synchronous" chat environments, i.e., online environments in which the gradual production of utterances cannot be observed by others. Unlike in face-to-face (F2F) communication, in quasi-synchronous chat it is difficult for participants to observe how postings are taken up by subsequent postings because there are no visual, auditory or kinesthetic cues indicating when someone decides to enter into the conversation (Murphy & Collins, 1997; Siemieniuch & Sinclair, 1994). As such, the analysis of methods used by participants to communicate F2F may not be appropriate in analyzing communication in a quasi-synchronous environment. One must engage in some form of content analysis to examine computer-mediated communication transcripts (Chen & Looi, 2007).

Content analysis—involving coding messages and counting the number of individual postings with given codes—is of limited use for studying interactions between messages and for analyzing the group processes resulting from such interactions (Jeong, 2003). This is an area in which traditional experimental studies often focused too much on quantitative measures of classifications of isolated utterances, ignoring the sequential structure of the discourse (Stahl, 2002; Suthers, 2006b).

Sequential analysis uses transitional state diagrams to illustrate the transitional probabilities between coded event categories. The categories are agreed upon by coders (with inter-rater reliability measured by Cohen's Kappa coefficient), and assigned using the grounded theory approach (Jeong, 2003).

Other types of analysis include the use of constructed *message maps* to illustrate the flow of an online discussion (Levin, Kim & M., 1990) and the use of an idea within a message as the unit of analysis (Henri, 1992), reinforcing the idea that the unit of analysis could possibly encompass an entire message constructed by an individual at a certain time during the discourse (Gunawardena, Lowe & Anderson, 1997; Rourke et al., 2001). The selection of the unit of analysis is based on the situation in which it is used (De Wever et al., 2006) and the granularity of the content to be analyzed (Chi, 1997).

Suthers (2005) proposed examining patterns of *information uptake* for the analysis of intersubjective meaning making, beginning with the identification of uptake acts in which one participant takes up another participant's contribution and acts on it. The basis of intersubjective meaning making is the process of communication requiring participants to establish a common ground, building from this common ground through adjustment and development in understanding (Rogoff, 1997).

The analysis of online conversations is typically a task done by researchers poring over data collected on the conversations. As discussed above, there is the additional ambiguity posed by non-adjacency of uptakes. In our work, we perform the analysis of information uptakes from the researcher's perspective, but in addition we explore the interpretations of uptakes by asking the participants to provide their own perspectives on which specific utterance or action they were responding to when they responded, and why. We recognize that the use of post-event analysis faces similar interactional troubles to face-to-face survey interviews (Hammersley, 2003; Lee & Roth, 2003; Suchman & Jordan, 1990); we consider the data from participants' interpretations as another data source to triangulate interpretations of the discourse with that of the researcher's interpretations. Situations where uptake information might be missed by researchers are identified, hence increasing the reliability of the identification of uptake relationships between postings.

The Chat Environment and its Participants

The design of a learning environment should allow students to articulate their understanding because students learn best when they are able to express what they have learned (Sawyer, 2006). The quasi-synchronous chat environment of VMT allows students to articulate their thinking and to collaborate to solve math problems. We used the VMT system with a target group of students (ages 17-18 years) from a junior college in Singapore (Stahl, Wee & Looi, 2007). They have a basic foundation in mathematics and are among the top 20% of their cohort in terms of academic ability. The students have received sufficient mathematical training that the level of mathematical background knowledge assumed in any problem used was compatible with their expertise. The transcripts in this chapter are extracted from samples of interactions of different online teams from this group of students. (We have slightly modified some of the wording within the textual postings for readability by an international audience.)

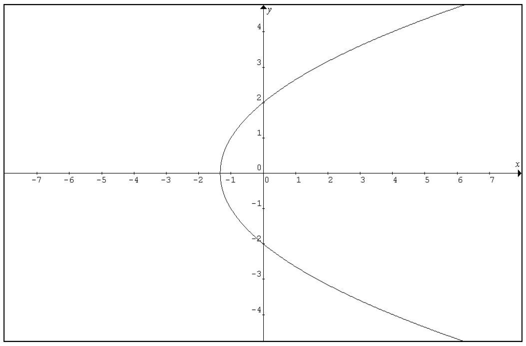
Mathematical Problem Designs

Three mathematical problem designs were used to construct problems for use with the VMT environment in the Singapore junior college. The problems are designed to complement the existing school curriculum, where students solve traditional close-ended (CE) math problems individually during lectures and tutorials (Stahl et al., 2007). The first type is known as the *open-ended* (OE) problem design, where there is more than one possible solution to the problem. The second type, called the *conceptual approach* (CA) problem design, focuses on the use of strategies to solve the problem rather than emphasizing the solution itself. This design provides the opportunity for students to articulate their interpretation of the problem as well as sharing methods of approaching the problem. The third type adopts the *guided collaborative critique* (GCC) problem design (Wee, 2007a), where students are guided through a proposed situation (including the problem solution) and through a critique of identified common conceptual errors.

Open-Ended Problem Design (OE)

Open-ended problems were designed to encourage students to reason mathematically about their problem-solving steps. OE designs lead to many possible answers. However, such designs are often perceived as not very useful in preparing students for standardized tests and examinations. There is a need to construct problems that not only prepare students academically for examinations but also strengthen their mathematical reasoning in the process. Figure 25-1 shows an OE problem that was used.

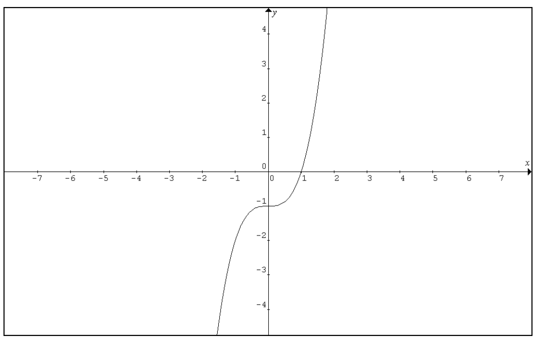
Diagrams 1, 2, 3 and 4 show four graphical plots. Select one plot that contains a function. Illustrate using mathematical proofs or otherwise, why the graphical plot selected is a function.



$f(x) = \pm\sqrt{3x+4}$

$x \in \Re$

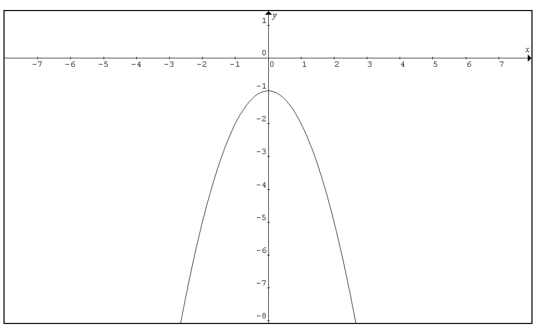
Diagram 1



$f(x) = x^3 - 1$

$x \in \Re$

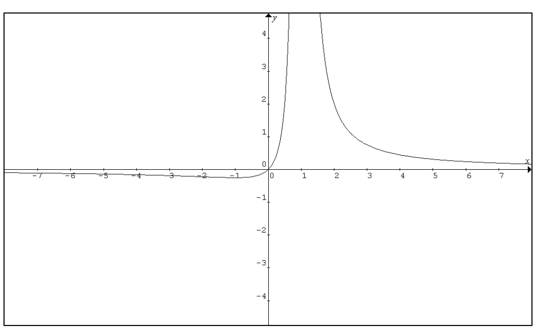
Diagram 2



$f(x) = -x^2 - 1$

$x \in \Re$

Diagram 3



$f(x) = \dfrac{x}{(x-1)^2}$

$x \in \Re$

Diagram 4

(a) Identify any graphical plot that is not a function. Illustrate using mathematical proofs or otherwise, why the graphical plot selected is not a function.

(b) Using the result obtained in part (a), restrict the domain of $f(x)$ such that $f^{-1}(x)$ exists.

(c) Find the domain and range of $f^{-1}(x)$.

(d) Given that $g(x) = 2x + 1$, $x \in (a,b)$ and using the $f(x)$ obtained in part (b), find suitable values for a and b such that $g(f(x))$ exists.

Figure 25-1. A sample OE problem.

Traditional Closed-Ended Problem/Conceptual Approach (CA) Problem Design

Initial versions of VMT problems used the traditional close-ended (CE) problem design. Such designs were adopted from textbooks where students were tasked to read a given problem and apply standard procedures to find the unique correct solution. However, the implementation of CE problem design in the chat environment was not effective in promoting quality mathematical reasoning between participants. One drawback of the CE problem design is that students tried to just type expressions, with limited mathematical reasoning. This prompted us to develop the CA problem design. The CA problem design gives students the opportunity to discuss the rationale or purpose of the approaches they take to solve the problem, thus developing their mathematical reasoning rather than simply presenting the solution itself. One advantage offered by the CA problem design is that students are given the opportunity to explore collaboratively mathematical concepts encountered when solving mathematical problems individually during class. Figure 25-2 shows a CA problem we used.

The functions f and g are defined by $f : x \rightarrow 4x^2 + 3$, $g : x \rightarrow 3 + e^{-2x}$, $x \in \Re$.
With the aid of the graph $y = f(x)$, explain why f is a 1-1 function. Find
(i) $f^{-1}(x)$, and
(ii) $f^{-1}g(x)$, giving the domain of each function.
Read the problem and collaboratively discuss the rationale for the mathematical concepts behind what is a 1-1 function with respect to $f(x)$, understanding that $R_f = D_{f^{-1}}$ where $f^{-1}(x)$ is the reflection of $f(x)$ about $y = x$ and understanding the relationship between the two domains, $D_{f^{-1}g(x)}$ and $D_{g(x)}$.

Figure 25-2. A sample CA problem.

Guided Collaborative Critique (GCC) Problem Design

The latest VMT problem design type, *Guided Collaborative Critique* (GCC) (Wee, 2007b), is constructed using a hybrid design that combines the merits of both CE and OE problem designs. The problem is first constructed using a CE design, but an erroneous solution is proposed for it. (The example analyzed in Chapter 9 is also of this type.) The choice of using the CE problem design to construct the problem is to familiarize students with examination-oriented questions while enabling them to evaluate, critique and repair the given erroneous worked-out solutions based on the OE problem design. The term "guided" refers to a sequence of structured steps in place to aid students in the analysis of the problem. The term "collaborative" emphasizes use of dialogue in the group problem-solving process to construct knowledge. The term "critique" is associated with the group's ability to locate errors embedded in the proposed (but erroneous) solution and collaboratively build arguments to substantiate their identification of the errors and defend the validity of the proposed repair. In the context of this research, an error is defined as a representation identified as mathematically inappropriate in the "proposed solution." Students not only collaboratively explore mathematical concepts learned in class, but also reason out the feasibility of their application in various GCC problems.

Embedded in the worked-out solution in the GCC problem in Figure 25-3 are three common errors found in student assignments. The first error requires the student to identify the common term as 3 and not 3^{-1} when factoring the term $(1+\frac{x}{3})^{-2}$. The second error is designed for students to realize that the expansion is only valid when $|x| < 3$ and not $|x| > 3$. The third error is the most complex of the three, requiring students to understand the need to take into account the $(-1)^r$ term when simplifying $\frac{(-2)(-3)(-4)...(-2-r+1)}{r!}3^{-r-2}x^r$. The students were required to collaboratively work within their group to locate the three errors in the proposed solution and discuss ways to repair the errors.

Expand $(3+x)^{-2}$ as a series of ascending powers of *x*, up to and including the term in x^3, expressing the coefficients in their simplest form. State the range of values of *x* for which the expansion is valid. Find also the coefficient of x^{25} in the form $\frac{k}{3^m}$ where *k* and *m* are constants to be found.

Proposed solution:

$$(3+x)^{-2} = 3\left(1+\frac{x}{3}\right)^{-2}$$

$$= 3\left[1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{3}\right)^3+\cdots\right]$$

$$= 3-2x+x^2-\frac{4}{9}x^3+\cdots$$

Expansion is valid for $\left|\frac{x}{3}\right|>1 \Rightarrow |x|>3$.

The general term is

$$(3)\frac{(-2)(-3)(-4)\cdots(-2-r+1)}{r!}\left(\frac{x}{3}\right)^r$$

$$= \frac{(2)(3)(4)\cdots(r+1)}{r!}3^{-r+1}x^r$$

$$= \frac{r!(r+1)}{r!}3^{-r+1}x^r$$

$$= (r+1)3^{-r+1}x^r$$

Identify the faults in the proposed solution and suggest repairs to fix it.

Figure 25-3. A sample GCC problem.

VMT Interaction Transcript

The VMT Replayer tool is a VCR-like interface used to reproduce the session so that it unfolds on the screen the same way that it did for the students. The VMT Replayer tool plays back the entire session, capturing the moment-by-moment interaction between the students as they post messages in the chat line and manipulate artifacts on the shared whiteboard. The interaction is also available to researchers as a log in the form of a spreadsheet, which is handy for analysis. Log 25-1 shows the interaction transcript of three participants (Lincoln, William and Smith) solving the OE designed math problem. Log 25-2 shows the interaction transcript of three other participants (Mason, Charles and Kenneth) solving a CA designed math problem. Log 25-3 shows the interaction transcript of three participants (Wane, Yvonne and Tyler) solving a GCC designed math problem. The first column shows the time that an utterance was posted or a graphic drawn. The second column shows the name of the participant. The third column shows the message posted by the participant in the VMT chat room. The message can take the form of text posted in the chat line or an artifact constructed on the shared whiteboard. The fourth column shows a contribution number assigned during analysis (we will come back to discussing the purpose of contribution numbers in a later section) and the action performed by the participant. The action performed is (by default) that the participants are typing into the chat line, unless otherwise indicated. Other possible actions include drawing on the shared whiteboard and using the referencing tool to link to another posting or artifact. Subsequent sections will illustrate how the interaction transcripts are used in the construction of the proposed chat interaction analysis model. Note that the first step in construction has already been performed in the following logs by assigning contribution numbers to individual postings or sets of sequential postings that form a single interactional move by one participant.

Log 25-1.

Time	Name	Message	Contribution
10:27:34	Lincoln	for qn E, the range of F is the domain of G	C86
10:27:44	William	Ya	C87
10:28:22	Smith	i thought domain of gf(x) equals to domain of f?	C88
10:28:27	Lincoln	so it 0 to -ve infinity	C89
10:29:12	Lincoln	no, that is for gf to exist first	C90
10:29:25	Lincoln	to prove that gf can happen	
10:29:58	Lincoln	then domain of gf is equal to the domain of f	
10:31:28	William	den wat is the range	C91
10:32:37	William	i think range is -2 to infinity	
10:32:38	Lincoln	no is the domain of f	C92
10:32:50	William	ooh	C93
10:32:54	Lincoln	that is the domain of gf	C94
10:33:01	Smith	to prove, gf does exist, range of f must be a subset of domain of g	C95
10:33:18	Lincoln	yah that is wat i am trying to say	C96
10:33:25	William	so wat the range of g	C97
10:33:40	Lincoln	sorry if i write the word equal just now, it is subset	C98
10:33:46	Smith	we r using diag 1 u know guys?	C99
10:33:54	William	ya	C100
10:34:11	William	so whats the range	
10:34:42	Lincoln	i thought is the domain of f that is the domain of g	C101
10:35:07	William	wat qn we doing	C102
10:35:13	Lincoln	it is the domain lah	C103
10:35:26	Lincoln	E wat	
10:35:26	William	Domain is all negative real values a is 0 b is -infinity	C104 [ref to WB]
10:36:18	William	ok the ahs is up	
10:36:25	Smith	we must first make diag a 1-1 function	C105
10:36:40	Smith	we take the top throw the bottom.	
10:37:02	Smith	range of f is a subset of domain of g, so we take the highest possible range of diagram 1 lo	
10:37:07	Lincoln	u are talking about question e right?	C106

10:37:19	Smith	yup	C107
10:37:21	William	ya	C108 Ref to C106
10:37:31	Lincoln	that will be infinite	C109
10:37:38	William	we take the bottom	C110 Ref to C108
10:38:05	Smith	take the top better	C111
10:38:16	Lincoln	i will take the top	C112
10:38:29	Lincoln	more comfortable	
10:38:36	William	ok	C113
10:38:44	Smith	so, a=0 b= +ve infinity	C114
10:38:44	William	b is infinity	C115 [Amend WB]

Log 25-2.

Time	Name	Message	Contribution
7:35:26	Mason	lets start	C1
7:35:46	Charles	so we need to draw the f	C2
7:36:35	Charles	hw to draw here	
7:36:40	Kenneth	draw the graph y=f(x), then use horizontal line to prove is 1-1	C3
7:36:41	Mason	then take a horizontal line test	C4
7:37:00	Mason	u dun have to solve the problem. Just say how u gonna solve it	C5
7:37:09	Kenneth	okay	C6
7:37:27	Charles	then Range of f inverse = domain of f	C7
7:37:54	Charles	Domain of f inverse=range f	
7:38:01	Kenneth	Yar	C8
7:38:07	Kenneth	then (i) done	
7:38:34	Charles	for f inverse g(x)	C9
7:38:42	Kenneth	domain of g = domain of f inverse g	C10
7:39:05	Charles	its the subset	C11
7:39:12	Mason	I think you have to test on the range of g and see if it fits the domain of f-1	C12
7:39:21	Kenneth	ops	C13
7:39:42	Mason	ken	C14
7:39:49	Kenneth	?	C15
7:39:53	Mason	dun draw such conclusion	C16
7:40:07	Kenneth	must test	C17
7:40:15	Mason	like domain of g=domain of f inverse g	C18
7:40:22	Mason	How u know?	
7:40:15	Charles	Domain of f inverse g(x)=Domain of g correct?	C19
7:40:22	Charles	then we can solve C20	
7:41:05	Kenneth	formula of composite functions lol C21 /Ref toC18[	
7:41:26	Kenneth	coz domain of f inverse g cannot exceed domain of g	C22 /Ref to C18
7:41:32	Mason	oh	C23
7:41:37	Mason	then i wrong	
7:41:38	Mason	sorry	
7:42:27	Kenneth	No need to actually work out? so we state method le	C24
7:42:45	Mason	en	C25
7:42:43	Kenneth	?	C26
7:42:45	Mason	1st one settle	C27
7:42:49	Mason	move on	

Log 25-3.

Time	Name	Message	Contribution
4:15:03	Wane	i cant remember the method for finding the coefficient	C1
4:15:08	Yvonne	yea	C2
4:15:16	Wane	do u remember tt formula we learnt in secondary school?	C3
4:15:18	Tyler	it's a binomial series	C4
4:15:28	Yvonne	same... but there is one mistake ler	C5
4:15:32	Tyler	use the binomial formula	C6
4:15:42	Wane	the more than sign	C7
4:15:43	Tyler	yeah step by step	C8
4:15:52	Yvonne	i not sure cause it's power to -2	C9
4:15:58	Wane	the first part is correct	C10
4:15:06	Yvonne	can enlighten me?	C11
4:16:16	Yvonne	no.... that first part is wrong ler	C12
4:16:19	Wane	then the modules x more than 3 is wrong	C13
4:16:23	Wane	it should be less than	
4:16:25	Tyler	first take out the 3	C14
4:16:33	Yvonne	when he take out constant, it will not be 3	C15
4:16:49	Yvonne	cos it's 3^-2	
4:16:52	Tyler	yeah. coz there is a a power to -2	C16
4:17:15	Yvonne	but the rest of the steps i'm not very sure	C17
4:17:18	Tyler	so the second line is not correct	C18
4:17:28	Tyler	$3\left(1+\frac{x}{3}\right)^{-2}$	C19/ Shared
Whiteboard / "3" outside the bracket is circled			
4:17:39	Yvonne	$3\left[1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^{2}+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{3}\right)^{3}+\cdots\right]$	C20/ Shared
Whiteboard / "3" outside the bracket is circled			
4:17:40	Tyler	see I circle it .right?	C21
4:17:44	Wane	that 3 is correct	C22
4:17:54	Yvonne	y?	C23
4:18:10	Tyler	no . that3 should also to the power to 3	C24
4:18:23	Yvonne	-2	C25/Ref to C23
4:18:42	Tyler	sorry.	C26
4:18:45	Wane	the formula is (a+ bx) power n	C27
4:19:12	Yvonne	the person took out the common factor	C28
4:19:26	Wane	Tyler is correct	C29
4:19:28	Wane	i over look it	
4:19:32	Tyler	but should take out the a.	C30/ Ref to C26
4:19:51	Yvonne	ok.... for the next part... did your spot any error?	C31
4:19:52	Tyler	coz -2 is a negative value	C32
4:19:54	Wane	the formula is a power n (1+ bx/a) ower n	C33
4:20:06	Tyler	so it cant use Wane's formula	C34/ Ref to C28
4:20:21	Wane	what?????	C35
4:20:34	Yvonne	i don't get it	C36
4:20:35	Wane	hold on	C37
4:20:35	Wane	wait	
4:21:16	Tyler	i mean if the power is a negative value. it should use(1+ax) to power of n	C38
4:21:42	Yvonne	ya... that's why they took out the common factor	C39
4:21:44	Tyler	see?	C40
4:21:53	Tyler	yes	C41

4:22:11	**Wane**	**tts what i was saying**	**C42**
4:22:27	**Yvonne**	**ok...**	**C43**
4:22:41	**Tyler**	**so move to the next line**	**C44**

Collaborative Interaction Model

The analysis of interaction transcripts is complex and time consuming. Our proposed model—called the Collaborative Interaction Model (CIM)—is designed to analyze relationships among contributions (graphical and textual postings). There is a high probability that the postings may appear in an order that obscures their response structure. It is not possible to shrink the time window for searching relations of relevance to adjacent contributions in order to reduce the complexity of analysis caused by this, because there is always a chance that any past contribution could be taken up again. Focusing the analysis on the relationship between adjacent postings is therefore in general insufficient for understanding relationships between the postings in a quasi-synchronous chat.

In CIM, chat postings are analyzed line by line; postings belonging to the same interactional unit are grouped together as a contribution and assigned a single contribution number. Contributions belonging to the various participants are represented by differently shaped nodes. The interaction betweens contributions are mapped using arrows to illustrate uptake relationships (Suthers, 2006a).

Jordan & Henderson (1995) pointed out that all events (in this case the occurrences in the discourse) of any duration are *segmented* in some way. They argued that researchers would be keen to understand the transition process of interaction between segments (known as *stages* in the CIM). The CIM model adapts the concept of segmentation to trace the development of knowledge construction across stage boundaries. Segmentation is constructed initially using the researcher's interpretations of the interaction transcript. This is then triangulated with the participant's interpretations, hence increasing the reliability of interpretations.

The CIM traces the development of knowledge construction in an online collaborative environment by mapping the interaction between participants (linking of contributions by uptake arrows) throughout the discourse. The model is applicable for a group of 3 to 5 persons. The object with a contribution number is known as a *node* in the CIM. The concept of contribution will be elaborated below. Each node shape (rectangle, oval or hexagon) represents one of the participants. Nodes represent contributions constructed. The model does not directly address design issues. It does not analyze the design of the software or compare it to other designs. The model is intended to help understand how learners interpret and build on each other's representations. It is used to trace emerging paths of knowledge construction. The CIM is a methodology that describes how groups collaborate in an online environment. This descriptive method could help instructional designers review different ways of improving tested collaborative interface designs.

Constructing the CIM

Chat postings (including constructions on the shared whiteboard) are coded into contributions (numbered in Logs 25-1, 25-2 and 25-3). The contributions are then mapped and linked to form the CIM (see Figures 25-4 and 25-5). The concept of uptake is defined as a situation in which a participant references or manipulates content in previous contributions (Suthers, 2006b), either their own or someone else's. Uptakes are indicated by arrows linking contributions in the CIM. The construction of this network of arrows takes place through two phases. The first phase occurs when the CIM is constructed based on the researcher's interpretation (see Figure 25-4 and Log 25-1). Figure 25-4 shows a segment of a three-person team Collaborative Interaction Model. Researchers discuss their interpretations of the interaction transcripts during data sessions. The second phase (see Figure 25-5) occurs when the CIM is triangulated with post-mortem interpretations made by the participants using a tool that we call the Individual Uptake Descriptor Table (IUDT).

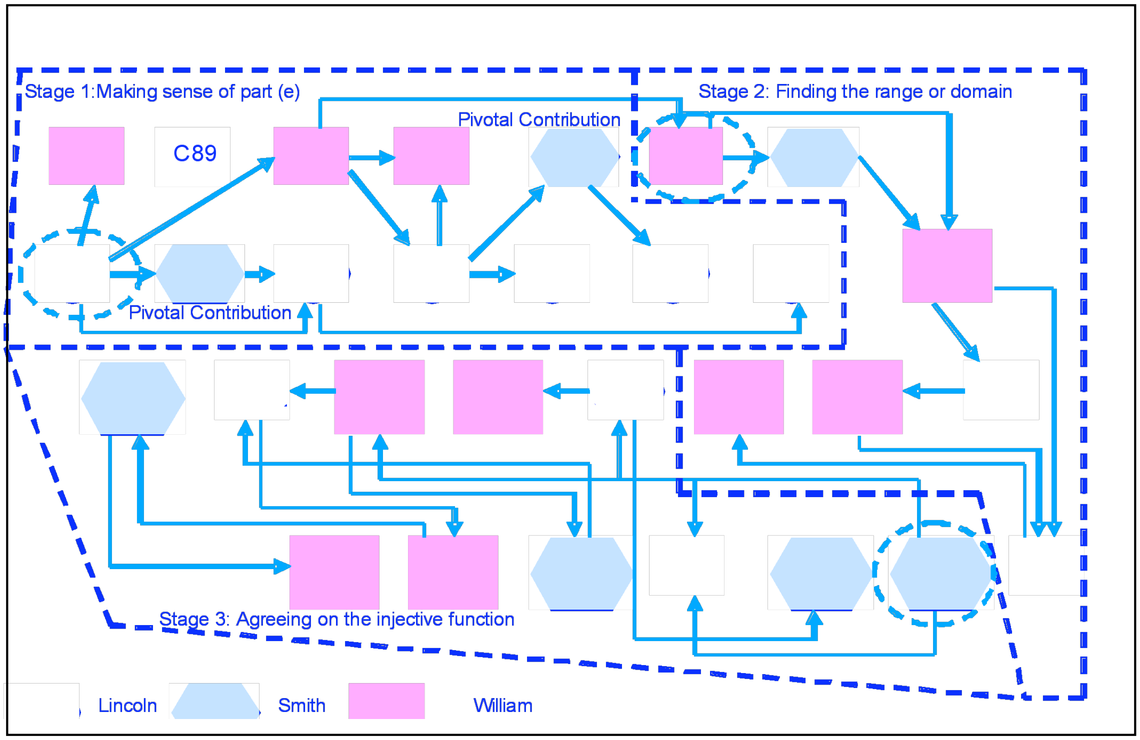


Figure 25-4. CIM before triangulation with IUDT.

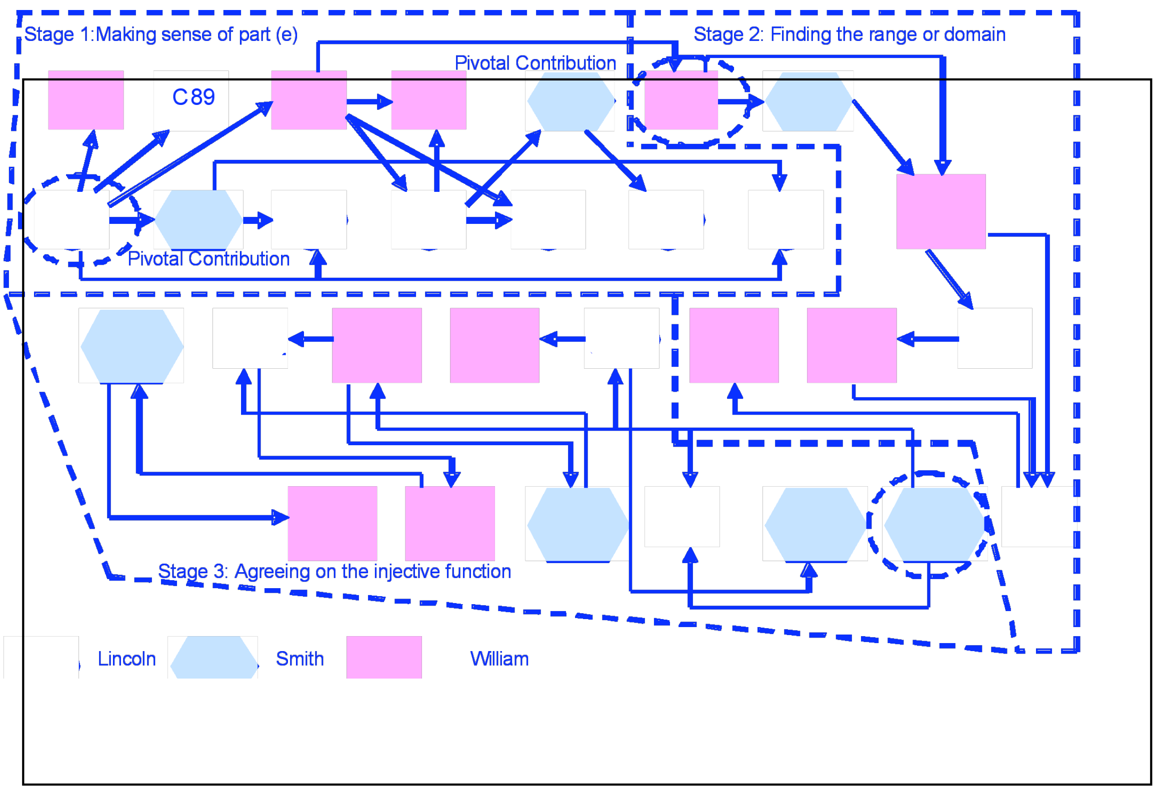


Figure 25-5. CIM after triangulation with IUDT.

Coding of a Contribution

Chat postings and whiteboard artifacts are coded in sequential order to form contributions, logical units from the participant's perspective. Sequential order is defined by the order of postings. A participant may type in a representation in the chat and then manipulate some artifacts on the shared whiteboard. When coding a hybrid interaction like this, the researcher has to take into account all the actions in sequential order. Assigning a logical contribution number is based on the researcher's interpretation of how participants defined the logical unit of their interactions. Each contribution is assigned a contribution number in the interaction transcript. In the CIM, participants are represented by differently shaped nodes. For example, in Figure 25-4, rectangles represent William's contributions, ovals represent Lincoln's contributions and hexagons represent Smith's contributions. Each node has a contribution number which represents a posting by a participant in the interaction transcript (Log 25-1).

Stages in the CIM

A *stage transition* is defined to occur when there is a shift of direction in the discourse. Events in temporal and spatial orientation can be segmented in various ways (Jordan & Henderson, 1995; Kendon, 1985); participants negotiate across segment boundaries. The boundaries are known as stages in the CIM, negotiated by two types of transitions: abrupt and seamless. An abrupt transition is defined as a sudden change due to a new proposal. Seamless means that the transition is smooth (e.g., participants have agreed to move on to a next stage). Figure 25-6 shows the CIM constructed from the GCC problem design (see Figure 25-3). It consists of six stages with three abrupt transitions and two smooth transitions. When no member takes up a prior contribution, the stage transition may be abrupt. For example, the transitions from stage 1 to 2, stage 2 to 3 and stage 3 to 4 are abrupt. The last contribution of each stage was not taken up by any member of the group. It will be useful for researchers to analyze why such contributions are not taken up. Unlike stage transitions mentioned earlier, the transition from stage 4 to 5 and stage 5 to 6 arise from pivotal contributions where the transition process is not abrupt.

Stage Transition

Interaction analysis classifies events of any duration to be segmented in some way. An event has an internal structure that is recognized and maintained by the participants. A transition from one segment (or stage) to another occurs once the segment reaches its boundary. The next segment is of a different "character." The notation of "character" is similar to what we called "direction," where the direction of each stage consists of contributions aligned by coherence. Some of the possible ways in which a stage transition can occur are illustrated in Figure 25-6.

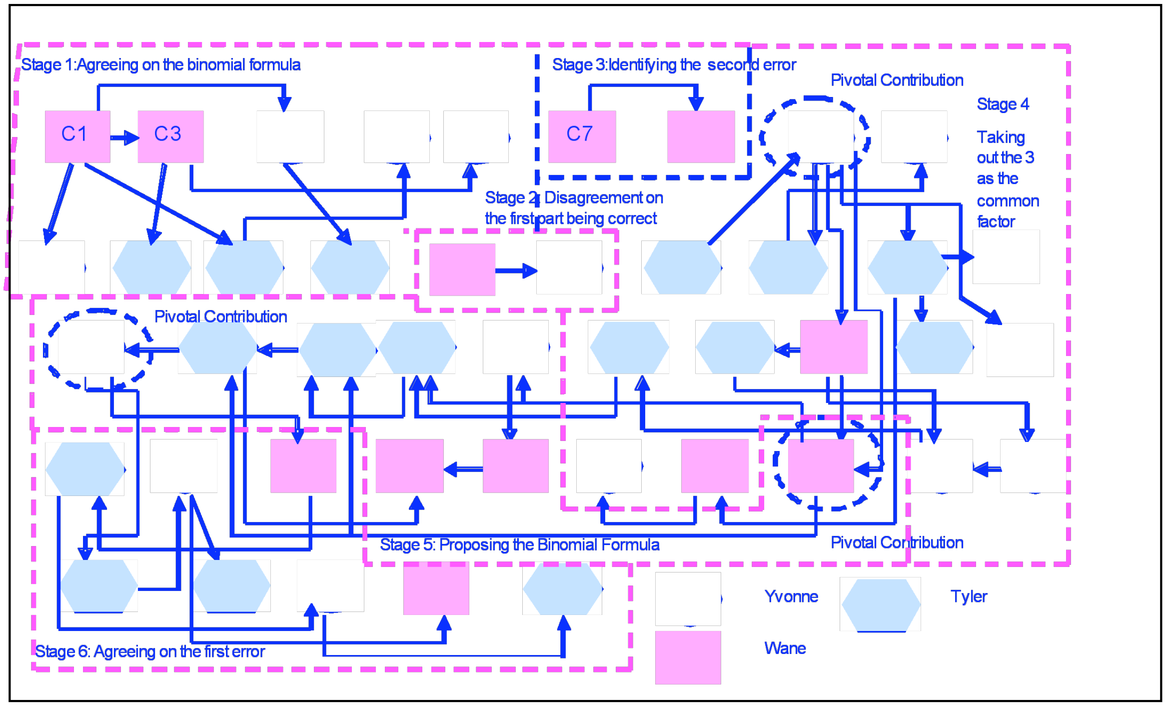


Figure 25-6. Stages in the Collaborative Interaction Model.

Consider the stage transition from stage 1 to 2. Yvonne takes up Tyler's "**it's a binomial series**" [C6] with "**i not sure cause it's power to -2**" [C9]. Wane proposed that "the first part is correct" [C10], stating a different direction to the conversation between Yvonne and Tyler, who were discussing the validity of the secondary school binomial formula. This caused an abrupt stage transition. Yvonne takes up Wane's proposal "**the first part is correct**" [C10] but rejects the claim by stating "**no.... that first part is wrong ler**"[1] [C12], informing Wane that there is a mistake in the first part of the proposed solution. Again there is an abrupt stage transition from stage 2 to 3, as Wane ignores Yvonne's proposal [C12] and continues with "**then the modules x more than 3 is wrong**" [C13]. Tyler proposed "**first take out the 3**" [C14].

Tyler's Individual Uptake Descriptor Table (IUDT) mentions that the question was reviewed and that he realized an error occurred when 3 was taken out of the term $3\left(1+\frac{x}{3}\right)^{-2}$. Wane's [C13] mentioned the second error while Tyler's [C14] mentioned the first error, leading to an abrupt stage transition from stage 3 to 4.

Stage transitions may also occur when participants propose a significant contribution resulting in a shift of direction in the discourse. For example, Yvonne's [C15] "**when he take out constant, it will not be 3, cos it's 3^-2**" is selected as a pivotal contribution due to the implication resulting from its construction. [C15] was taken up by Wane who rejected Yvonne's claim and counter proposed, "**that 3 is correct**" [C22]. [C15] was also taken up by Tyler who agreed with Yvonne 's [C15], "**yeah. coz**

[1] The expression, *ler*, is an emphatic term derived from Chinese and commonly used by Singapore students.

there is a a power to -2" [C16] and "**so the second line is not correct**" [C18], explaining to Wane that there is indeed an error and concurrently agreeing with Yvonne. The construction of [C15] enables the participants to take up and manipulate [C15] constructively through argumentation and agreement, forming a basis for knowledge construction. Wane's contribution, "**the formula is (a+ bx) power n**" [C27], is selected as a pivotal contribution as well as the contribution nearest to the boundary between stage 4 and 5 because the formula "**(a+ bx) power n**" does not appear to be coherent with the direction of stage 4, which focuses on having the power -2 assigned to 3 when it is taken out of the term $3\left(1+\frac{x}{3}\right)^{-2}$. The above two cases illustrate a smooth transition arising from a pivotal contribution, where participants readily take up and manipulate this significant contribution, and thereby take the discourse in another direction.

Stage Reversal

A *stage reversal* occurs when participants revert to an earlier direction in the discourse. In a similar sense, the probability of an occurrence of a stage reversal is dependent on the group's motivation in returning to issues discussed in the previous stage. The accuracy of the knowledge constructed in the earlier stages may also result in a stage reversal applied in later chat segments. A stage reversal could have occurred when participants require knowledge constructed in previous stages to solve tasks in the current stage. Researchers should analyze how group interaction results in a stage reversal. Figure 25-5 shows that stage 1 shares a similar direction to that of stage 4. Both directions (stages 1 and 4) focused on making sense of an error found in the term $3\left(1+\frac{x}{3}\right)^{-2}$. In stage 4, Tyler's "**first take out the 3**" [C14] appears to be coherent with "**i not sure cause it's power to -2**" [C9] by Yvonne, where both contributions mentioned the first error.

Uptake of Contributions

Our study refines the notation of uptake (Suthers, 2006b) as not just building on another group member's contribution, but also interpreting that existing contribution based on the new contribution. The manipulation of contributions involves not only the action of working on the contribution, but also the interpretation that motivates the action. By identifying the rationale of interpretations, researchers can understand the objective of the manipulation leading to the new contribution. Through this identification, researchers are able to identify how group members interpret other interpretations (their own or others'), and understand the purpose of their manipulation and why this manipulation is essential to construct a new contribution. In the CIM, the uptake is represented by the arrow linking two contributions. Uptake is a function of the following variables: (1) Participants must interpret contributions that are related somehow to their prior understanding, making a connection between a prior understanding and the current interpretation in order to construct a new understanding. (2) Prior understanding is achieved from previous contributions or knowledge constructed prior to the discourse. Uptakes resulting in knowledge constructed from previous contributions form the basis of interpretation, but knowledge constructed prior to the discourse such as previous encounters with similar types of problem also contribute actively to the interpretation. (3) Language and cultural representations are mutually dependent and they form the vehicle of communication in the discourse. Language and cultural representations are embedded in the contribution, forming part of the interaction and affording a meaning-making process somewhat different from that of another group of a different cultural and language background. Uptakes encompass not only information related to the tasks, but also the language and culture of the participant.

Pivotal Contributions

A significant contribution known as a *pivotal contribution* shifts the emergence of meaning-making patterns into new stages. The concept of stages simplifies the analysis of different knowledge construction patterns in the discourse. Figure 25-7 shows the CIM with two stages (see Log 25-2). The first stage shows how the participants attempt to show that the function f(x) is one-to-one. The second stage shows how participants use the mathematical definitions to establish relationships between the range/domain of f(x) and that of the composite function. Contribution [C2] in Figure 25-7 and Log 25-2 was selected as a pivotal contribution because it steers the discourse into the direction of showing f(x) as a one-to-one function. Contribution [C7] was also identified as a pivotal contribution, shifting the group's focus from showing f(x) as a one-to-one function to using knowledge of composite functions to find the range/domain of f(x).

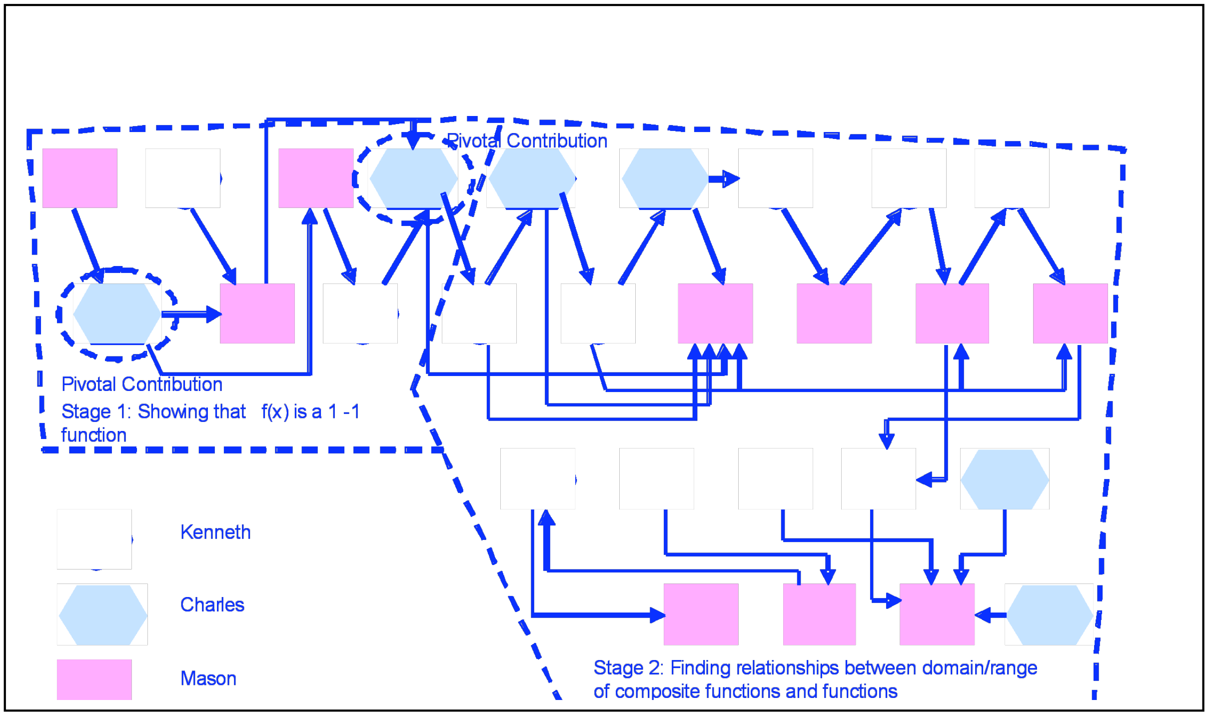


Figure 25-7: Collaborative Interaction Model (Mason, Charles and Kenneth).

The emergence of meaning-making patterns leading to the construction of the pivotal contribution and patterns of knowledge construction diverging from the pivotal contribution form the basis for analyzing how shared meaning-making is achieved at a group level, rather than at an individual level—i.e., across multiple contributions by multiple group members.

Individual Uptake Descriptor Table (IUDT)

Postings in a quasi-synchronous chat environment may arrive out of sequence and this makes it challenging for researchers to identify accurately the uptake relationships among postings. The CIM is designed to increase the reliability of identifying uptake relationships through the use of *individual uptake descriptor tables* (IUDTs). Figure 25-5 shows three uptake arrows ([C86] to [C98], [C88] to [C98] and [C91] to [C94]) not identified by researchers before the triangulation process with IUDTs. The IUDT (see Table 25-1) consists of three columns; "Each chat line you typed," "Whose and what chat lines did you see that made you type the chat line?" and "What were your other thoughts?" The IUDT is to be constructed within 24 hours of the chat session. The first column indicates the chat lines typed by the participants. The second column shows the representations the participants were interpreting prior to the construction of the chat posting. The representations could be the participant's own or other participants'. The third column indicates the rationale behind the construction of the chat posting.

Table 25-1. Lincoln's Individual Uptake Descriptor Table.

	Each chat line you typed.	Whose and what chat lines did you see that made you type the chat line?	What were your other thoughts?
61	**No the domain of F**	William: **I think range is -2 to infinity**	**Wrong answer given by William**
62	**That the domain of GF**	William: **I think range is -2 to infinity**	
63	**Sorry if I write the word equal just now when I suppose to write subset**. [C98]	Lincoln*:* **For qn E, the range of F is the domain of G** [C86] Smith: **I thought domain of GF equals to the domain of F**. [C88]	**I make a typing error**.

Let's take the following case where researchers missed an information uptake relationship. Table 25-1 shows a segment of Lincoln's IUDT. Before Lincoln constructed the posting, "**Sorry if I write the word equal just now when I suppose to write subset**" [C98], Lincoln was interpreting his previous posting, "**For qn E, the range of F is the domain of G**" [C86] and Smith's posting, "**I thought domain of GF equals to the domain of F**" [C88]. Without Lincoln's IUDT, researchers would merely be guessing at what led to the construction of [C98]. Referring to Log 25-1, researchers would have attempted to locate "**equal**" in earlier postings, to match Lincoln's apology that "**equal**" was mentioned when it was supposed to be "**subset**" [C98]. The most recent posting where Lincoln mentioned "equal" is found in "**then domain of gf is equal to the domain of f**" [C90]—which was mathematically correct, causing a confusion as to why Lincoln apologized. When reviewing Lincoln's IUDT*,* the term "**equal**" [C98] was referring to "**the range of F is the domain of G**" [C86] as a mistake. The mathematical condition for a composite function $gf(x)$ to exist is that the range of $f(x)$ is a subset of the domain of $g(x)$. Lincoln was also attempting to address Smith's confusion, "**I thought domain of GF equals to the domain of F**" [C88], of his posting, "**For qn E, the range of F is the domain of G**" [C86], by correcting it to a "**subset**" [C98].

The use of IUDTs faces many of the same interactional troubles as face-to-face survey interviews (Hammersley, 2003; Lee & Roth, 2003; Suchman & Jordan, 1990). In a group of three chat participants, the researchers asked each of them to complete an IUDT individually, and then triangulated their own (researchers') interpretations together with each of the chat participant's. In addition, a focus group was formed to further probe conflicting interpretations as a group. As in the discussion of the IUDT in Table 25-1, the participant's perspectives provided the researchers with opportunities to identify missed interpretations, thereby increasing the reliability of the representation of uptake relationships between interactions.

Discussion

Collaborative learning analysis is the fundamental motivation for the development of the CIM. The model provides a systematic approach to analyze contributions in quasi-synchronous chat environments. The following describes the characteristics of the CIM.

Generality of the CIM

The CIM is designed to analyze quasi-synchronous interaction transcripts across various disciplines. We have applied the model to three different math problem designs. In ongoing research, more interaction transcripts will be analyzed using the CIM, further exploring the generality of the CIM.

Triangulation of Interpretations

The construction of the CIM was based on several data sessions conducted to analyze the interaction transcripts. The data were analyzed from the researchers' perspective and triangulated with the participants' individual uptake descriptor tables (IUDTs). The IUDTs were constructed from the participants' perspectives within 24 hours of the chat session and served to assist researchers in triangulating interpretations of the interaction transcript after data sessions. Subsequent research will explore issues related to the development of the CIM using our methods with the objective of seeking objectivity and validity in the construction process.

Unit of Analysis

The CIM proposes uptakes as the unit of analysis. The *IUDT* is designed to help researchers understand the motivations for the construction of the uptake from the participants' perspectives. This is insufficient to understand the group knowledge construction process, since the *IUDT* is designed to capture information from an individual perspective. Further analysis of the relationships among uptakes is required for researchers to understand the moment-to-moment interaction between participants before any conclusion on group knowledge construction can be drawn.

Stages

The CIM divides groups of contributions into stages. The concept of stages relates the analysis of the discourse to its respective directions. Each stage represents a different direction in the discourse and a change of stage indicates a shift of direction. The construction of meaning is embedded in the interactions. The segmentation process, where contributions are clustered into different stages, allows researchers to explore the negotiation process directed by the group in a particular area during problem solving.

Pivotal Contribution

This study was implemented using three different types of problem design: Open-Ended (OE), Conceptual Approach (*CA*) and Guided Collaborative Critique (*GCC*). The chat interaction of these three problem designs was analyzed using the CIM. It identified "pivotal moments," known as pivotal contributions, which exerted major effects on the outcome of the discourse. Pivotal contributions are currently identified from the researchers' perspective. Ongoing work attempts to triangulate pivotal contributions from the researchers' perspective with pivotal contributions from the participants' perspective.

Level of Analysis

The CIM model provides a framework for analysis of textual contributions at both the micro level and the macro level for appropriate understanding of the ways group meaning making is achieved. The CIM captures the moment-to-moment interaction between participants through the analysis of uptakes at the micro level. The segmentation of the flow of knowledge construction by stages and pivotal contributions is intended to inform the understanding of group cognition and functionality at the macro level.

Problem Design

The CIM is primarily designed to map out interactions in the quasi-synchronous VMT environment. A good problem design should promote effective mathematical conceptual discourse. For example, the use of the *GCC* problem design promotes awareness of common conceptual errors in specific math problems. Through discussion of such errors, students will become prepared to encounter such errors in similar future problems. Educators can use the CIM to provide feedback to students during a post chat session. For example, representations of stages can be used to explain how students negotiate mathematical concepts during problem solving, or pivotal contributions can be used to acknowledge a student's contribution of a useful math proposal.

Educator's Tool

The CIM can also assist teachers in understanding interaction transcripts (how students interpret and manipulate mathematical representations in the stages) and in reflecting on their teaching. The analysis can help groups of teachers devise alternative approaches to teach a given topic. In Figure 25-3, stage 1 shows that there is a possible confusion in using binomial formulas as taught in secondary school when students reach junior college (refer to table 25-3). Teachers can clarify this concept to the students by differentiating between positive n and negative n powers. Teachers may also explicitly distinguish what is taught in secondary school from what is taught in junior college to avoid conceptual confusion in preparation for related lessons.

Conclusion

This research proposes an approach that builds on the concepts of information uptakes to understand group cognition in small-group problem solving. It provides a structural view to the uptakes, with arrows in the model linking contributions representing uptakes. The linking of contributions affords a deeper analysis of the way one individual's contribution is influenced by its uptake or interpretation by another participant's contribution. From the model, we distill the notion of a pivotal contribution as one that is central to the group's knowledge-building or problem-solving process. A sequence of postings forms the elemental cell of interactional meaning making. Shared meaning is constructed across several postings of more than one participant, and the unit of meaning making is the interaction itself, which is a group accomplishment. In subsequent research we will further elaborate the coding framework of the CIM to more fully operationalize the key ideas discussed in this chapter.

Three different mathematical problem designs were adopted in the construction of VMT problems: the open-ended (OE), the conceptual approach (CA) and the guided collaborative critique (GCC) problem designs. Through the constructed CIM models, we would like to further explore whether different problem types engender different types of meaning-making paths, and investigate how and why.

A further contribution of our work is the exploration of triangulation of data, including the interpretation of uptakes by the participants themselves, individually and as a focus group. In the transcripts we looked at, we shared some incidents where uptake information was first missed by researchers. When participants suggested them later, the researchers did re-consider their analysis. We will continue to explore these methods as a way of increasing the reliability of identifying uptake relationships between interactions, and of drawing more accurate CIMs.

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