The Organization of Collaborative Math Problem Solving Activities across Dual Interaction Spaces

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Abstract. In this paper, we focus on the organization of activities that produce graphical representations on the shared whiteboard of a CSCL system with dual interaction spaces called VMT Chat, and the ways those representations are used as interactional resources by small groups during their collaborative math problem solving work. In particular, we will investigate how actions performed in one medium inform the actions performed on the other, and how participants coordinate their moves across dual mediums to make their actions mutually intelligible to each other.

Keywords: Dual interaction spaces, interaction analysis, shared representations

Introduction

Computer-Supported Collaborative Learning (CSCL) is a recently emerging paradigm in the field of educational technology which is "…centrally concerned with meaning and practices of meaning making in the context of joint activity and the ways in which these practices are mediated through designed artifacts" (Koschmann, 2002). Recent advances in the information and communication technologies have opened up new avenues for supporting and studying the practices of meaning making at various collaborative learning settings. Dual Interaction Spaces (DIS), which typically bring together two synchronous communication technologies such as a text-chat and a shared workspace, have been widely used to support collaborative learning activities online (Dillenbourg & Traum, 2006; Muhlpfordt & Wessner, 2005; Jermann, 2002; Soller & Lesgold, 2003). The way such systems are designed as a combination of two technologically independent communication mediums bring significant interactional consequences for the users (Stahl et al., 2006; Muhlpfordt, 2006). In particular such systems require users to organize their actions across both interaction mediums in intelligible ways, so that they can sustain their joint work as a group in a DIS environment (Stahl, 2006b).

Despite the popularity of DIS applications in the CSCL literature, there are only a few studies about how small groups organize their interaction in these environments. A recent workshop held at CSCL 2005 Conference in Taipei on DIS highlighted the need for systematic analysis of interactions afforded by such systems (Dillenbourg, 2005). One of the proposals included a modeling based approach to interaction analysis called Object-oriented Collaboration Analysis Framework (OCAF), which attempts to identify patterns in the sequence of categorized actions of dyads that produced objects on the shared task space (Avouris et al., 2003; Komis et al., 2002). The tasks included construction of diagrams with well defined ontological elements such as entities, relationships, and attributes. This allowed authors to model the correct solution for each task and match it against each dyad's diagram for evaluation purposes. The model is mainly used to gather structural properties of interactions, and to compute representations that display how actions were distributed across dual spaces and how they were related to each other for a specific task.

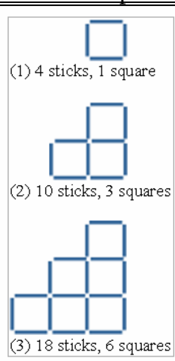
Another approach to the analysis of interactions in a DIS environment involves extending discourse analytic methods to code actions occurring in both interaction mediums. For instance, Jermann and Dillenbourg (2005) employ a coding scheme to study the correlation between planning moves in the chat and the success of subsequent manipulations performed on the shared simulation in the Traffic Simulator environment. The study reported that dyads that coordinated their planning and execution moves across both mediums performed better in that task. Dillenbourg and Traum (2006) also employ a similar methodology to study the relationship between grounding and problem solving in a DIS environment. The authors studied how a DIS environment with a shared whiteboard and a text chat mediated the problem solving work of dyads who collaboratively worked on a murder-mystery task. The authors hypothesized that the whiteboard would be mainly used to disambiguate dialogues in the chat window via basic illustrations (i.e. the napkin model). However, they found that the dyads used the whiteboard for organizing factual information as a collection of text boxes, and the chat was mainly used to disambiguate the information stored on the whiteboard (i.e. the mockup model). They attributed this outcome to the nature of the task (which requires users to keep track of many facts about the murder case) and the difference of the mediums in terms of the persistency of their contents.

In this paper we will try to build on this line of inquiry by employing an ethnomethodologically informed approach to analyze the interactions taking place in a DIS environment called VMT Chat. In particular we will focus on the organization of activities that produce graphical representations on the shared whiteboard, and the ways those representations are used as interactional resources by the groups as they collaboratively work on an open-ended math problem. Through detailed analysis of excerpts taken from VMT Chat sessions we will investigate how actions performed on one space inform the actions performed on the other, and how participants coordinate their actions across both interaction spaces. By documenting the methods enacted by participants to address these interactional challenges with available features of the system, we will attempt to build on the findings of earlier studies by highlighting some of the important affordances of DIS environments that have not yet been explicitly articulated in the CSCL literature.

Data and Methodology

The data excerpts we used in this paper are selected from a series of experimental chat sessions conducted at the Virtual Math Teams Project. The Virtual Math Teams (VMT) project is an NSF-funded research program through which researchers at the College of Information Science and Technology and the Math Forum investigate innovative uses of online collaborative environments to support effective K-12 mathematics learning. In an effort to provide a more coherent presentation we used excerpts from a single session of a team of 3 students that participated in the VMT Spring Fest event. This event brought together several teams from the US, Scotland and Singapore to collaborate on an open ended math task on combinatorial patterns. During their first session all the teams were asked to work on a particular pattern made up by sticks (Table 1). For the remaining 3 sessions they were asked to come up with their own shapes, describe the patterns they observe as mathematical formulas, and share their observations with other teams through a wiki page. This task was chosen because of the possibilities it afforded for many different solution approaches ranging from simple counting procedures to more advanced methods involving recursive functions. Moreover, the task had both algebraic and geometric aspects, which would potentially allow us to observe how participants would put many features of the VMT Chat system into use.

Table 1: Task description for the VMT Spring Fest



N	Sticks	Squares
1	4	1
2	10	3
3	18	6
4	?	?
5	?	?
6	?	?
...	...	...
N	?	?

1. Draw the pattern for N=4, N=5, and N=6 in the whiteboard. Discuss as a group: How does the graphic pattern grow?

2. Fill in the cells of the table for sticks and squares in rows N=4, N=5, and N=6. Once you agree on these results, post them on the VMT Wiki

3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

The VMT Chat system has two main interactive components that conform to the typical layout of other DIS systems: a shared drawing board that provides basic drawing features on the left, and a chat window on the right (Figure 1). One of the unique features of this chat system is the referencing support mechanism that allows users to visually connect their chat postings to previous postings or objects on the board via little arrows (see Figure 1 for an example of message-to-whiteboard reference) (Muhlpfordt & Wessner, 2005).

Studying the meaning making practices employed by the users of CSCL systems inevitably requires a close analysis of the collaborative process itself (Dillenbourg et al. 1995; Stahl, Koschmann & Suthers, 2006). In an effort to investigate our research questions we considered the small group as the unit of analysis (Stahl, 2006a), and adapted Conversation Analysis (CA) methods to conduct micro-level analysis of group interactions that took place in the VMT Chat environment (ten Have, 1999; Psathas, 1995; Garcia & Jacobs, 1998; O'Neil & Martin, 2003). In particular, our analysis will be informed by the findings of social studies of science (SSS) on scientists' use of representations as part of their professional discovery work. Although there are obvious differences between scientists and students in terms of the nature of things they discover and the practices of inquiry they follow, Koschmann and Zemel (2006) highlighted striking similarities between both cohorts in terms of the way they organize their discovery work. More specifically, the authors found that both cohorts went through episodes "…of noticing, of directing partners' attention, and of seeking, negotiating, and securing ratification of an understanding." (Koschmann & Zemel, 2006, p356). Hence, motivated by the reported interactional similarities, the findings of SSS regarding scientists' use of representations (Woolgar & Lynch, 1990) and the situated work practices of mathematicians (Livingston, 1987; 1999) will be of particular interest to our study.

We conducted numerous data sessions where we collaboratively analyzed the excerpts presented in this paper. During these sessions we used the VMT Player tool, which allows us to replay a VMT Chat session as it unfolded in real time based on the time-stamps of actions recorded in the log file. The order of actions we observe with the player as researchers exactly matches the order of actions experienced by the users. However, the temporal difference between actions we observed could differ in the order of micro-seconds from what the users had experienced due to factors such as network delays affecting the delivery of packages to clients, and the rendering performance of the user's personal computer. In other words, although we were not able to exactly reconstruct the chat from the perspective of each participant, we had a sufficiently good approximation that allowed us to study the sequential unfolding of events at each session, which is crucial in making sense of the complex interactions taking place in a collaborative software environment (Koschmann et al., 2005; Cakir et al., 2005).

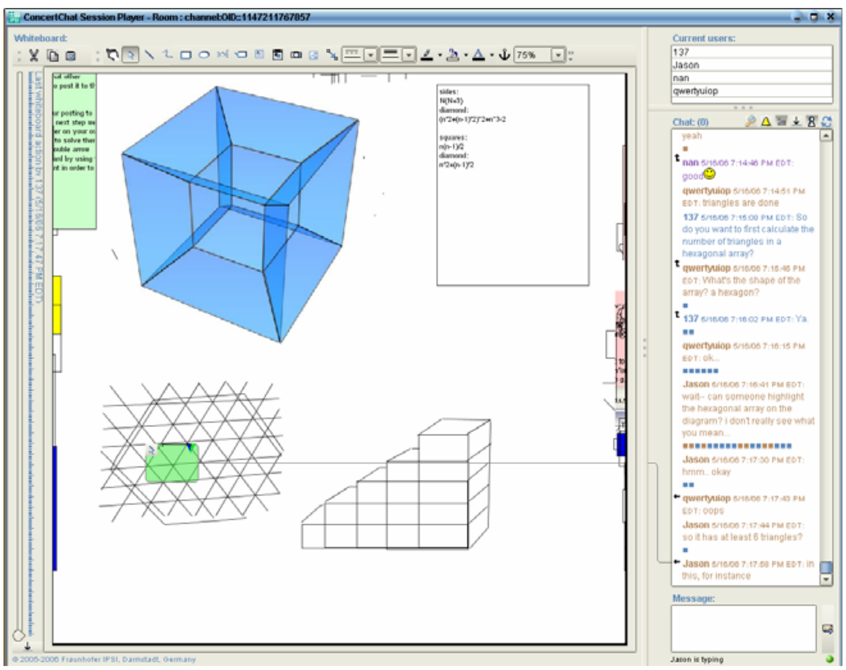


Figure 1: The VMT Chat environment

Analysis

In this section we will present our observations regarding how participants related their actions across dual interaction spaces during their joint problem solving work. In particular we will highlight how whiteboard objects were used as interactional resources during a math activity, how both spaces differ in terms of their affordances for supporting group interaction, and how these differences are used in a complementary way by team members to sustain their collaborative problem solving work in mutually intelligible ways.

Availability of the Production Process

Our first observation is that, whiteboard and chat contributions differ in terms of the availability of their production process. As far as chat messages are concerned, the participants can only see who is currently typing, but not what is being typed until the author decides to send his/her message. A similar situation applies to atomic white board actions such as drawing a line or a rectangle. Such actions simply appear as a single action on the shared space. However, the construction of most shared diagrams includes multiple atomic steps, and hence the sequence of actions that produced these diagrams is available for other members' inspection.

The availability of the drawing process can have interactionally significant consequences for math problem solving chats due to its instructionally informative nature. The whiteboard affords an animated evolution of the shared space, which makes the visual reasoning process manifested in drawing actions publicly available for other members' inspection. For instance, the episode illustrated below presents an interesting case where one of the members' drawing actions had informed the subsequent drawing actions performed by the other.

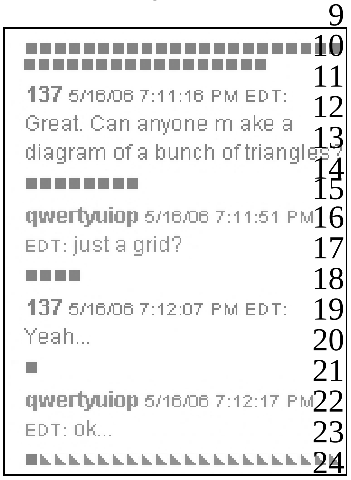


Figure 2: Excerpt 1

The excerpt shown in Figure 2 is taken from the beginning of this group's 3rd session at the VMT Spring Fest event. There are currently 3 members in the room: 137, Qwertyuiop and Jason. The drawing actions at the beginning of this excerpt were the first math problem solving related moves of the session. The little boxes in the excerpt are awareness messages that indicate actions performed on the whiteboard. We introduced different shapes like squares and triangles to make it clear to the reader who performed each action. From now on squares and triangles will be used to indicate whiteboard actions performed by 137 and Qwertyuiop respectively.

At the beginning of this excerpt we observe a series of drawing actions performed by 137 (Figure 3 below shows the evolution of this effort until 137's message at 7:11:16). 137's actions on the whiteboard included the drawing of a hexagon first, then 3 diagonal lines, and finally lines parallel to the diagonals and to the sides of the hexagon whose intersections eventually introduced some triangular and diamond shaped regions. Moreover, 137 also performed some adjustment moves (for instance between stages 4 and 5 in Figure 3) to make sure that 3 non-parallel lines intersect at a single point, and the edges of the hexagon are parallel to the lines introduced later as much as possible. Hence, this sequence of drawing actions suggests a particular organization of lines for constructing a hexagonal shape.

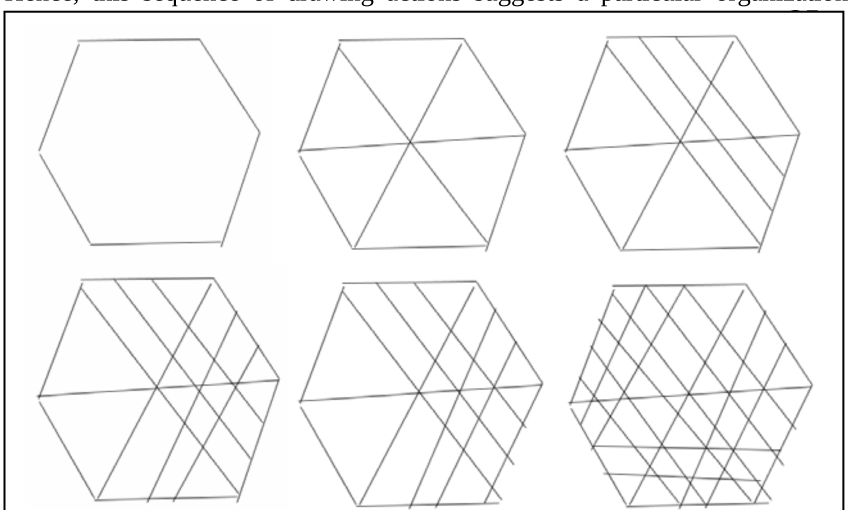


Figure 3: The evolution of 137's drawing on the board

137's chat posting which comes after the drawing episode suggests that he considers his drawing inadequate in some way. He makes this explicit by soliciting help from other members to produce "a diagram of a bunch of triangles" on the board, and then removing the diagram he has just produced (the boxes following this posting correspond to deletion actions). By removing his diagram 137 makes that space available to other members for the projected drawing activity. Qwertyuiop responds to 137's query with a request for clarification regarding the projected organization of the drawing ("just a grid?"). After 137's acknowledgement Qwertyuiop performs a series of drawing actions that resembled the latter stages of 137's drawing actions, namely starting with the parallel lines tipped to the right first, then drawing a few parallel lines tipped to the left, and finally adding horizontal lines at the intersection points of earlier lines that are parallel to each other (see Figure 4 below). Having witnessed 137's earlier actions, the similarity in the organizations of both drawing actions suggest that Qwertyuiop has appropriated some aspects of 137's drawing strategy, but modified/re-ordered it in a way that allowed him to produce a grid of triangles as requested by 137.

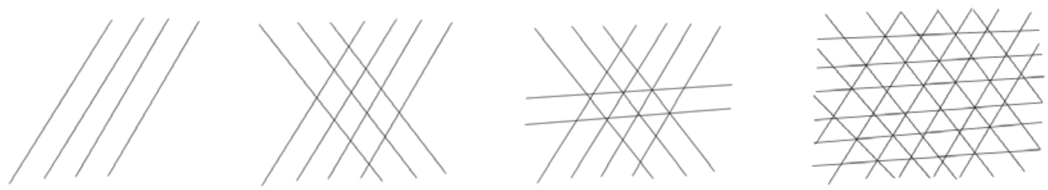


Figure 4: The evolution of Qwertyuiop's drawing of the triangular grid as per 137's request

The key point we would like to highlight in this episode is that the production of 137's earlier steps was available to other members' inspection as an ordered sequence of actions. 137 did not provide any explanation in chat about his drawing actions, or about the shape he was trying to draw. Yet, as we have seen in his subsequent performance on the board, the orderliness of 137's actions seemed to have informed Qwertyuiop. Note that Qwertyuiop could have come up with a grid of triangles that have a very different organization here. But the way he organized his actions is in extraordinary compliance with what 137 tried earlier, which suggests that he has appropriated some aspects of 137's earlier performance. In other words he was able to notice a particular organization in 137's drawing actions through his reading work (Livingston, 1995). Moreover, as we can see in the following excerpt, 137 proposes the group to consider a question based on Qwertyuiop's drawing (at 7:15:08, Figure 5) and then locks[1] the drawing to the background (the whiteboard action preceding his posting at 7:16:02, Figure 5). Thus, 137's subsequent use of this drawing provides us additional evidence that Qwertyuiop's diagram served as an adequate response to 137's query.

Mutability of Chat & Whiteboard Contents

Another interactionally significant difference between the two interaction spaces is the mutability of their contents. Once a chat posting is contributed, it cannot be changed or edited. Moreover the sequential position of a posting cannot be altered later on. If the content or the sequential placement of a chat posting turns out to be interactionally problematic, then a new posting needs to be composed to repair that (Garcia & Jacobs, 1998). On the other hand, the object-oriented design of the whiteboard allows users to re-organize its content by adding new objects and by moving, annotating, erasing, reproducing existing ones. For instance, the way 137 repaired his drawing in excerpt 1 above by moving some of the lines he drew earlier to make sure that they intersect at certain points and they are parallel to the edges of the hexagon illustrates this point.

Past and Future Relevancies Implied by Shared Drawings

As part of an ethnomethodological study of cognitive scientists' whiteboard use during design meetings in a face to face setting, Suchman conjectured that "…while the whiteboard comprises an unfolding setting for the work at hand, the items on the board also index an horizon of past and future activities" (1990, p317). In other words, what gets done now informs the relevant actions to be performed subsequently, and what was done previously could be reproduced or reused depending on the circumstances of the ongoing activity. We have observed a similar pattern in students' use of the whiteboard in our software environment.

For instance 137's first line in the excerpt below illustrates this point (Figure 5). This excerpt follows the one we considered in the previous section, where the group has established a grid filled with triangles after a failed attempt to embed a grid of triangles inside a hexagon. Given the group's recent experience with the production of the shared drawing, 137 proposes the group to calculate "the number of triangles" in a "hexagonal array" as a possible question to be pursued next at 7:15:08. This is the first time someone specifically mentioned a hexagonal array in this session, although a hexagon was previously drawn as part of the failed drawing. Given the way he formulated his proposal, 137 seems to consider that

[1] In the VMT Chat system when an object is locked it cannot be moved around unless one of the members unlocks it. This is especially useful when a user wants to annotate an existing diagram with additional drawings.

there are enough referential and historical resources currently available to the group to make the proposed course of action intelligible to other members.

Marking Relevant Objects on the Shared Space

Bringing relevant mathematical objects to other members' attention often requires a coordinated sequence of actions performed on both interaction spaces. The episode following 137's proposal in excerpt 2 presents a perspicuous setting to illustrate this observation. After 137 posted his proposal, both Qwertyuiop and Jason posted queries for clarification at 7:15:45 and 7:16:41 respectively, which indicate that the available referential resources were insufficient for them to locate what 137 is referring to with the term "hexagonal array". Jason's query is particularly important here since it calls for a response to be performed on the shared diagram, i.e. at the other interaction space. Following Jason's query 137 begins to perform a sequence of drawing actions on the shared diagram. He adds a few lines that gradually begin to enclose a region filled with triangles on the board (see Figure 6 below). In the mean time Qwertyuiop also performs a few drawing actions near the shared drawing, but his actions did not introduce anything noticeably different since he quickly erased what he drew each time.

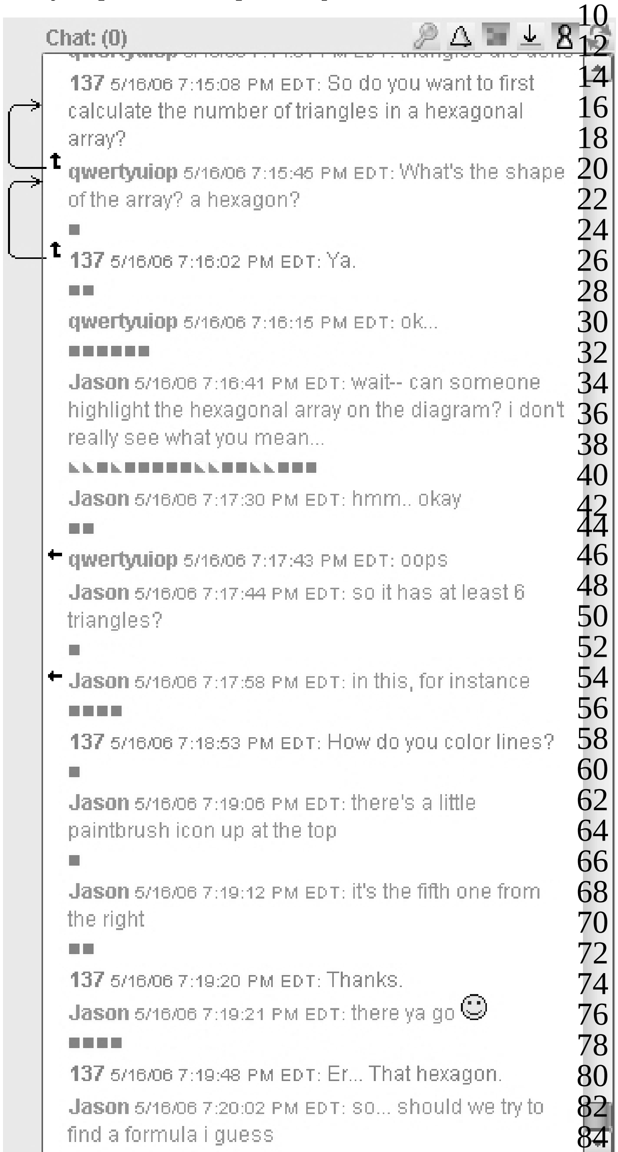


Figure 5: Excerpt 2 (referential arrows are super-imposed on the snapshot for illustration purposes).

When the shared diagram reached the stage illustrated by the top right frame in Figure 6 below, Jason posts the message "hmmm… okay" at 7:17:30, which can be read as a public display of a noticing and an endorsement. Note that nobody has posted a chat message since the last posting from Jason himself. Thus, this posting seems to be a response to the ongoing drawing activity on the board. In other words, Jason seems to be treating the evolving drawing on the shared diagram as an adequate response to his earlier query for highlighting the hexagonal array on the board. Although Jason explicitly endorsed 137's drawing as an adequate illustration, the blue boxes in the chat stream that appear after Jason's acknowledgement at 7:17:30 show that 137 is still oriented to the whiteboard. Then, at 7:18:53 he solicits other members help regarding how he can change the color of an object on the board, which opens a side sequence about a feature of the system. Jason's response to 137 at 7:19:06 provides a verbal description of the icon associated with the button for changing the brush color. Then at 7:19:12 he elaborates further on his description by indexing the same button as an element of the linear list of icons on the top bar (see Figure 1 above). This exchange is interesting for two reasons. First it is an example where one member instructs the other about a relevant feature of the system which seems to be required for marking the target object. Second, the objects of reference in this case went beyond the contents of whiteboard and chat contributions, and reached to other elements of the interface as relevant indexical resources. 137's subsequent performance where he changes the color of an edge of the hexagon from black to blue, and Jason's congratulative response at 7:19:21

show that the instruction succeeded. Once 137 finished coloring all the edges of the hexagonal shape he posts “that hexagon” at 7:19:48, which can be read as a reference to the shape enclosed by the blue contour, and as a response to other members’ earlier request for clarification.

In this excerpt 137 used the color contrast between the triangular grid and the contour as a highlighting method to make the hexagon visible to others on a grid of triangles. In short, the explicit marking of the hexagon on the board through a sequence of actions that took place in both interaction spaces allowed this group to achieve indexical symmetry (i.e., to establish a common system of reference) with respect to what is referred by the term hexagonal array (Hanks, 1996; Zemel et al., in prep). As the final posting from Jason in this episode illustrates, once the group achieved indexical symmetry, they oriented themselves to a discussion about the mathematical properties of this array.

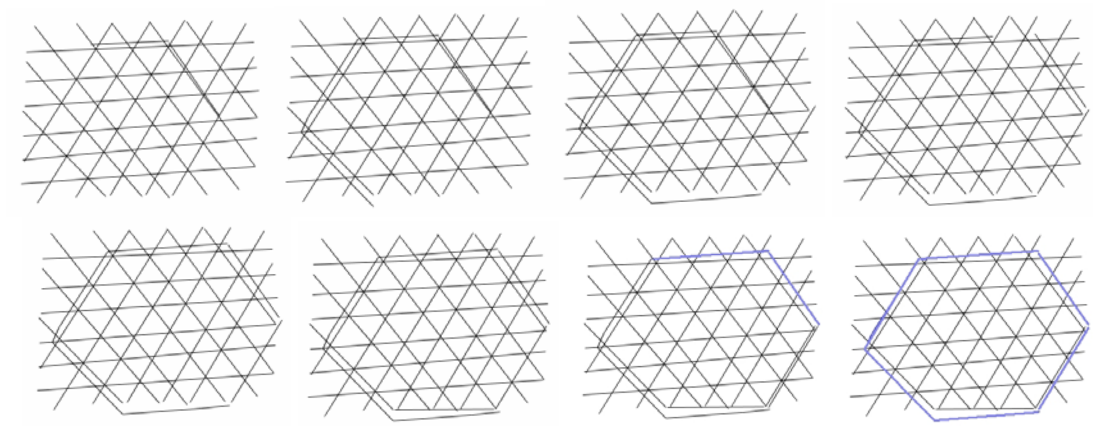


Figure 6: Snapshots from the sequence of drawing actions performed by 137

Chat vs Whiteboard Contributions as Referential Resources

Chat postings and objects posted on the whiteboard differ in terms of the way they are used as referential resources by the participants. The content of the white board is persistently available for reference and manipulation, whereas the chat content is visually available for reference for a relatively shorter period of time. This is due to the linear growth of chat content which replaces previous messages with the most recent contributions at the bottom of the chat window. Although one can make explicit references to older postings by using the scroll-bar feature, the limited size of the chat window affords a referential locality between postings that are visually proximal to each other. This visual locality qualifies the whiteboard as the more persistent medium as an interactional resource, although both mediums technically offer a persistent record of their contents.

In all of the excerpts we have considered so far, the shared drawing has been used as an indexical resource within a sequence of distinct activities. For instance, in previous excerpts the group has oriented themselves to the following activities: (1) drawing a triangular grid, (2) formulating a problem that relates a hexagonal array to a grid of triangles, and (3) highlighting a particular hexagon on the grid. In addition to these, in the excerpt following the one we covered in the prior section, the group discusses the possibility of splitting up the hexagon into 6 large triangles as a strategy to solve the problem proposed in excerpt 2 (see Figure 7). As the group oriented to different aspects of their shared task, the shared diagram on the board was modified and annotated accordingly. Yet, although it has been modified and annotated along the way, the availability of this shared drawing on the screen and the way participants organize their discussion around it highlights its persistent characteristic as a referential resource. In contrast, none of the chat postings in these excerpts were attributed a similar referential status. As we have seen at each episode the postings responded/referred to either recently posted chat messages or to the objects on the shared space.

Deriving Generalizations from Representations of Specific Instances

The drawings on the board have a figurative role in addition to their concrete appearance as illustrations of specific cases. The particular cases captured by concrete, tangible marks on the board are often used as a resource to investigate and talk about general properties of the mathematical objects indexed by them. In other words, “…inscriptions of the whiteboard are conceptual in that they stand for phenomena that are figurative, hypothetical, imagined, proposed or otherwise not immediately present, but they are also concrete – visible, tangible marks that can be pointed to, modified, erased and reproduced." (Suchman, 1990, p315).

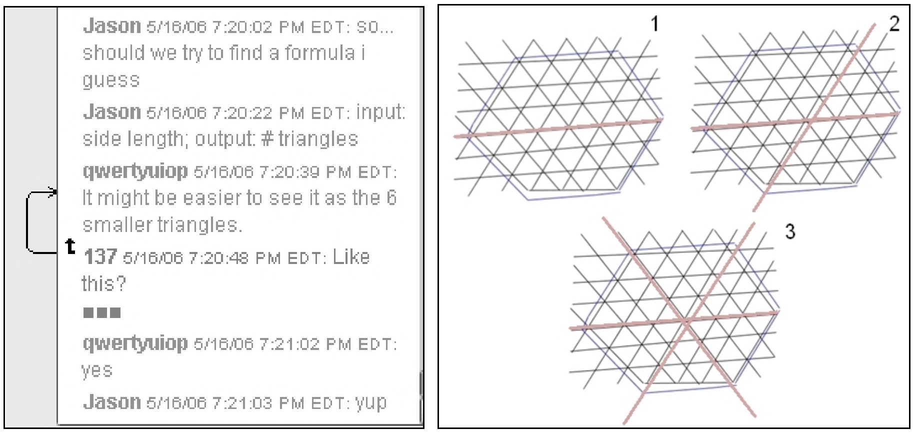


Figure 7: The episode where 137 proposes a particular way of splitting the hexagon into 6 parts. The image on the right corresponds to the sequence of 3 whiteboard actions represented as 3 blue boxes in the chat transcript.

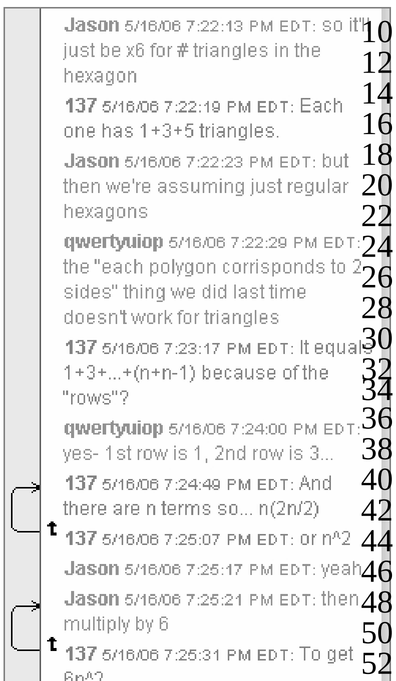


Figure 8: Excerpt 4

For example, in the excerpt displayed in Figure 8 the group co-constructs a general formula to compute the number of triangles in a hexagonal pattern based on their observations on the specific case represented by the shared drawing. This episode follows the one illustrated in Figure 7 above, where the group members constituted the problem they will be working on (see the first two postings from Jason in Figure 7), and then considered dividing the diagram into 6 pieces as a strategy to approach that problem. At 7:22:13 Jason relates this partitioning move to the problem at hand by stating that the number ("#") of triangles in the hexagon will equal 6 times ("x6") the number of triangles enclosed by each partition.

In the next posting 137 seems to be indexing one of the six partitions with the phrase "each one". Hence, this posting can be read as a proposal about the number of triangles included in a partition. The sequence of numbers in the expression "1+3+5" calls others to look at a partition in a particular way. Note that 137 could have simply said there are 9 triangles in each partition, but instead the way he organizes the numbers in summation form informs others about a particular way of counting the triangles. In other words, he highlights a particular orderliness in the organization of triangles that form a partition (Livingston, 1999). Moreover, the sequence includes increasing consecutive odd numbers, which informs a certain progression for the growth of the shape under consideration.

About a minute after his most recent posting, 137 proposes an extended version of his sequence. The relationship between the sequence for the special case and this one is made explicit through the repetition of the first two terms. In the new version the "…" notation is used to substitute a series of numbers following the second term up to a generic value represented by "n+n-1" which can be recognized as the n^{th} odd number. Hence, this representation is designed to stand for something more general than the one derived from the specific instance on the board. 137 attributes this generalization to the concept of "rows", and solicits others' assessment regarding the validity of his claim. The concept of rows seems to serve as a pedagogic device that attempts to locate the numbers in the sequence on the generalized hexagonal pattern, yet 137 is not explicit about how this can be done (see Figure 9 below for an illustration of the generalized hexagonal pattern problem posed by this group).

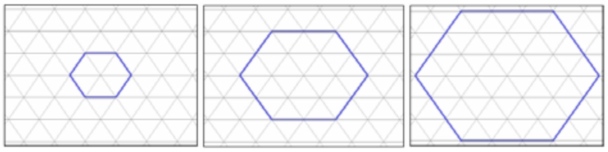


Figure 9: A reconstruction of the first three iterations of the geometric pattern this group considered during the session. For stages 1,2, and 3 the hexagonal shape has 6.(1)=6, 6.(1+3)=24, 6.(1+3+5)=54 triangles respectively. As the group discovered, when n equals the stage number the number of triangles are given by the formula 6n^2.

Qwertyuiop's endorsement to 137's proposal comes in the next line. He also demonstrates a row by row iteration on a figurative hexagon where each number in the sequence corresponds to a row of triangles in a partition. In other words, Qwertyuiop elaborates on 137's statement by explicitly articulating his understanding of the relationship between the rows and the sequence of odd numbers. This serves as an evidence of indexical symmetry within this group with respect to the generalization offered by 137. Then 137 evaluates the summation of n odd numbers, and together with Jason conclude that the number of triangles would equal 6*n^2 for a hexagonal array made of triangles (see Figure 9 below).

Discussion

In this study we attempted to highlight how shared representations on the whiteboard were used as interactional resources in collaborative math problem solving activities, and discussed some of the differences between the affordances of each interaction space in a DIS environment. In particular, we mentioned interactional consequences of the availability of a shared drawing's production process, and the figurative use of representations to mediate the discussion between specific and general aspects of the task at hand. We also highlighted how shared references to objects on the board are established through a coordinated sequence of actions across both mediums. Finally, we pointed out how the difference between the two mediums in terms of the growth of their contents qualified whiteboard as the more persistent medium, and how the availability of a persistent shared drawing indexed a horizon of past and future activities.

Our observations do not contradict with the results reported in earlier studies, yet due to the complexity of the shared task and the size of our groups we have observed more complex relationships between actions performed on each medium. Dillenbourg and Traum (2006, p147) offered two models to describe the relationship between the whiteboard and chat in a problem solving setting; namely the napkin and the mockup models. We have observed that in the context of an open ended math task the groups exhibit both uses depending on the contingencies of their ongoing work in a complementary fashion. For instance, during long episodes of drawing actions the chat served as an auxiliary medium to solicit help from other members to complete the drawing task, whereas when the group was discussing a strategy to address the problem the whiteboard was used to quickly illustrate the ideas.

Our data set allow us to study mathematical objects co-constructed by students as social/interactional phenomena, which also conforms to the recent discursive approaches in the Math Education community (Sfard 2000; Dorfler, 2002; Meira, 1995). Our analysis allowed us to observe and highlight some of the important processes involved with the mathematical meaning making activities of small groups mediated by a DIS system. Coming to a better understanding of these processes will inform the design of software features that will better support collaborative activities of small groups in a DIS environment, and advance our understanding of mathematical objects and the practices that produce them. In our future work we plan to elaborate more on these aspects of collaborative math problem solving activities.

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