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Computer-Supported Math Discourse Among Teachers and Students

This full research-and-development project designs, develops and tests an interrelated system of technological, pedagogical and analytic components to provide a range of opportunities for middle- and high-school students to engage in significant mathematical discourse (DR K-12 challenge 2); it catalyzes and supports these opportunities by enhancing the ability of in-service teachers to engage in, appreciate and foster math-problem-exploration and math-discourse skills in their students (DR K-12 challenge 3). The project addresses the core STEM discipline of mathematics by motivating the identification, comprehension and enjoyment of mathematical discourse skills through socially interactive, collaborative learning experiences involving pedagogically organized series of stimulating, skill-appropriate problems using computer-based visualization/exploration and small-group math-problem discussion.

The project's design-based-research approach crafts a socio-technical *educational model* to provide a comprehensive, practical package of tools and techniques for classroom teachers and students, which integrates and refines a number of mutually supportive components: (a) *Innovative technology*: A custom, open-source virtual learning environment that integrates synchronous and asynchronous media with the first multi-user dynamic-math-visualization application. (b) *Curricular resources*: Problem-based learning topics in specific areas of mathematics designed to help teachers tune rich math problems to local texts or curriculum and to guide student exploration. (c) *In-service teacher professional development*: Practicing teachers in online masters programs are mentored to understand and model the innovative technologies and pedagogies by doing collaborative problem posing/exploring/solving and engaging in collaborative reflection on the math discourse in their logged interactions. (d) *Middle- and high-school students*: The teachers introduce the model, technology and resources into their classrooms.

The project builds on and integrates previous work of the PIs, including: the discourse-analysis-based theory of group cognition (Stahl, 2006); the Virtual Math Teams learning environment developed, analyzed and evaluated in (Stahl, 2009b); curricular materials and dynamic math visualization software of GeoGebra, adapted to flexible multi-user collaborative learning; online professional development and online mentoring of in-service math teachers at the Math Forum and at the Drexel and Rutgers-Newark schools of education; and the adaptation of conversation analysis to text-based chat interaction analysis, designed to highlight how collaborative problem solving or group knowledge building takes place. The project adapts components that have been explored, prototyped, or piloted by the PIs to classroom use. Project key personnel and Advisory Committee members bring expertise and experience in educational software R&D; math problem-set adaptation, dissemination and mentoring; in-service math teacher training; online math resources, collaborative learning, problem-based learning and dynamic math; design-based educational research management and evaluation; theory of knowledge building in small groups and in online communities. They also bring opportunities for national deployment and scaling up.

Intellectual merit. This project integrates leading-edge cyber-learning-environment technology incorporating innovative collaborative math exploration tools with educational approaches based on current directions in the learning sciences. It approaches this through a systematic iterative process of co-evolving the technology and curricular resources in the context of engaging, reflective collaborative-learning experiences of significant mathematical discourse by in-service teachers and their students. It thereby advances theory, technology and practice within real-world educational settings to forge a coherent research-based approach to math education appropriate to today's challenges and potentials.

Broader impact. The project designs, tests, integrates, evaluates and disseminates technology, curricular resources, pedagogical methods and analytic tools for use in math-teacher professional-development programs, classrooms of math students, home-schooling networks, online schools and the Math Forum community (over three million visits per month). Project results will support the use of math exploration technology within collaborative math-discourse approaches at diverse schools nationally through their spread to in-service teacher-training programs and services—bringing practical cyber-learning of math to at-risk and isolated math students. It documents the potential impact on both teachers and students of this computer-supported math-discourse approach quantitatively and qualitatively.

Computer-Supported Math Discourse Among Teachers and Students Project Description

Mathematics education in the future faces enormous opportunities from the availability of ubiquitous digital networks, from innovative educational approaches based on theories of collaborative learning and from rich resources for interactive, online, dynamic math exploration. The fact that more and more teachers and students are learning online—with distance education, online masters programs, home schooling, online high schools, etc.—makes the incorporation of virtual collaborative learning environments a natural trend. A major issue in realizing these opportunities on a broad scale in schools is empowering teachers to appreciate and engage in the new approaches, and supporting them with appropriate tools, models and resources for practical instructional usage.

This project therefore proposes to develop a model of professional development and a suite of supports for math teachers. It will design, test, evaluate and refine a virtual learning environment that integrates synchronous and asynchronous media with an innovative multi-user version of a dynamic math visualization and exploration toolbox. Online teams of in-service teachers will be introduced to the collaborative exploration of Common Core State Standards-based math topics in this environment. They will then be guided in reflection on their own team’s discourse with the use of chat-replaying tools. As they become familiar with the use of the technology and with the nature of collaborative math discourse, some of the trained teachers will mentor other teachers through a similar process of engagement. Also, they will introduce their students—primarily in diverse urban schools—to experiences of mathematical exploration and to reflection on math-team discourse. The model of math teacher professional development and of student collaborative math learning centers on the *production of significant math discourse*.

Theoretical Framework: Math Cognition as Math Discourse

To mathematicians since Euclid, math represents the paradigm of creative intellectual activity. Its methods set the standard throughout Western civilization for rigorous thought, problem solving and argumentation. Many of us teach math in part to instill in students a sense of deductive reasoning. Yet, too many students—and even some math teachers—end up saying that they “hate math” and that “math is boring” or that they are “not good at math” (Boaler, 2008; Lockhart, 2009). They have somehow missed the intellectual math experience—and this may limit their lifelong interest in science, engineering and technology. According to a recent “cognitive history” of the origin of deduction in Greek mathematics (Netz, 1999), the primordial math experience in 5th and 4th Century BC was based on the confluence of labeled geometric diagrams (*shared visualizations*) and a language of written mathematics (*asynchronous collaborative discourse*), which supported the rapid evolution of math cognition in a small community of math discourse around the Mediterranean, profoundly extending mathematics and Western thinking.

The vision behind our project is to foster *communities of math discourse* in networks of math teachers, in classrooms of K-12 math students and in online communities associated with the Math Forum. We want to leverage the potential of networked computers and dynamic math applications to catalyze groups of people exploring math and experiencing the intellectual excitement that Euclid’s colleagues felt—refining and testing emerging 21st Century media of *collaborative math discourse* and *shared math visualization* to support math discourse in both formal and informal settings and groupings. Those members of the project team who teach math teachers masters-level courses and professional-development workshops—and others—have found that many people teaching K-12 math have had little experience themselves participating in processes of mathematical exploration and discovery (Krause, 1986; Livingston, 1999; Silverman & Thompson, 2008). This project is designed to provide teachers with first-hand experiences and to mentor them in guiding their students to engage in rich math discourses that go beyond generating numeric answers to supply math reasoning and to draw conceptual connections (Briedenbach et al., 1992; Carlson, 1998; Carlson et al., 2002; Monk, 1992; Thompson, 1994).

The learning sciences have transformed our vision of education in the future (Sawyer, 2006; Stahl, Koschmann & Suthers, 2006). New theories of mathematical cognition (Bransford, Brown & Cocking, 1999; Brown & Campione, 1994; Greeno & Goldman, 1998; Hall & Stevens, 1995; Lakatos, 1976; Lemke, 1993; Livingston, 1999) and math education (Boaler, 2008; Cobb, Yackel & McClain, 2000; Lockhart, 2009; Moss & Beatty, 2006), in particular, stress collaborative knowledge building (Bereiter, 2002; Scardamalia & Bereiter, 1996; Schwarz, 1997), problem-based learning (Barrows, 1994; Koschmann, Glenn & Conlee, 1997), dialogicality (Wegerif, 2007), argumentation (Andriessen, Baker & Suthers, 2003), accountable talk (Michaels, O'Connor & Resnick, 2008), group cognition (Stahl, 2006) and engagement in math discourse (Sfard, 2008; Stahl, 2008). These approaches place the focus on problem solving, problem posing, exploration of alternative strategies, inter-animation of perspectives, verbal articulation, argumentation, deductive reasoning and heuristics as features of *significant math discourse* (Maher, Powell & Uptegrove, 2010; Powell, Francisco & Maher, 2003; Powell & López, 1989).

To learn math is to participate in a mathematical discourse community (Lave & Wenger, 1991; Sfard, 2008; Vygotsky, 1930/1978) that includes people literate in and conversant with topics in mathematics beyond basic arithmetic. Learning to “speak math” is best done by sharing and discussing rich math experiences within a supportive math discourse community (Papert, 1980; van Aalst, 2009). By articulating thinking and learning in text, students make their cognition public and visible. This calls for a reorientation of the teaching profession to facilitate dialogical student practices as well as requiring content and resources to guide and support the student discourses. Teachers and students must learn to adopt, appreciate and take advantage of the visible nature of collaborative learning. The emphasis on text-based collaborative learning can be well supported by computers with appropriate computer-supported collaborative learning (CSCL) software, such as that prototyped in the Virtual Math Teams (VMT) Project (Stahl, 2009b).

Research Project Goal, Hypothesis and Components

Project Goal

*To incrementally refine a research-based, classroom-tested model of computer-supported, resource-supported **math education** through **shared visualizations** and **collaborative discourse** by groups of mentored **teachers** and groups of their **students**—by designing, developing and testing: (i) a **discourse-based model** of math-teacher professional development and mentoring support; (ii) customized **technology** for computer support of **shared math visualization** and **joint exploration**; and (iii) adaptable, standards-based math-content teaching **resources** for middle-school and high-school students, guidelines for group **collaboration** and **accountable talk**, tools for **reflection** on discourse and networks of on-going **mentoring** relationships for math teachers.*

Research Hypothesis

The project is based on an hypothesis, which it will test, concerning how to *increase the quality and quantity of significant math discourse* among math teachers and K-12 students:

Indicators of math learning (by groups of teachers and groups of their students)—such as group discussion of math content, problem posing/exploring/solving, explanation of math moves, visualization or investigation of multiple representations, and reflexive analysis of group math work—can be increased through (i) a math-discourse-based **model of in-service teacher professional development** supported by and integrated with use of (ii) a multi-user version of **dynamic mathematics technology** integrated in a rich online learning environment to support shared visualization and joint exploration of mathematical topics and (iii) **mentoring relationships, collaboration and accountable talk guidelines, and curricular resources** for online professional-development courses, K-12 classes and formal and informal online math communities.

This hypothesis is intended to guide iterative cycles of trial and analysis in design-based research (design, develop and test—not to prove efficacy and effectiveness). It will assess the effect of the combination of project components—because in such a socio-technical system the effect of introducing the technology is highly dependent upon the mentoring and the use of appropriate resources.

The hypothesis centers on measurements of *group* math discourse rather than on assessment of *individual* learning of math content—in accordance with the socio-cultural theory that effective individual math learning can be an indirect product of *participation* in group math discourse (Lave & Wenger, 1991; Sfard, 1998; 2008; Stahl, 2006; Vygotsky, 1930/1978). Vygotsky's notion of the *zone of proximal development* suggests that students may be able to engage in mathematical work within groups at a level that they will not be able to engage in for a couple years as individuals—and that such group work can be essential for the individual development in the long run (Vygotsky, 1930/1978, pp. 84-91). As a result, there is a need to assess the educational effectiveness of group interactions as such, beyond pre/post tests of the individuals. In addition, the striking finding within CSCL research of *productive failure* (Barron, 2003; Kapur & Kinzer, 2009; Patak et al., 2011; Schwartz, 1995) shows that there can be a paradoxical inverse relationship between measures of successful learning by small groups versus by the individual members of those groups because of group processes that reveal deep mathematical relationships but that do not lead immediately to high test scores of the individuals. For these reasons, *the project evaluates its goal in terms of the quantity and quality of the math discourse* that takes place during the small-group problem-solving interactions, looking for hypothesized increases for groups as they participate and in successive project years as the model, technology and resources are iteratively developed.

(i) Model of Math Education

The proposed project will design, develop and test a *model* of math education through collaborative math problem proposing/exploring/solving, by involving in-service teachers in first-hand mathematical experiences and helping them to reflect on their own learning experiences. Then they will try out the model with their students, while receiving mentoring and support from the project. The collaborative model of math education stresses math discourse. In this project, groups of teachers and groups of students will do math problem solving collaboratively and then reflect on the logs of their discourse to identify key moves. We propose using teachers' and their students' original mathematical conversations as “didactic objects” (Thompson, 2002) designed to support “decentering” (Wolvin & Coakley, 1993) and “collective reflection” (Cobb et al., 1997) on particular aspects of their math discussion. The discourse-centered model of math education will structure learners in small teams and will provide mentoring to guide the team's mathematical exploration, discourse and learning. Math Forum staff and other project team members will provide initial mentoring to the first cohorts of teachers, who will in turn mentor subsequent cohorts of teachers as well as students in their own classes. A permanent support network will be established to provide sustainability of project accomplishments. The teachers who are trained in this project will be encouraged—initially by paying them—to participate in teacher networks, including national and international networks of teachers, supporting broadening dissemination of the discourse model of math education.

(ii) Online Math Collaborative Learning Environment

The proposed project will design, develop and test two forms of *technology* to support math learning with collaborative and interactive tools for cyberlearning: (a) computer-supported collaborative learning (CSCL) software and (b) dynamic mathematics (software that allows users to manipulate geometric diagrams, equations, etc.). (a) CSCL provides virtual learning environments in which teams of students can interact synchronously and asynchronously to build knowledge together. This student-centered approach has many advantages, including increased motivation, sharing of skills, engaging in significant discourse and practicing teamwork. This project will adapt and extend the Virtual Math Teams (VMT) environment already prototyped and tested by the PIs (Stahl, 2009b). (b) Dynamic math (such as Geometer's Sketchpad, Mathematica, Cabri or GeoGebra) has already profoundly impacted math education (Goldenberg, 1995; Hoyles & Noss, 1994; King & Schattschneider, 1997; Laborde, 1998;

Myers, 2009; Scher, 2002), with Geometer's Sketchpad and GeoGebra used in many US classrooms and globally. Yet, research on math education has not analyzed how students use dynamic math tools in sufficient detail (compare Çakır, Zemel & Stahl, 2009; Stahl, 2009b). GeoGebra (<http://www.geogebra.org>) is an open-source system for dynamic geometry, algebra and beginning calculus—including trigonometry, conics, matrices, graphing and Euclidean constructions. It offers multiple representations of objects in its graphics, algebra and spreadsheet views that are all dynamically linked, making GeoGebra a particularly flexible tool for exploration. Working with the developers of GeoGebra, this project will provide the first multi-user version of dynamic math, so that teacher teams and student teams can explore math collaboratively; it will integrate this into the larger VMT virtual collaborative-learning environment with text chat and wiki to support persistent discourses about math—that can be shared, reflected on and researched.¹

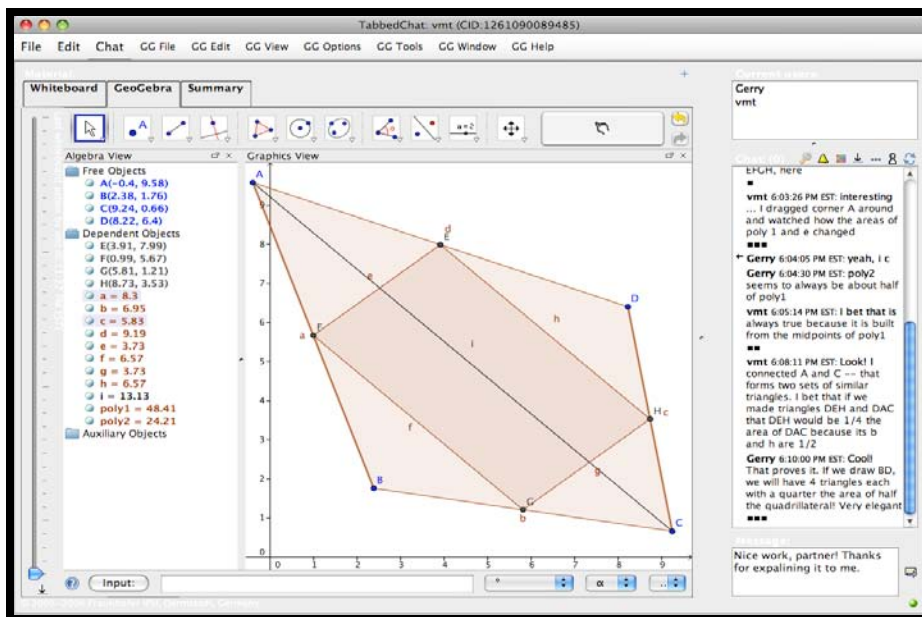


Figure 1. A demo (not real student interaction data) GeoGebra construction created and discussed collaboratively in a proof-of-concept multi-user prototype of the project's learning environment, based on the VMT system. The VMT system includes (not shown here): a Lobby with social networking and tools for teachers, integration with a wiki, and Web browsers.

(iii) Curricular Resources

The proposed project will design, develop and test **resources** to support teachers and students in their interactive explorations of rich math problems (e.g., open-ended problems with multiple possible solution approaches and many potential extensions to explore). Three kinds of resources are: (1) Curriculum packages in domains of K-12 math, building on existing NSF-funded and community-based sources (see <http://dynamicgeometry.com>, <http://keypress.com/x5582.xml> and <http://geogebra.org/en/wiki>). The curriculum will be based largely on classroom-tested problems using dynamic-math software and integrated with popular math textbooks (e.g., *Everyday Mathematics*, *Investigations in Number, Data and Space*, *Mathematics in Context*, *Connected Mathematics*, *Interactive Mathematics Program*, *Core-Plus Mathematics*, *Simms Integrated Mathematics* and textbooks from McDougal Littell or Glencoe), but adjusted by experienced Math Forum staff for collaborative online usage. It will be aligned with the recommendations of the *Common Core State Standards for Mathematics* and the new NCTM volumes, *Focus in High School Mathematics: Reasoning and Sense Making in Algebra/Geometry*. Teachers will be mentored in adapting the content of their local curriculum to collaborative online student exploration,

¹ For a demo of the prototype system, go to <http://vmt.mathforum.org/VMTLobby>. Log in as “guest” with password “guest”. The Lobby should open showing the List of All Rooms. Select Project “VMT Research”. Click on “Apply filters”. Open “Geometry”. Open “Polygons”. Click on “GeoGebra Demo Room” Eventually a JavaWebStart chat room should open. Explore its different tabs and functions.

whether using GeoGebra or not. (2) Guidelines, suggestions and examples for collaborative learning, knowledge building and math exploration will be published. This will feature “accountable talk” guidelines for math discourse. (3) Training resources in understanding online math discourse will be developed to help teachers and students identify examples of productive inquiry moves, etc., to foster reflection on logs of their math discourses. These broad categories of resources will encapsulate the expertise of the project team in problem design, collaboration mentoring and discourse analysis, producing documents that can be used by a gradually growing community of math teachers and students. The content of these resources will build on experience at the Math Forum, the VMT Project, the teacher professional-development programs at Drexel and Rutgers and the related research literature. The content will be elaborated, tested, evaluated and refined—and then published as project deliverables.

Results from Prior NSF Support

The proposed project grows out of the successful Virtual Math Teams (VMT) Project. This is a several-year NSF project (awards DUE-0333493, IERI-0325447, SBE-0518477, DRL-0723580) that developed an open-source virtual learning environment for math students. The system integrated a social-networking portal, synchronous text chat, a shared whiteboard, an asynchronous wiki, a referencing tool, mathML expressions and a web browser. Student actions and chat postings are automatically logged to be replayed for analysis. Over a thousand student-hours of piloted usage were logged. A qualitative micro-analytic approach to interaction analysis was developed based on ethnomethodologically inspired conversation analysis (Garfinkel, 1967; Sacks, 1962/1995; Stahl, 2009a; 2009c; Zemel, Çakir & Stahl, 2009). A large number of publications have appeared from the project (see <http://GerryStahl.net/vmt/pubs.html>), including 2 books (Stahl, 2006; 2009b) and 6 doctoral dissertations (Çakir, 2009; Litz, 2007; Mühlpfordt, 2008; Sarmiento-Klapper, 2009; Wee, 2009; Zhou, 2010).

The VMT Project pioneered the study of online collaborative math discourse—both its nature and modes of computer support for it. The 28 studies in (Stahl, 2009b) present some of the most important of the 169 publications related to the project. They include a number of dissertation-level case studies of interactions in the VMT environment by middle-school, high-school and junior-college students, which analyze: how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face; how the structure of text chat interaction differs from spoken conversation; how the media of graphical diagrams, textual narratives and symbolic representations can be intimately interwoven to build deep math understanding; how deictic referencing is important to establishing shared understanding; how students co-construct a joint problem space; how collaborative meaning making and knowledge building are accomplished in detail; how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and how a methodology based on interaction analysis can be used for a science of group cognition.

The VMT Project was structured as design-based research, with the technology, research and theory co-evolving through dozens of iterations. The VMT Project demonstrated both the practicality of the proposed project and the need for it. While the VMT Project prototyped a rich cyber-learning environment and studied student interaction, it did not develop the range of supports that we know are needed for classroom use: robust software, problem sets, guidelines, etc. Furthermore, it did not include a dynamic-math component. The VMT Project provides a solid starting point for the proposed project and documents the need for further technological development, enhanced support for dynamic math, curricular models and training of in-service teachers. The design, development and testing of these logical next steps are needed to enable a powerful and innovative form of math education to be offered in a practical form to K-12 schools through education schools and to the public through the Math Forum.

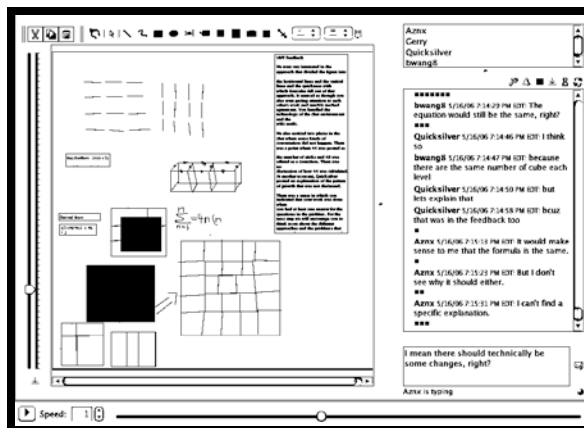
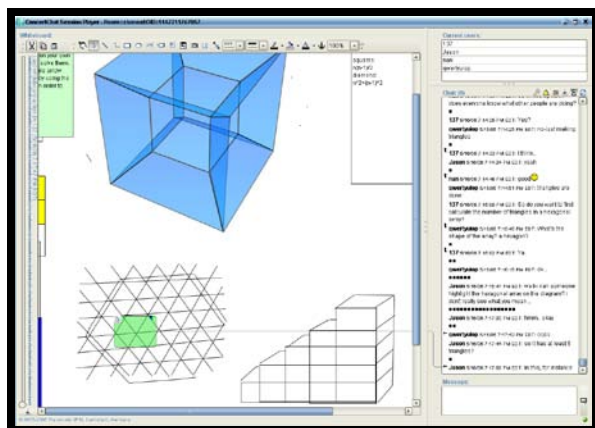
Prior NSF support of the Math Forum has developed a successful approach to online mentoring of math teachers and their students. Since 1993, the Math Forum has mentored over 100,000 students, conducting hundreds of workshops, summer institutes and school-improvement contracts. Recently, it has successfully completed the Virtual Fieldwork, Online Mentoring, and Teacher Workshop Model projects (NSF DUE-0717732, DUE-0127516 and DUE-0532796). Mixed-methods studies of these have shown

the surprising result that the online mentoring of K-12 pre- and in-service teachers had a more positive effect for teachers with low math self-efficacy (Renninger et al., in press). This is due to the non-linear and flexible format of online discussion—suggesting that online collaboration may well help at-risk math students at least as much as those with higher math self-efficacy. Math Forum approaches are making inroads with a population of people who most would think will not change (Renninger et al., 2010). In the proposed project, Math Forum workshops for teachers will complement and feed teachers into the courses at Rutgers and Drexel. The workshops will also train mentors and seed the on-going teacher network.

Research and Development Design

The proposed project adopts an iterative design-based-research approach to design, develop and test innovative curriculum materials, technologies, teaching methods and models for teacher in-service professional development and K-12 student instruction. The project develops a socio-technical educational model that evolves and integrates a number of mutually supportive components, each of which has previously been explored in a preliminary way by one or more of the PIs. However, the components have not previously been integrated into a scalable model of math education. The proposed project brings together the PIs, other necessary senior staff and advisors with the resources to begin to systematically test, refine, validate and disseminate the integrated model. There are several areas of work:

(a) A model of math education as computer-supported math discourse. The model incorporates: (b) innovative technology for collaborative math discourse, (c) support for shared mathematical visualizations and (d) curricular materials to stimulate and guide math discourse. The model includes three successive project targets: (e) in-service teacher professional development, (f) middle- and high-school math education and (g) broader virtual math-discourse communities.



Figures 2 and 3. Images of actual student online collaborative work on patterns. In Figure 2, a student points from a chat message to a smallest hexagon pattern composed of 6 triangles illustrating VMT's unique integration of chat and whiteboard with its deictic reference tool. Figure 3 shows the Replayer tool interface across the bottom.

(b) Innovative technology for collaborative math discourse. The VMT Project developed a research prototype of a custom, open-source virtual learning environment that integrates synchronous (text chat, shared whiteboard, dynamic math exploration, shared web-browser) and asynchronous (a community wiki, a social-networking portal) media to support math visualization and collaborative discourse by virtual math teams. This prototype was adequate for extensive testing in multiple iterations, as well as limited use by select teachers in their classrooms as part of research trials. As part of the proposed project, we will implement, test and refine new interfaces for teachers, mentors and administrators. These will allow *teachers* to register a number of students at once, set up multiple copies of interaction rooms for multiple small groups of students, monitor activity in rooms, respond to problem behavior online and review reports of student activity. New functionality will also make it easier for *students* to document

their online work (e.g., in the project wiki or in Word documents, Excel spreadsheets and PowerPoint slides) with log excerpts and images of constructions. Support for *researchers* will facilitate researchers in the project as well as colleagues outside the project to easily replay sessions of student interaction.

The VMT Project was widely recognized as an important example of synchronous support for online collaboration and was studied by several international researchers (GerryStahl.net/vmt/pubs.html); it is expected that the proposed project will be of even more interest, particularly within the math education research community. The VMT Replayer allows complete replay of a user session, including all actions and system notices, as though the session was digitally video-recorded. The researcher's view is guaranteed to be identical to the user's view since it is generated from the same data as sent to a client computer. The log information will be made available in convenient textual formats for student reflection and reporting as well as for researcher analysis. New functionality to be explored includes automated feedback agents and displays, increased integration so math objects can be moved easily from the synchronous tabs (chat, whiteboard, summary, GeoGebra, web browsers) to asynchronous components (wiki pages, email, documents), as well as refinement of the interface. The system will be released as open source on SourceForge so that others can deploy it on their own servers or extend the software to meet their own educational needs. The Math Forum will maintain the system as a permanent service, so that users can easily create topics for chat rooms and invite other users to collaborate.

(c) Support for shared mathematical visualizations. The project will port GeoGebra—a comprehensive and well established application for dynamic-math exploration—to the virtual learning environment described above. It will make the application fully multi-user. It will integrate the application in a tab of the environment (see Figure 1 above). As previously described, GeoGebra is a particularly appropriate dynamic-math application for this project because its source code is freely available as open source, there is an active international development community to support on-going development, the lead developer and the founder are committed to consult on this project, the application supports a wide range of math from algebra and geometry construction to calculus and 3-D, GeoGebra has won international prizes, it has been translated into about 50 languages and it has received on-going NSF support. Like all other dynamic-math applications, GeoGebra currently exists only as a single-user application. While users can send their static constructions to each other, display screen images, or awkwardly include a view of the GeoGebra application within other environments through screen sharing (e.g., in Blackboard, Moodle, Elluminate, etc.), only one person can dynamically manipulate the construction. Our port converted GeoGebra to a client-server architecture, allowing multiple distributed users to manipulate constructions and to all observe everyone's actions in real time. Every action in the GeoGebra tab will be immediately broadcast by the server to all collaborating clients (and logged in detail for replay and research). We have been exploring turn-taking mechanisms (see Figure 4) to avoid conflicts in the construction and modification of GeoGebra drawings; although it is important in synchronous chat to allow multiple users to type simultaneously, we have found that it is natural for a group to allow one member at a time to change a graphical construction and for group members to take turns editing and rearranging.

Incorporation of GeoGebra in the VMT environment framework allows users to engage in text chat while manipulating the construction. Importantly, users can graphically point from a chat posting to an area of the construction that they want to index (see Figure 2)—an important support for math discourse that is unique to VMT. They can also scroll back and forth through the history of the GeoGebra construction, animating its evolution—a powerful way to explore many mathematical relationships. In addition, a complete record of the collaborative construction is

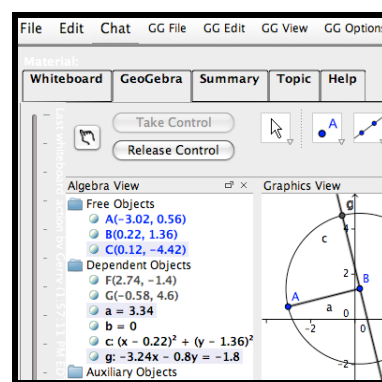


Figure 4. The GeoGebra tab with turn-taking button to avoid conflicts.

available to the participants, their teachers and project researchers, allowing them all to analyze and reflect upon the complete

interaction, including the construction actions synchronized with the chat. We have already completed a prototype port of GeoGebra to VMT in order to confirm its feasibility. It provides an exciting collaborative experience. The port now needs to be engineered in a robust way, incorporating all of the GeoGebra functionality (including import and export compatible with standard GeoGebra and Geometer's Sketchpad to facilitate sharing of constructions, and a full menu system to support learning by new users). In Year II of the project, we will incorporate the extended GeoGebra 4.0 functionality that will be released by then, including support for inequalities and CAS (computer algebra system like Mathematica, Maple, or the TI-Nspire CAS calculator). The project will produce a refined and tested multi-user version of GeoGebra and will release it as open source.

(d) Curricular materials to stimulate and guide math discourse. Problem-based learning (PBL) materials in areas of mathematics like algebra, combinatorics and geometry will be adapted from existing high quality curricula and piloted. These materials will define challenging math problems for collaborative online group exploration and help teachers to tune them to local student capabilities. The materials will allow students to explore rich but accessible problems taken from topic domains covered in their textbooks and in the Common Core State Standards. The PBL approach involves mentors who are trained to guide student exploration and to steer collaborative student groups to address their joint learning issues (Barrows, 1994; Hmelo-Silver, 2004; Hmelo-Silver & Barrows, 2008; Koschmann, Glenn & Conlee, 2000). Project team members and others have developed some model math problems (Krause, 1986; Math Forum & Wolk-Stanley, 2003a; 2003b; 2004a; 2004b; Powell, Lai & O'Hara, 2009). The Math Forum has years of Problems-of-the-Week in several areas of school mathematics, which can be adapted to online group collaboration. Much curriculum has been developed with NSF funding for dynamic-math applications like GeoGebra and Geometer's Sketchpad, including lessons tied to state standards and intended to support popular textbooks through student hands-on exploration. The project will facilitate classroom teacher use of such resources in this new learning context. The team has already prototyped a series of problems that consecutively explore issues of combinatorics; along with the problems, a teachers' guide contains concrete suggestions on how to adapt the problems for different kinds of student teams (Powell, Lai & O'Hara, 2009). The problems in this document were tested in the VMT Project and in high-school classrooms of teachers studying at Rutgers. Sets of problems correlated to textbooks and to the Common Core State Standards will be compiled, some taking advantage of GeoGebra. Additional resources will be developed to train teachers and students in mentoring techniques, in collaboration skills and in math-discourse skills. All these resources will be tested and produced in publically available online documents as project deliverables. These and other math problems will be incorporated in the VMT Lobby's library of Topics, to be available to students in home-schooling and informal-learning situations.

(e) In-service teacher professional development. To effectively change education in schools, teachers must be prepared to understand and to learn how to model use of the innovative technologies and pedagogies. Practicing teachers rarely find time to engage in learning processes capable of transforming their teaching practice and they seldom are able to introduce major new approaches in their highly constrained curricula. This project therefore involves in-service teachers when they have scheduled time to pursue masters-level professional-development courses. It starts by involving them during their regular courses (taken online) in online collaborative problem solving using the project's software technology and curricular approach—(a), (b) and (c) above. Later course work involves them trying out what they have learned back in their own classrooms, within the context of their current curriculum; the project provides mentoring and resources to support this effort.

Both Drexel and Rutgers-Newark offer masters-level teacher-professional-development programs and courses in math education in online modes. The fact that these teachers will already be studying together online creates an ideal setting for the use of an online learning environment with dynamic-math support. These graduate programs have been designed, taught and directed by project co-PIs Silverman and Powell. The proposed project will allow these programs to develop, test and adopt the educational model of computer-supported math discourse. This model will be pioneered at these two schools of

education, providing a collaborative interaction that will produce a more generalized result than would development at a single institution. It will also permit extended utilization of the online medium by, for instance, having teachers from both institutions working together on math topics in small groups and having them mentor teachers from each other's institution. In the later years of the project, this model will be disseminated to other schools of education, partially through Advisory Committee members. The Math Forum has effectively implemented a similar model, incorporating its Online Mentoring Project modules into teacher education programs around the country.

The initial plan at *Drexel University* is to build on the existing MS in Mathematics Learning and Teaching (MS-MLT) program, which is already exclusively offered online. This program in math education was originally developed by co-PI Silverman and is taught primarily by him and Math Forum staff. For the first cohort of students under this project, Drexel will offer MTED775, "Special Topics: Supporting Math Learning through Computer-Supported Collaborative Discourse." This course will be one required math-education elective for MS-MLT students and an elective for other professional-development students. Then two new education courses will be developed to make this model a part of the regular course offerings of the School of Education: MTED 651 (which will focus on teachers personally engaging in computer-supported, resource-supported collaborative discourse and reflection on both their activity and their learning) and MTED 652 (which will focus on supporting teachers to incorporate computer-supported, resource-supported collaborative discourse in their classes). MTED 652 will include resource development for teachers' classroom implementations. Each of these courses—which have been approved at Drexel pending funding of this project—will carry 3 quarter-credits.

The initial plan at *Rutgers-Newark* is to engage two cohorts each year of practicing teachers in a revised version of the online course in "Mathematics and Instructional Technology" taught by co-PI Powell. The goals of the course are three-fold: (1) to familiarize in-service teachers with the mathematical problem-solving and problem-posing activities of the online problem-exploration units in which their students will engage; (2) to deepen in-service teachers' thinking about the effects of the collaborative environment on their own and their students' thinking about mathematics (math objects, relations among objects and dynamics among relations), math reasoning and problem-solving heuristics; (3) to focus in-service teachers' instructional attention on understanding and facilitating students' discourse in mathematics. To accomplish these goals, the course will engage in-service teachers in a sequence of tasks, beginning with familiarizing them with the project online environment through involving them in mathematical activities using it, then engaging them in reviewing their session logs and finally having them plan how they will implement the model in their teaching.

(f) Middle- and high-school math education. The in-service teachers will introduce the technology and curricular resources that they used in their university classes into the classes they teach, often mixing students from different schools or cities in online teams to take advantage of being part of an online discourse community and to motivate the use of online media by students in face-to-face classrooms. The teachers will take the logs of their students' interactions back to their professional-development sessions for on-going group analysis. They also will engage their students in reflection on their own logs, discussing how the math discourse surfaces mathematical insights and conceptual connections.

The curricular resources adapted by the project are designed to support classroom math activities by enhancing and reinforcing the core objectives covered in textbook readings and instructor-led activities. Resources include adaptation options and guidelines to help teachers tune problem sets to complement their core activities. For instance, the research-based textbooks, *Mathematics in Context* and *Discovering Geometry*, which are used in the Philadelphia public school system, stress student investigation in order to construct conceptual understanding of key math concepts and the *Common Core State Standards for Mathematics* recommend that "students consider the available tools [such as] dynamic geometry software...to explore and deepen their understanding of concepts" (p.7). The project model builds on this approach, providing opportunities for students to explore and discuss topics online with peers from their own or other schools. The model provides: tools for dynamic, multi-user, graphical exploration; visual and numeric feedback on quantitative and qualitative changes during exploration; and a record of the

exploration and accompanying discourse, which students can replay, reflect on and incorporate in reports—e.g., pasting log excerpts or screen images in their documents.

Reflection on interaction logs by teachers and students primarily involves trying to follow the problem-solving path of participants and to notice critical collaboration moves. They will be encouraged to look for examples of accountability to the group, to standards of math reasoning and to the characteristics of their math objects. They will look for instances where someone poses a productive inquiry that initiates effective group exploration—or where the group fails to come up with a useful proposal or fails to take up a proffered proposal. Examples will be culled and shared on the project wiki.

Although many project activities center on teacher professional development, the ultimate goal is to *increase the quality and quantity of both teacher and student mathematical discourse*. Therefore, teacher professional development will be oriented to improving the math discourse of their students. While the primary indicator of project success will be the identification of desirable mathematical discourse moves during problem solving by teachers and students, the project will also be concerned with changing student conceptions of math. It will survey a sample of teachers and students before and after their involvement in the project to compare self-reports of attitudes about math and about approaches to math instruction. In addition, some teachers and students will be asked as a final part of their course work to compose a brief reflection paper on their learning experience.

Most of the in-service teachers in the project come from the Philadelphia, Camden, Newark, New Brunswick and New York City areas. Thus, many of the classrooms that will be involved in the program are inner-city K-12 schools with high proportions of educationally at-risk and economically disadvantaged students; others are from near-by suburban and private schools with contrasting student populations. The project educational model will therefore be tested in diverse, real-world settings.

Because teacher and student work on math problems will all take place in the online software environment, complete detailed logs will be available to the project staff, as well as to the students and teachers themselves. The logs can be reviewed and studied in detail with the Replayer software, as well as with various formats of log printouts. This will not only facilitate reflection by students and teachers on their own work, but also permit the documentation of interesting cases for teacher instruction and detailed analysis for project evaluation. The project will compile a portfolio of instructive case studies.

(g) Broader math-discourse communities. Once teachers studying at Drexel or Rutgers and their students become involved in online collaborative dynamic geometry and math discourse, teams will be set up that involve students from online schools, home-schooling networks or the Math Forum virtual community. This will yield data for generalizing project findings as well as stimulate the spontaneous generation of self-organizing communities of math discourse. This will primarily take place through contacts and presentations by project staff and the teachers who have been trained, as well as through the Math Forum and its large user community (3 million visits/month). The project technology and resources will be made publically available as an integral part of the Math Forum services in Years IV and V of the project. The VMT software environment is designed to support the viral spread of user communities across the Internet; the proposed project is intended to form a critical mass of users and topics to catalyze that process. The model of computer-supported math discourse will become institutionalized at Drexel and Rutgers, will be taken to other schools of education through Advisory Committee members and personal contacts of project staff, through Math Forum outreach, through the extensive active GeoGebra user community and through presentations at educational conferences and in related journals.

(h) Group cognition theory. When small groups engage in collaborative problem posing, exploring and solving, they can accomplish cognitive tasks interactively or transactively as a group. The project will analyze logs of student math work, shared visualizations and reflective discourse, using conversational analysis and statistical methods to study how students build on each other's utterances, constructions and actions to accomplish mathematical cognition. Building on past work on group cognition (Çakır, Zemel & Stahl, 2009; Koschmann, Stahl & Zemel, 2009; Stahl, 2006; 2010a; 2010b), this will provide a contribution to theory of situated and distributed cognition. In particular, analysis of the use of GeoGebra in a fully logged multi-user online environment with guidance in math discourse moves will pioneer in

the development of theory of cognition in groups using dynamic-math tools, providing insight into math learning generally. Case studies and other findings with theoretical implications will be published.

Project Phases, Milestones, Deliverables

	Year 1	Year 2	Year 3	Year 4	Year 5
Model	Design, development, testing of courses for teachers	Integrate technology and resources fully in courses	Refine model based on formative evaluation	Publish articles about model	Disseminate model to other schools of education, etc.
Technology	Debug VMT; multi-user GeoGebra 3.3; menu system; implement full logging	New VMT functionality; multi-user GeoGebra 4.0; teacher admin & monitoring supports	Release VMT as a Math Forum service; automate statistical analysis	Support VMT Open Source; develop feedback of analysis to participants	Disseminate VMT servers
Resources	Pilot teacher resources; develop student resources	Test teacher and student resources	Evaluate use of resources	Publish resources	Disseminate resources
Curricular materials	Review existing materials for GeoGebra and Geometer's Sketchpad	Compile problem sets aligned with standards and textbooks	Evaluate use of materials	Publish materials in formats for teachers, home schooling, distributed schooling	Disseminate materials
Teachers	Pilot model with 10 teachers in Drexel and Rutgers courses and 20 teachers in Math Forum workshops	Implement model with 35 teachers in Drexel and Rutgers courses and 40 teachers in Math Forum workshops	Evaluate model with 50 teachers in Drexel and Rutgers courses and 40 teachers in Math Forum workshops	Continue training with 60 teachers in Drexel and Rutgers courses and 40 teachers in Math Forum workshops	Evaluate teacher training in Drexel and Rutgers courses; continue training 40 teachers in Math Forum workshops
Students	Pilot with 25 students	Involve 750 students of teachers in courses and workshops; log series of sessions by student small groups	Involve 750 students of teachers in courses and workshops; log series of sessions by student small groups	Involve 750 students of teachers in courses and workshops; log series of sessions by student small groups	Evaluate changes in significant math discourse of student groups over time: within group and across cohorts
Mentoring	Prepare mentoring materials based on previous Math Forum mentoring projects	Pilot mentoring of teachers with 2 outstanding teachers	Increase to 5 teacher mentors	Increase to 10 teacher mentors	Increase to 15 teacher mentors
Theory	Validate coding scheme	Analyze discourse moves in logs	Conduct in-depth case studies and interviews	Compile best practices case studies	Develop theory of math group discourse

Evaluation

Formative evaluation is a constant process built into the design of the project. As a design-based research effort, the over-riding research hypothesis listed at the start of this project description will be addressed by designing and exploring an iteratively refined solution—and by documenting its impact on the quantity and quality of math discourse by teachers and students. The interlocking components of the project will be reviewed at weekly project team meetings. Team meetings will include interaction analysis data sessions (Jordan & Henderson, 1995; Stahl, 2010a), in which the group collaboratively discusses new data from logs of teachers or students—and makes design decisions for refining the co-evolving components. The project team will discuss what seems to be working and what does not. It will decide what to modify for the next iteration. The project is complex, with many dependencies among its components and many shifting contextualities. A flexible approach like design-based research is needed to respond to a continuous formative evaluation and on-going project modification.

The explicit evaluation effort will include semi-annual formative-assessment reports documenting: (a) project progress, (b) improvements in project outcomes and (c) plans for the next half year. The external Advisory Committee (AC) will review, discuss and respond to each report. The AC will meet annually to discuss project progress with the project team. The AC has expertise in mathematics education, research evaluation, teacher training, problem-based learning, conversation analysis, CSCL and virtual communities. Most AC members have been PIs on successful NSF grants in the learning sciences. The AC includes: **Sharon Derry** (Wisconsin), **Cindy Hmelo-Silver** (Rutgers-New Brunswick), **Christopher Hoadley** (NYU), **Timothy Koschmann** (Southern Illinois), **Mary Marlino** (UCAR), **Kay McClain** (Arizona State), **K. Ann Renninger** (Swarthmore), **Lauren B. Resnick** (LRDC, CMU), **Carolyn Penstein Rosé** (CMU), **Anna Sfard** (Haifa & Michigan State), **Wesley Shumar** (Drexel), **Tamara Sumner** (Colorado), **Daniel D. Suthers** (Hawaii), **Alan Zemel** (SUNY Albany). The external evaluator is **Sukey Blanc** (Senior Research Associate with Research for Action), who has led evaluations on projects such as the Metro Math MSP.

As discussed above, the research hypothesis focuses on the quantity and quality of math discourse at the group unit of analysis. Theories of the zone of proximal development, productive failure and group cognition argue that learning-related processes and phenomena at the group level may be different from those at the individual level. Other research has documented the efficacy of dynamic-math visualization tools for *individual learning*; for instance, a study of geometry students in eleven Florida schools revealed a significant difference in the FCAT mathematics scores of students who were taught geometry using Geometer's Sketchpad compared to those who used the traditional method—regardless of differences based on SES or gender (Myers, 2009). The proposed project has a different focus. The PI and colleagues have developed coding schemes and analysis approaches oriented to the *group unit of analysis* based on conversation analysis of adjacency pairs and longer sequences (Sacks, 1962/1995; Schegloff, 2007; Stahl, 2009b, Chs. 20, 22, 23, 26; 2011b; Stahl et al., 2011). This approach serves both quantitative and qualitative analysis, by simultaneously specifying the structure of meaningful discourse moves and providing countable categories of group interaction units, in order to document changes over time—comparing discourse characteristics in selected time slices within teams or across cohorts.

The project will automatically produce raw data in the form of log files of participant online interactions. The log files are anonymous, but allow tracking of individual users through consistent login handles. The VMT environment is instrumented to capture all user actions in the chat and whiteboard—this will be extended to multi-user GeoGebra. A database of all sessions is automatically maintained and provides spreadsheet logs in handy formats and Replayer files. Software tools will be used for automated and manual log analysis of discourse measures and their evolution during training. While low-level group processes (e.g., number, length and rate of chat postings and drawing actions in different time slices) can be tracked automatically and analyzed statistically, higher-level math-discourse processes have to be interpreted manually. The PI has on-going, NSF-supported collaborations with Carolyn Rosé of Carnegie-Mellon University's intelligent tutoring group, exploring software agents in the VMT environment to provide student guidance and also investigating computer support for coding discourse moves in text chat,

to aid and supplement manual analysis. Raw and coded logs will be maintained in a database to facilitate analysis of changes over time for groups across sessions and across successive cohorts of participants.

Quantitative analysis—based largely on the coding of discourse moves in teacher and student VMT logs—will track changes in key measures of significant math discourse. The project hypothesis will be operationalized as predicting an increase in specific measures as a given group works in the VMT environment during time slices across an academic term. Logs of the following groups involved in the project will be evaluated: (a) in-service teachers participating in Math Forum workshops, (b) teachers working together as part of teacher professional development course work, (c) students guided by their teachers, (d) students working with other students as part of school classes and (e) students interacting with others informally at other schools or globally.

Discourse will be coded and measured along the following dimensions: (1) volume of discourse and level of participation, (2) percentage of on-task math discourse, (3) use of representations, (4) integration of chat and drawing, (5) use of accountable talk moves, (6) adoption of socio-mathematical norms and practices, (7) speaking meaningfully with explanation and argumentation, (8) involvement in posing, exploring and solving problems and (9) additional dimensions to be developed based on project experience. The theory of math learning through participation in math discourse (Sfard, 2008) specifies important mathematical discourse moves, such as encapsulation, reification, naming, routines, deeds, explorations and rituals. The theory of accountable talk (Michaels, O'Connor & Resnick, 2008; Resnick, 1988) specifies discourse moves that promote accountability to the group, to standards of math reasoning and to the characteristics of the math objects. Speaking meaningfully in math discourse “implies that responses are conceptually based, conclusions are supported by a mathematical argument and explanations include reference to the quantities in the problem context [as opposed to a focus on merely] describing the procedures and calculations used to determine the answer” (Clark, Moore & Carlson, 2008, p.298). Socio-mathematical norms include what counts as an acceptable, a justifiable, an easy, a clear, a different, an efficient, an elegant and a sophisticated explanation (Yackel, 1995; Yackel & Cobb, 1996). Mathematical practices emerge from interaction, are taken up by participants and are applied repeatedly (Medina, Suthers & Vatrappu, 2009; Stahl, 2011a). These dimensions of significant math discourse are associated with typical sentences and discourse moves that can be identified by coders. A coding scheme will be validated with acceptable inter-rater reliability, as in (Stahl, 2009b, Chs. 22, 23; 2011b).

Detailed interaction analysis of selected cases will show *how* the math discourse actually evolves. Quantitative analysis can establish the statistical significance of changes in learning outcomes, but it generally does not provide much insight into the mechanisms of change; these mechanisms will become visible in detailed case studies in which the specifics of the interactions can be studied. By combining quantitative and qualitative analysis of discourse transformations, the project evaluation will determine how the online interaction involves engagement in significant mathematical discourse. This will help researchers to determine what to try in subsequent cycles of research and will allow evaluators to judge project progress.

Summative evaluation will assess the degree to which the discourse of teams of teachers and of students reveals—through the quantity and quality of their math discourse—increased understanding and improved practice of mathematics. It will make sure that project products (software, mentoring guides, problem sets, masters courses, analysis tools, best practices case library and analyses of case studies from the data corpus) have been produced and made publicly available. It will assess the effectiveness of these products based on the analyses of their use by teachers and students as logged in the data corpus, using quantity and quality of the facilitated math discourse as a measure of success.

In addition to the quantitative and qualitative analysis of changes in significant mathematical discourse by groups of teachers and students involved in the project, there will be ethnographic observations of participants. The observations—including pre/post surveys, open-ended interviews and reflection reports—will be primarily conducted by co-PI Khoo and External Evaluator Blanc, both trained cultural anthropologists. The goal of these observations will be to establish—as much as possible from the user perspective of the project participants—the effectiveness of project interventions (the

pedagogical model, the technology, the resources). Interviews with students and teachers will explore their changed attitudes toward mathematics and their insights into the nature of mathematical reasoning. This will be triangulated with the analysis of the math discourse of the same participants in specific time slices. Ethnographic observation of teachers will additionally explore to what extent they have come to feel that teaching math-discourse skills is key to fostering student math learning; to what extent they try to use the project model, technology and resources in their regular teaching; to what extent they intend to stay involved in support networks. The summative evaluation will report on these issues as well as the timely accomplishment of project tasks, training levels, dissemination efforts and project deliverables.

Dissemination

The primary avenues of dissemination will be: (a) through the Math Forum, (b) through Schools of Education, (c) through teacher professional associations, (d) through GeoGebra and dynamic math user communities and (e) through virtual learning communities, including home schooling and online schools.

(a) By the end of the project, the technology and the resources developed through the project will be publicly available as services of the Math Forum. The Math Forum has been the premier online resource for mathematics teaching and learning for over 16 years. It has three million visits to the site each month; its digital library contains over a million web pages, mostly user generated (as a forerunner of the Web 2.0 philosophy). Public services (which typically started from NSF-supported research projects) have been made sustainable through support from Drexel University, fee-for-service programs and teacher training contracts. The Problem-of-the-Week (PoW) is the Math Forum's core service and is subscribed to by many school districts. It is primarily oriented toward problem solving of challenging math problems by *individual* students. The result of the proposed project would be to extend this service with open-ended math problems for *groups* of students to explore collaboratively online. Teachers using the PoW service would be encouraged to involve their students in the new service, initially interacting with classmates, but eventually joining cross-school, national and international virtual math teams. Math Forum services typically support both formal and informal mathematics learning by teachers and students (Renninger & Farra, 2003; Renninger & Shumar, 2002b; 2004; Shumar & Renninger, 2002).

(b) Several of the co-PIs and Advisory Committee members (e.g., Powell, Silverman, Derry, Hmelo-Silver, Hoadley, Koschmann, McClain, Renninger and Sfard) teach at schools of education across the country—and are in contact with math educators at many more. The project accomplishments will influence the teacher professional-development programs in these centers. Teachers who are involved in the teacher professional-development components of this project will also spread project findings as early adopters at their graduate programs and K-12 schools. Ready access to project resources, models and technology at the Math Forum will facilitate general dissemination of innovative math education—including through the popular teacher discussion forums on the Math Forum website—to additional teacher professional development programs.

(c) The PIs and Math Forum are active in NCTM, AERA, PME, and PMENA and will present project findings at the annual conference for teachers of mathematics. Additionally, project researchers are prominent in the learning science communities around the ICLS, CSCL and other academic conferences and publish prolifically in academic and practitioner journals, books and conferences.

(d) Because it provides the first multi-user version of a dynamic-mathematics application, the project will be well known within the worldwide communities of GeoGebra and Geometer's Sketchpad users. The project technology will all be available as open source, so that other researchers and developers can build on it, modify it and install versions on their own servers. (The project technology is built on VMT and GeoGebra, both already available as open source at SourceForge.) Teachers, trainers and researchers who do not have the technical expertise to do this, can simply use the environment that is on the Math Forum servers; they can develop their own curriculum for it and can readily access detailed user logs from it. Features for administration of chat rooms will be built in to support local administration.

(e) For the sake of sustainability beyond the proposed project and to support further scale-up, it is important to establish an on-going network of teachers in the form of self-organizing communities (Renninger & Shumar, 2002a). As discussed above, this will begin with mentoring relationships between cohorts of teachers going through the project professional development. The mentoring relationship will grow into a mutual support network, in which teachers from the programs at both Drexel and Rutgers will share questions, case studies, best practices, curriculum, etc. Later in the project, this growing local network will connect with national and international teacher networks, such as Tapped-In (<http://tappedin.org>), the Knowledge Building Teacher Network (Chan, van Aalst & Law, 2009) and the Institute for Knowledge Innovation and Technology (<http://ikit.org>). These networks will disseminate use of the project services widely. We are aware of the issues in trying to build sustainable virtual learning communities (Barab, Kling & Gray, 2004) and will use an iterative approach. In addition, dissemination efforts will target organizations, consortia and networks of home schooling and of online schools.

Expertise

The proposed project brings together an interdisciplinary team of researchers, led by the PIs:

PI, Gerry Stahl: PI on the VMT Project. Author of *Group cognition: Computer support for building collaborative knowledge* and *Studying virtual math teams*. Founding editor of *International Journal of Computer-Supported Collaborative Learning*. He will have overall responsibility for the project.

PI, Arthur Powell: Chair of the Department of Urban Education at Rutgers-Newark and Associate Director of the Robert B. Davis Institute for Learning at Rutgers-New Brunswick. Specializes in problem solving, deductive reasoning and heuristics in math education. Expertise in analysis of learning in digital video. Primary responsibility for teacher professional development at Rutgers.

Co-PI, Jason Silverman: Faculty member at the School of Education, Drexel University. Developed and teaches the online masters degree program in Mathematics Learning and Teaching at Drexel. Primary responsibility for teacher professional development at Drexel.

Co-PI, Stephen Weimar: Director of the Math Forum since 1994. Established track record as PI on multiple successful NSF grants. Responsible for integration with Math Forum services.

Co-PI, Sean Goggins: Brings a decade of collaborative and social software design and development team leadership. He will be primarily responsible for automated and statistical data analysis.

Co-PI, Michael Khoo: Trained in anthropology, he evaluated components of NSF NSDL digital libraries. He will coordinate the internal formative evaluation component of this project.

Annie Fetter: Co-founder of the Math Forum. Directs the Problem-of-the-Week. Has done professional development and written curriculum for the Geometer's Sketchpad software since it was created. She will be involved in training and mentoring the teachers and coordinating the classroom usage.

Sukey Blanc: Trained in urban anthropology, she studies mathematics and science education, educational equity and school reform. She is Senior Research Associate with Research in Action, a Philadelphia-based non-profit organization engaged in education research and evaluation, which since 1992 has worked with public school districts, educational institutions and community organizations to improve educational opportunities for those traditionally disadvantaged. She will work with the Advisory Committee and will be responsible for external formative and summative evaluation.

The Math Forum. This well established math education site, MathForum.org, has its office at Drexel University with program and technical staff to run services and to maintain the Internet technology. The staff has extensive experience in mentoring math teachers, training new mentors, designing math resources and supporting a huge user community. Most of the program staff are experienced classroom math teachers. The technical staff will be responsible for software development during the project and then for maintaining the project software during and beyond the lifetime of the project.

The Advisory Committee. The AC brings expertise in math education; educational psychology; quantitative analysis of learning outcomes, motivation and attitudes; problem-based learning theory and analysis; CSCL; and online communities of learners. (See attached letters.)

Computer-Supported Math Discourse Among Teachers and Students

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