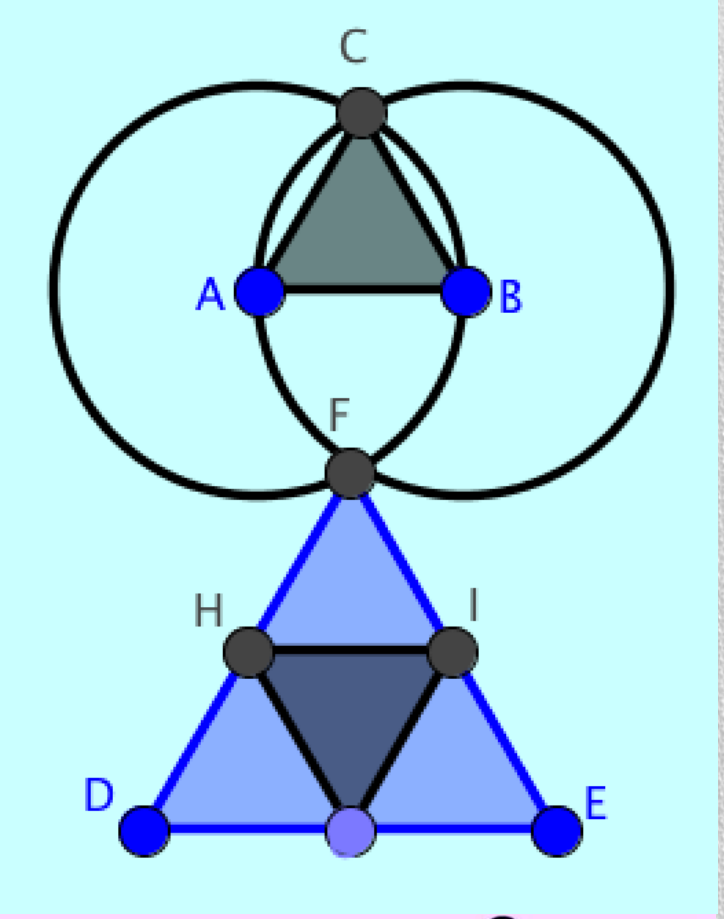


DESIGNING A LEARNING ENVIRONMENT FOR GEOMETRY

LinCS Seminar, Göteborg

October 9, 2013

Gerry Stahl

Dimensions of Design

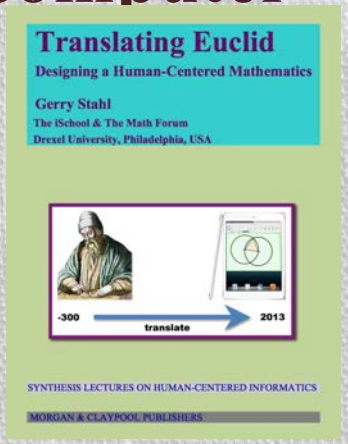
1. Viewing the whole design agenda
2. Translating the history of geometry
3. Understanding the philosophy of geometry knowledge
4. Mastering the mathematics
5. Building the technology
6. Supporting collaboration
7. Researching the learning processes
8. Theorizing the approach & resources
9. Defining the pedagogy
10. Developing the curricular resources
11. Design-based research of human-centered geometry

1. Viewing the whole design agenda

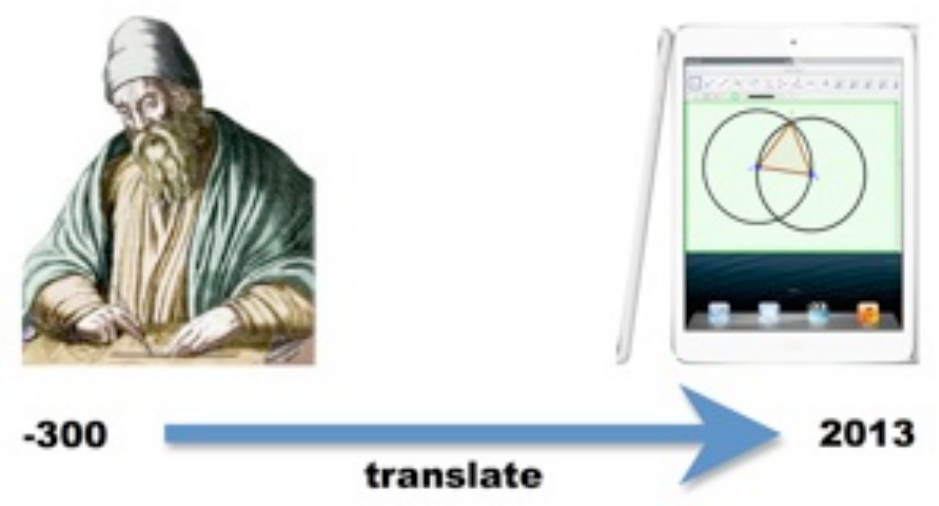
How should one translate the classic-education approach of Euclid's geometry into the contemporary vernacular of social networking, computer visualization, and discourse-centered pedagogy?

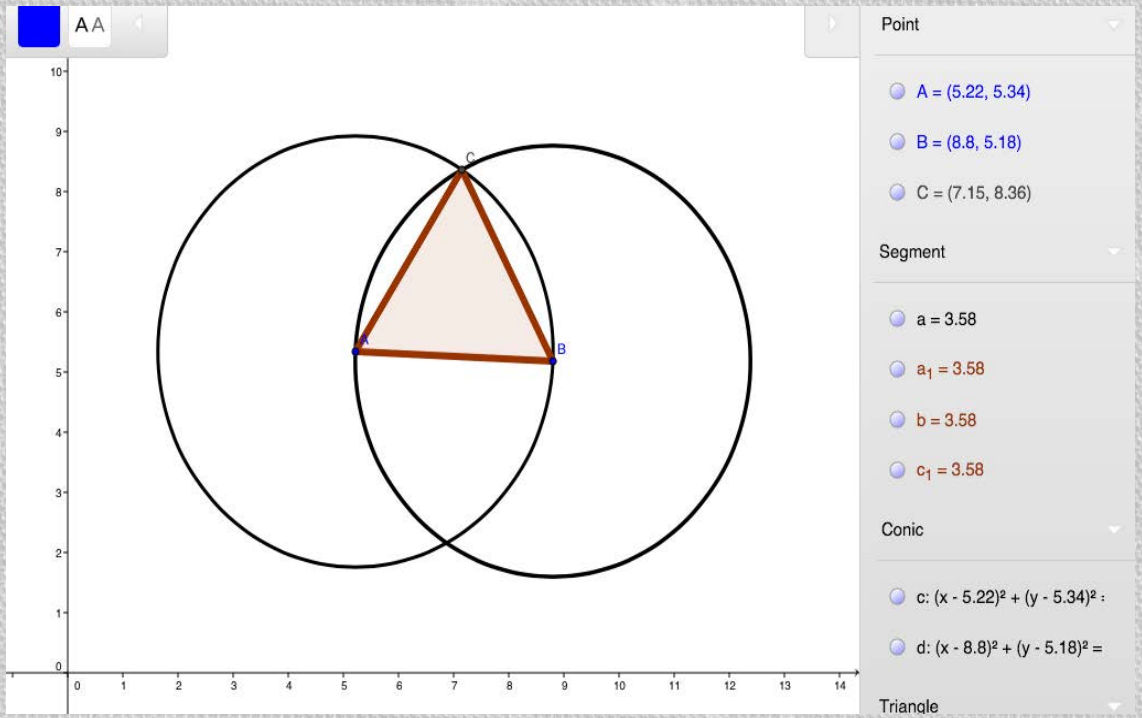
A multi-dimensional, iteratively evolving design-based research approach to designing a human-centered, 21st century geometry education using computer-supported collaborative learning

11 chapters of *Translating Euclid* book



Translating from clay tablets to iPad tablets





2. Translating the history of geometry

- **Early Greeks explored using persistent diagrams and specialized text in small, distributed community**
- **Euclid systematized a set of postulates that built on each other to construct and prove properties of figures. Established deductive argumentation**
- **Successive further bureaucratization of proofs as objective truth**
- **Axiomatization and formal logic. Euler extensions. Non-Euclidean geometries**
- **Dynamic geometry using computer interface**

3. Understanding the philosophy of geometry knowledge

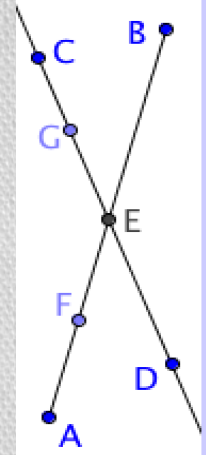
- **Alienation of geometric objects from human creators – starting with Plato's world of ideas**
- **Abstraction from the construction process enabled progress, but reified the constructs into other-worldly, mental objects**
- **The social construction of geometry was obscured by a focus on individual thought and knowledge; a widespread ideology of individualism**

4. Mastering the mathematics

- **Dynamic geometry offers a potential to retrieve the human-centered nature of geometry as a product of creative-discovery**
- **Collaborative dynamic geometry offers a potential to retrieve the social nature of geometry as a product of community discourse**
- **At the core of dynamic geometry is the concept of *dependency*. Proof can be understood as a consequence of constructed dependencies, based on a dialectic of discovery and creation**

Move

An example of what you will construct:

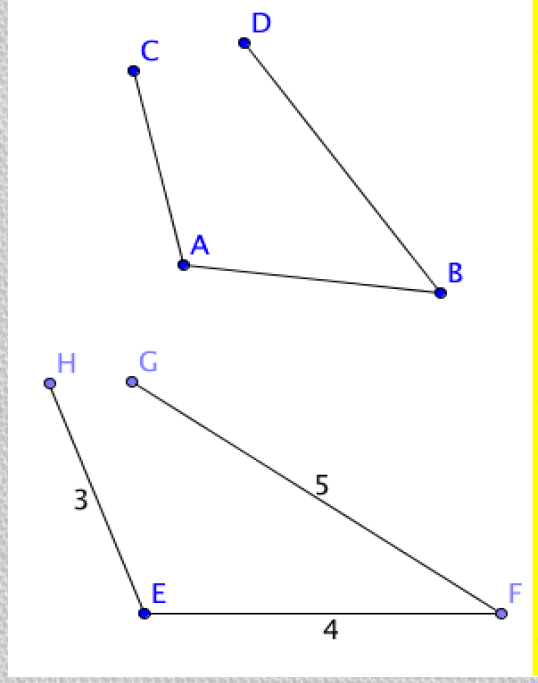


Construct dependent objects

Take turns controlling the construction.

1. Select the Segment Tool and click on two points to construct a segment like AB.
2. Select the Line Tool and click on two points to construct a line like CD that crosses the segment.
3. Select the Intersection Tool and construct a point where the line cuts the segment.
4. Construct another point on the segment and another point on the line, like F and G.
5. Drag each point, the line and the segment.
6. Discuss in the chat how each object is free, constrained or dependent on other objects.

Move



If one triangle is congruent to another, then all its angles and all its sides are dependent on the corresponding angles and sides of the other triangle.

Given three segments -- AB, AC, BD -- for constructing a triangle, how many angles or sides do you have to constrain to fully constrain the triangle?

The three segments EF, DH, FG have been constructed with the Segment-with-Given-Length-from-Point tool to constrain their lengths. How many triangles can you construct with these segments?

What do you conclude?

5. Building the technology

- Geometer's Sketchpad, Cabri, etc. pioneered the creation of dynamic geometry
- GeoGebra makes it freely available and integrated with other math (3-d, algebra, statistics, spreadsheet, trig, calculus, etc.)
- VMT provides a collaboration environment and integrates the first multi-user dynamic geometry

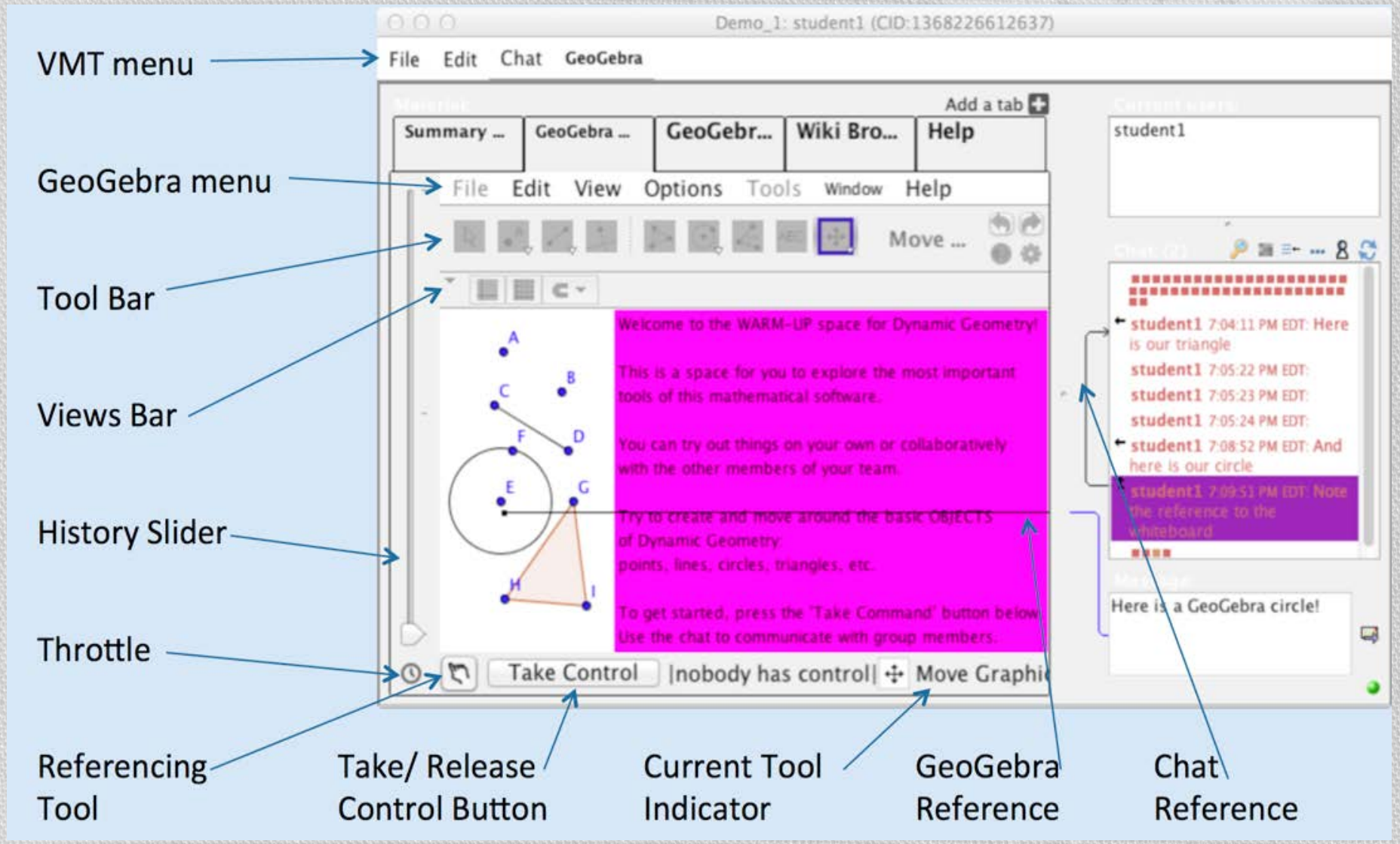
- The next dozen slides describe the design of the Virtual Math Teams (VMT) online environment

Virtual Math Teams (VMT)

- **VMT is an online environment for students to work on and discuss math problems synchronously.**

- **VMT combines support for dynamic geometry with media for collaborative learning.**

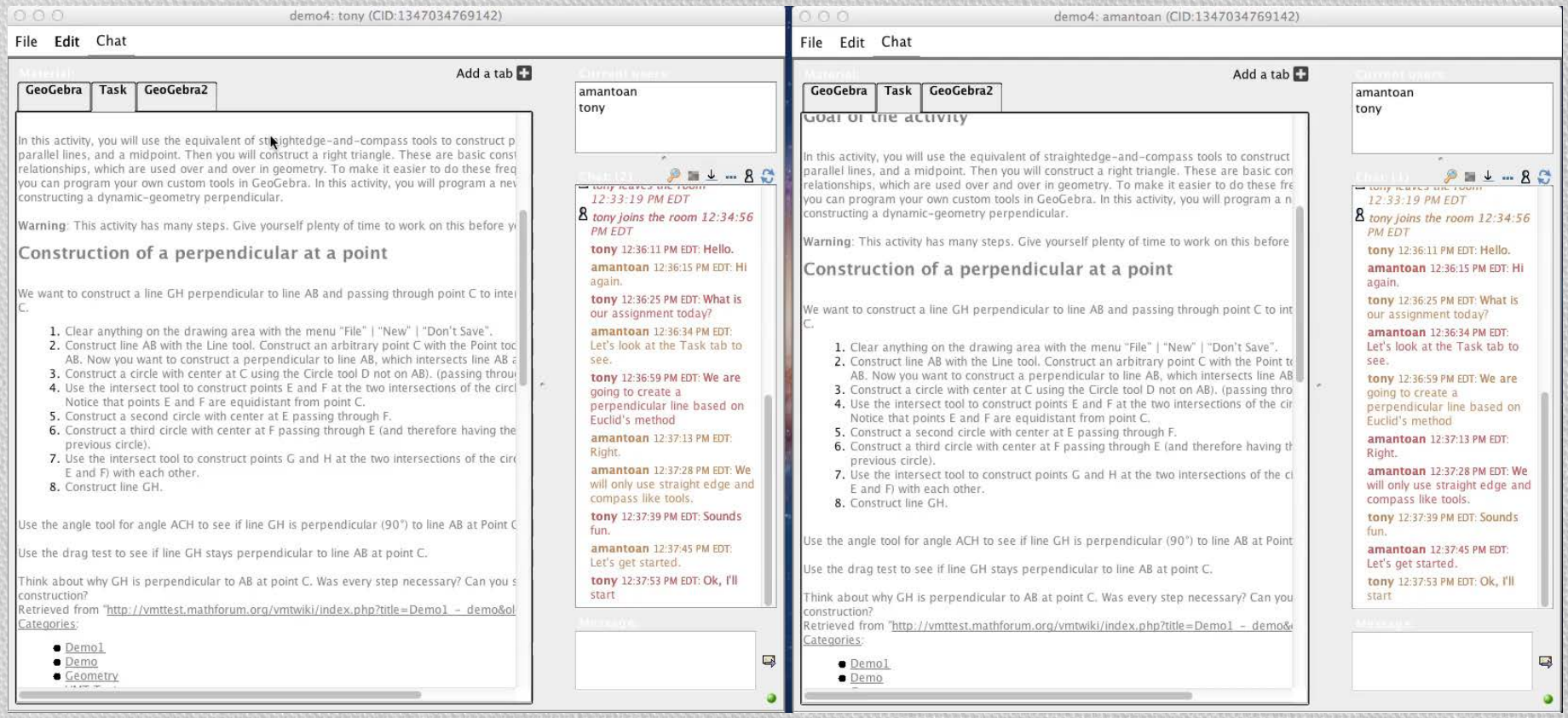
VMT with multiple GeoGebra tabs



Integration with GeoGebra

- **Remote students can synchronously work on a shared construction together.**
- **Users can take turns manipulating the construction.**
  - Adding, deleting, modifying and moving objects
- **The construction will stay in sync on each user's screen.**
- **Users can chat about the problem as they work.**

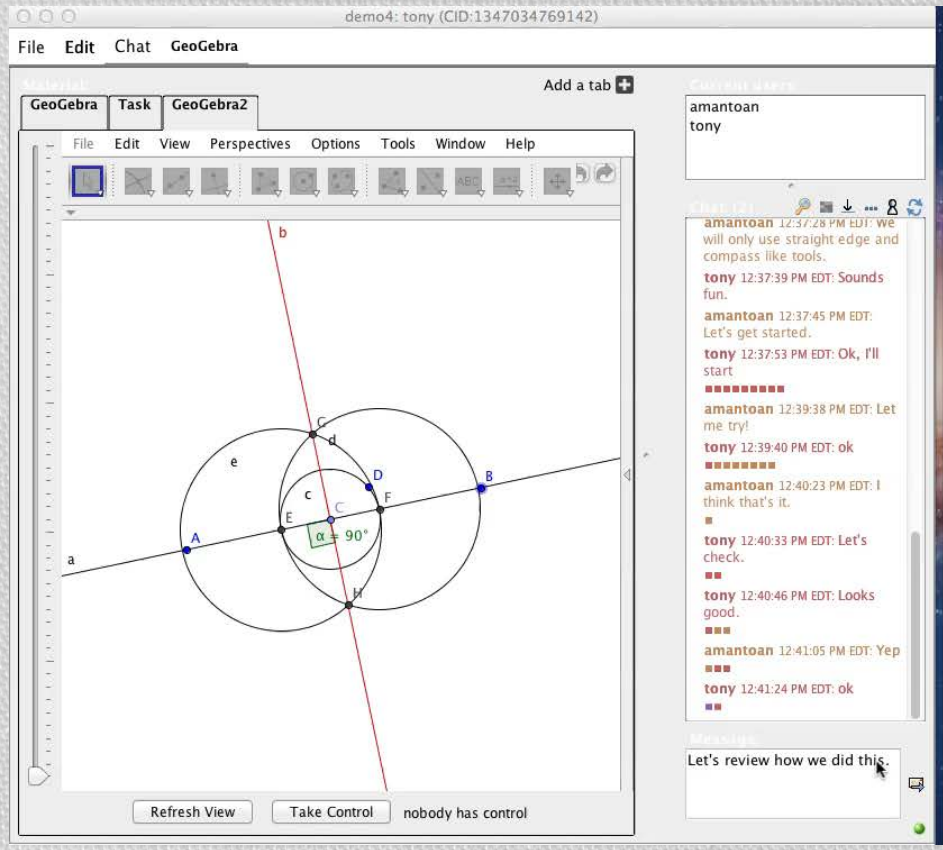
Multi-user GeoGebra

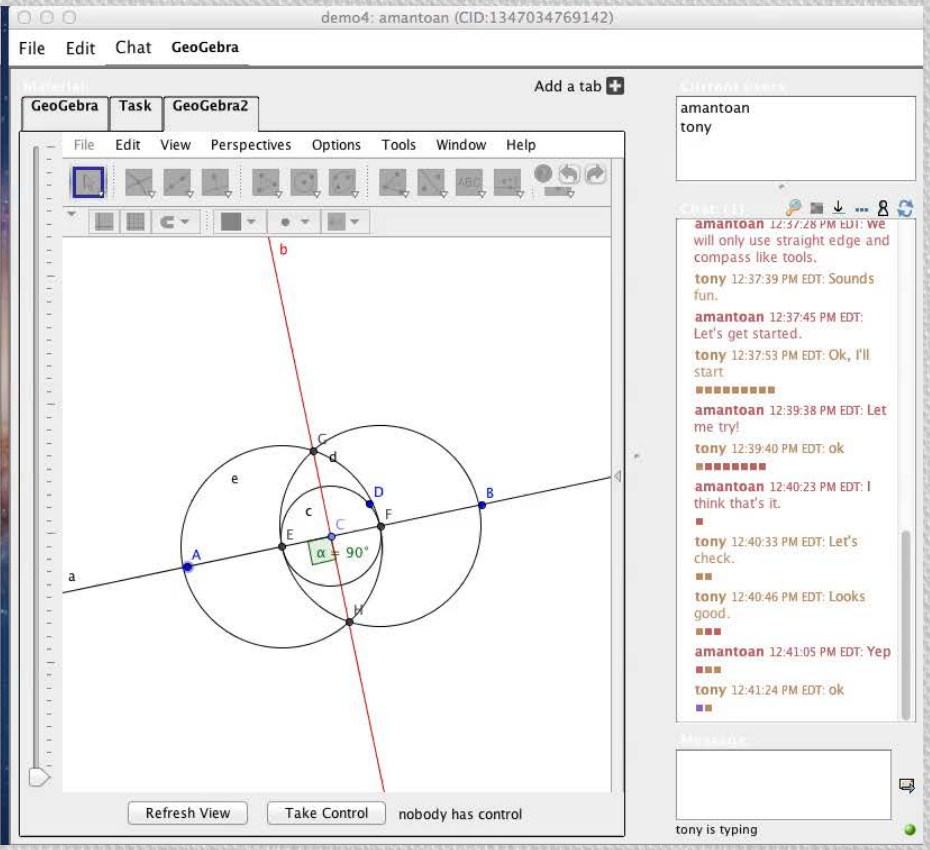


History Tracker

- **Built-in history tracker allows users to scroll back and forth in time to see how the construction developed.**
- **Shows everything that happened including style changes and object movements.**
- **Each GeoGebra workspace is tracked separately.**

History Tracker In Action





Other Shared Tools

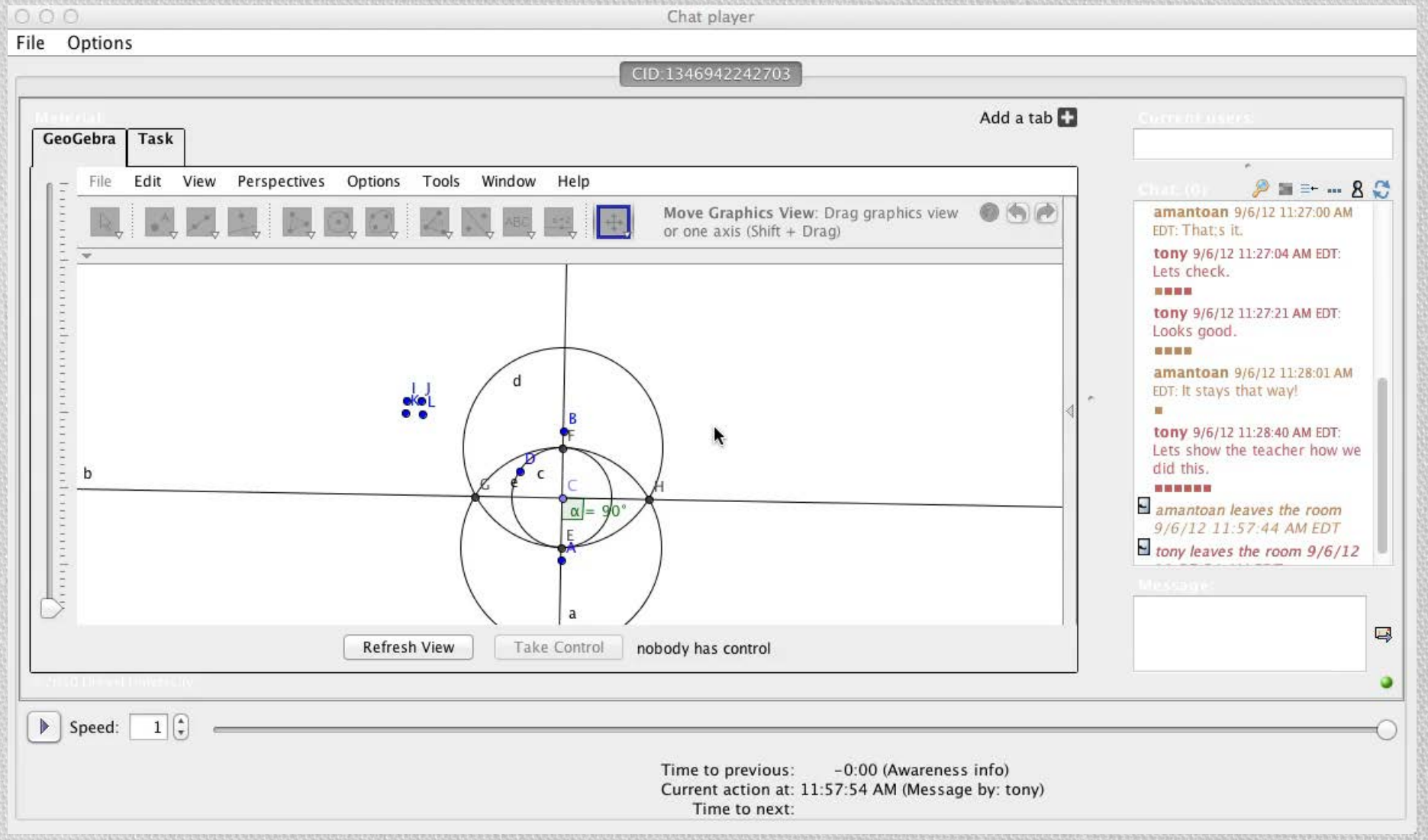
- **Shared Whiteboard**
  - Users can doodle on the shared whiteboard
  - Draw simple shapes
  - Summarize work, draft shared statements, note observations or hypotheses in text boxes
  - History of the whiteboard is also tracked.

- **Web browser**
  - Simple web browser
  - Can be used to show instructions for the student's assignments or other related information on the web.

VMT has built in tools for session analysis

- **VMT records every chat, and action in the session.**
- **Provides a session replayer to go back and forth through the session step by step.**
- **Chat, whiteboard, and GeoGebra events all playback in the order they occurred in the original session.**

The VMT Session Replayer



VMT has built in tools for session analysis

- **Spreadsheet log files can be downloaded for each VMT room – by anyone: students, students from other teams, parents, teachers, researchers.**
- **Log files and the VMT Replayer provide unique insights for teachers and researchers.**
- **Allows teachers to go back and see anything that teams did in the chat rooms.**

Session Log Files

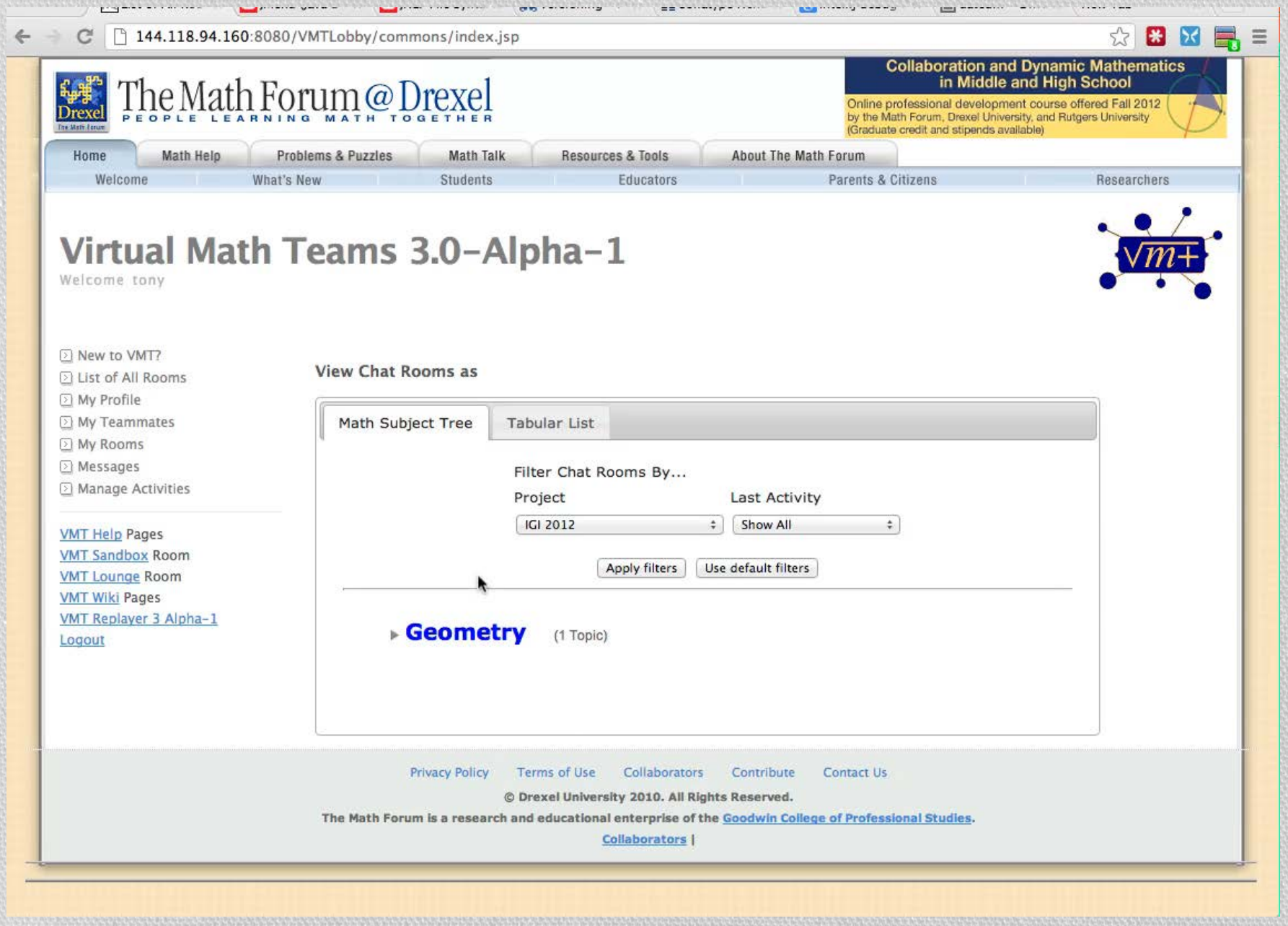
	A	B	C	D	E	F	G	H
1	Line	Date	Start Time	Post Time	Duration	EventType	tony	amantoan
2	1	09/06/2012		10:38:42	00:00:00	chat	joins the room	
3	2	09/06/2012		10:39:54	0:1:12	chat		joins the room
4	3	09/06/2012		11:24:49	0:44:55	chat	Hello	
5	4	09/06/2012		11:25:07	0:0:18	chat		Hi. Lets start by looking at the task description.
6	5	09/06/2012		11:25:11	0:0:4	chat	Ok.	
7	6	09/06/2012		11:25:31	0:0:20	chat	Alright, I'll start.	
8		09/06/2012		11:25:42	0:0:11	Geogebra:GeoGebra	added point:Point "A"	
9		09/06/2012		11:25:44	0:0:2	Geogebra:GeoGebra	added point:Point "B"	
10		09/06/2012		11:25:44	0:0:0	Geogebra:GeoGebra	added line:Line "a"	
11		09/06/2012		11:25:50	0:0:6	Geogebra:GeoGebra	added point:Point "C"	
12		09/06/2012		11:25:52	0:0:2	Geogebra:GeoGebra	added point:Point "D"	
13		09/06/2012		11:25:52	0:0:0	Geogebra:GeoGebra	added conic:Circle "c"	
14		09/06/2012		11:26:01	0:0:9	Geogebra:GeoGebra	added point:Point "E"	
15		09/06/2012		11:26:02	0:0:1	Geogebra:GeoGebra	added point:Point "F"	
16	7	09/06/2012		11:26:18	0:0:16	chat		Great, I'll take it from here.
17		09/06/2012		11:26:29	0:0:11	Geogebra:GeoGebra		added conic:Circle "d"
18		09/06/2012		11:26:33	0:0:4	Geogebra:GeoGebra		added conic:Circle "e"
19		09/06/2012		11:26:40	0:0:7	Geogebra:GeoGebra		added point:Point "G"
20		09/06/2012		11:26:43	0:0:3	Geogebra:GeoGebra		added point:Point "H"
21		09/06/2012		11:26:52	0:0:9	Geogebra:GeoGebra		added line:Line "b"
22	8	09/06/2012		11:27:00	0:0:8	chat		That;s it.
23	9	09/06/2012		11:27:04	0:0:4	chat	Lets check.	
24		09/06/2012		11:27:15	0:0:11	Geogebra:GeoGebra	added angle:Angle "?"	
25	10	09/06/2012		11:27:21	0:0:6	chat	Looks good.	
26								
27								

Sheet1

VMT Is Publicly Available

- **VMT is open source.**
- **Our Math Forum VMT server is available for all to use – vmt.mathforum.org**
- **Anyone may set up their own VMT server.**

Create your own topic rooms



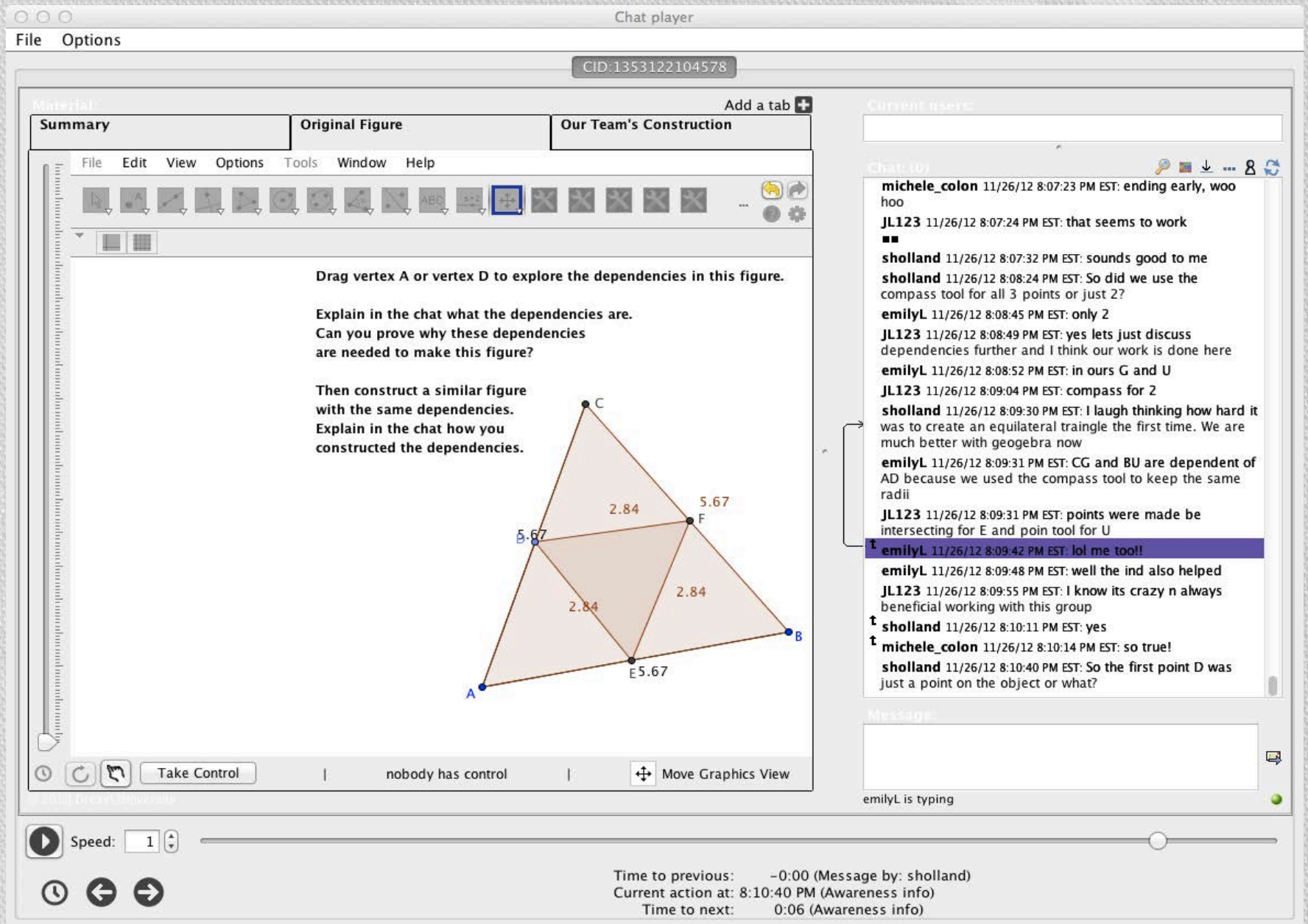
6. Supporting collaboration

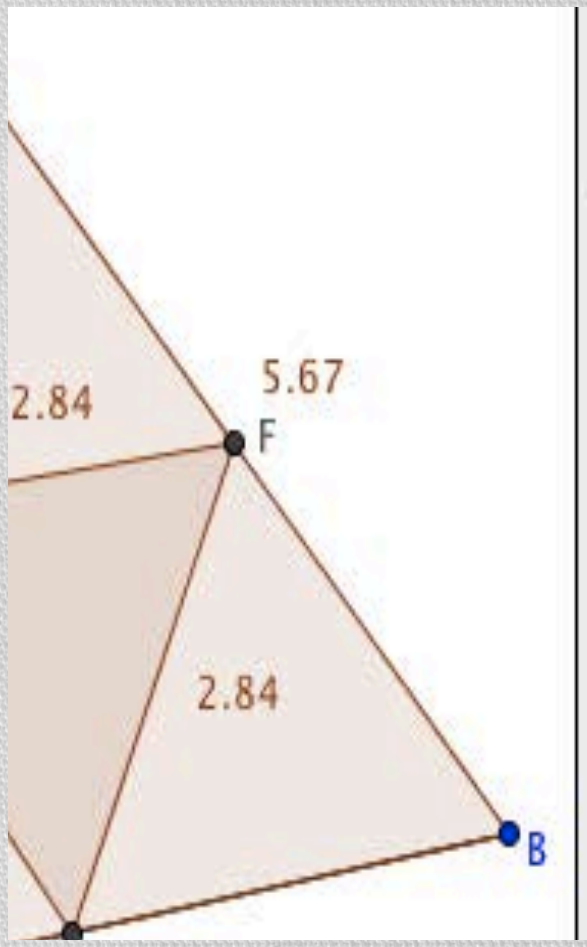
Virtual math teams are guided by carefully designed and tested curricular resources

They drag points to discover, discuss and reflect on dependencies in figures

They discuss how to construct figures with the needed dependencies and take turns constructing and testing the figures

Teachers collaborate on inscribed triangles





sholland 11/26/12 8:09:30 PM EST: I laugh thinking how hard it was to create an equilateral traingle the first time. We are much better with geogebra now

emilyL 11/26/12 8:09:31 PM EST: CG and BU are dependent of AD because we used the compass tool to keep the same radii

JL123 11/26/12 8:09:31 PM EST: points were made be intersecting for E and poin tool for U

emilyL 11/26/12 8:09:42 PM EST: lol me too!!

emilyL 11/26/12 8:09:48 PM EST: well the ind also helped

JL123 11/26/12 8:09:55 PM EST: I know its crazy n always beneficial working with this group

sholland 11/26/12 8:10:11 PM EST: yes

michele_colon 11/26/12 8:10:14 PM EST: so true!

Students collaborate on inscribed squares

gles | Squares | Hexagons

File Edit View Options Tools Window Help

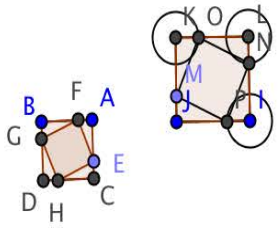
Move Graphics View:
Drag graphics
view or one axis (Shift +
Drag)

Take turns dragging vertex A of Quadrilateral ABDC and vertex E of Quadrilateral EFGH.

Chat about dependencies you notice and what you wonder about this figure.

Construct a Quadrilateral inscribed in a Quadrilateral that behaves the same as this one.
Chat about how you are constructing and why.

Note that the Compass tool is available by pulling it down from the Circle tool in the tool bar.



Take Control | nobody has control | Polygon

Chat: (0)

cheerios 3/4/13 4:00:42 PM EST i made a line segment which was if than i used the perpendicular line tool and made 2 lines on each side then used the compass tool and clicked on each point and then the center vertex was i and then made a another circle except the center vertex is j and connected all the points

fruitloops 3/4/13 4:01:07 PM EST correct

cheerios 3/4/13 4:01:15 PM EST then used to polygon tool and then hid the circles and lines

fruitloops 3/4/13 4:01:36 PM EST and we used the circles to make the sides equal because the sides are their radius

■■■■■■■■

fruitloops 3/4/13 4:02:39 PM EST point m is like point e because it moves around

■■■

fruitloops 3/4/13 4:02:48 PM EST and its the same color

■■■■■■■■■■■■■■■

fruitloops 3/4/13 4:04:14 PM EST good!!

■■■■■■■■

fruitloops 3/4/13 4:04:40 PM EST now hide the circles

Message:

7. Researching the learning processes

Students are asked to identify chat log segments that show effective collaboration and to reflect on what they noticed and wondered about

Teachers are asked to select and reflect on student chat log segments – and to discuss how to improve the resources, approach and experience

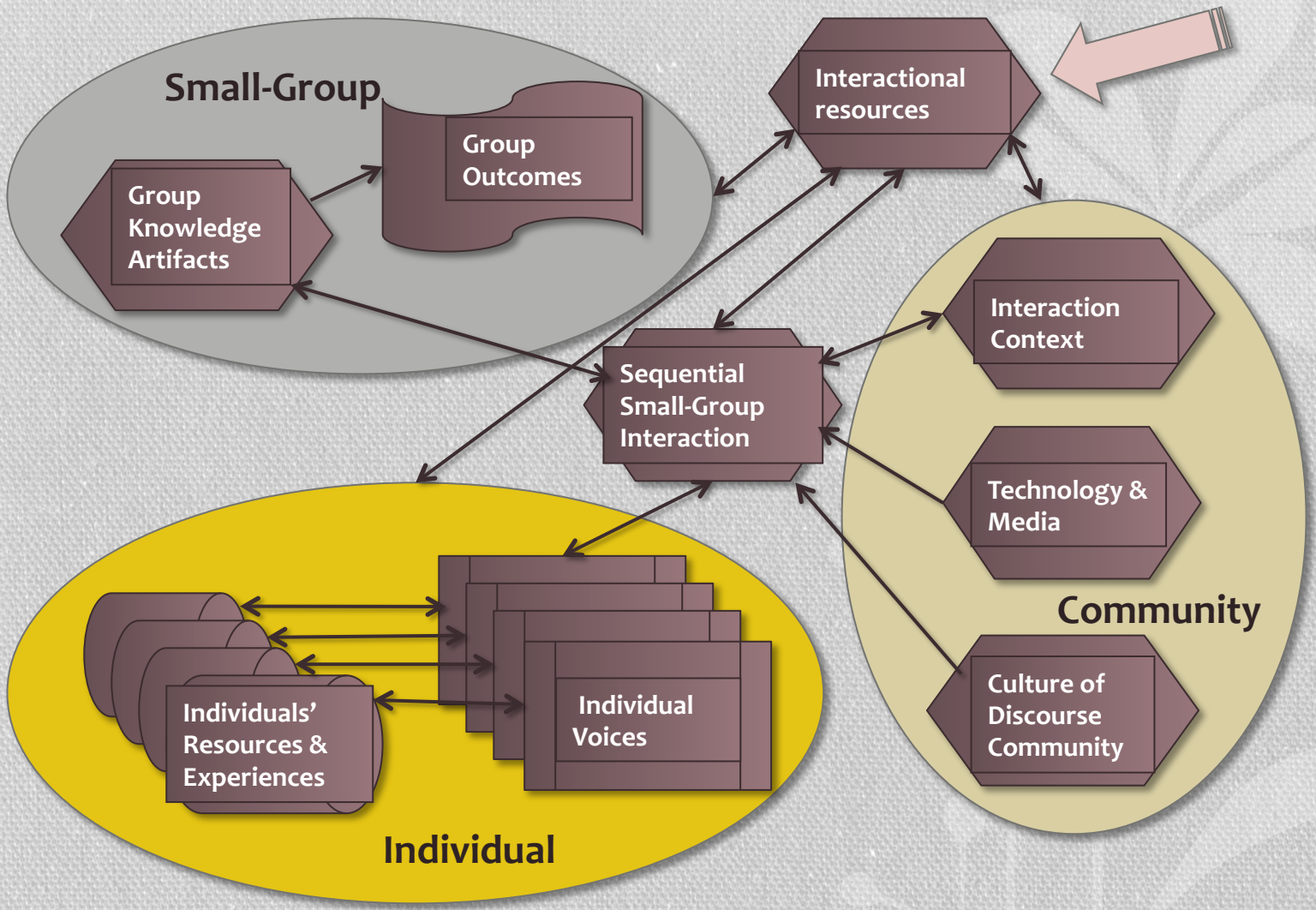
Researchers analyze logs and replayer to understand strengths and weaknesses of resources and to investigate how student teams collaboratively master dynamic geometry practices, skills and understanding

8. Theorizing the approach & resources

The ideology of individualism views geometry as a realm of mental ideas that individuals must master using logic.

But geometry is a product of the community of mathematicians, math educators and math students during the past 2,600 years; it can be learned collaboratively by teams discovering and creating under the guidance of teachers/educators

There is validity both to the importance of individual mental effort and socio-cultural practices. There should be support at individual, group and community levels



Levels of analysis connected by interactional resources

My recent theoretical work

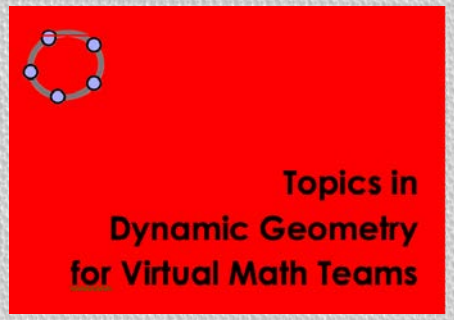
- **Group cognition as focus of analysis on the inter-personal inter-action**
- **Theory of resources mediating learning**
- **Unity of levels of analysis/interaction:**

Individual, team, community (class, schooling, math)

- **Theory implications of analyses of collaborative dynamic geometry learning**

10. Developing the curricular resources

Although the VMT Project was funded to just develop the technology and analyze its effectiveness, the real problem is to design the pedagogy: approach & resources



We developed a set of about 18 "topics" in a workbook format that included tutorial "tours" of the technology. Each topic was intended for about a one-hour online, synchronous, collaborative session. Each topic included 3 to 10 GeoGebra tabs with guiding tasks

Pedagogical focus

The approach emphasis includes:

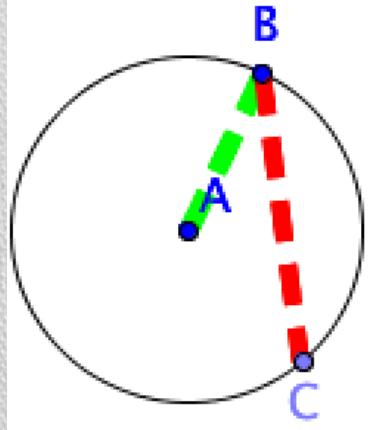
1. **The importance of experiencing first-hand the actual doing of mathematics: exploration, noticing, discovering, wondering, conjecturing, creating, designing, constructing, explaining, understanding, proving, teaching**
2. **Resisting the tradition of accepting on authority the definitions and understandings of geometric objects**
3. **Resisting the temptation to use GeoGebra just to illustrate geometric facts with pretty figures or flashy simulations**
4. **Guiding teachers and students to design their own constructions, including the definition of custom tools**
5. **Emphasizing the role of dependencies in dynamic geometry**

Beginning topics

1. **Warm up – login and create points**
2. **Messing with dynamic geometry – dragging points; copying & adding segment lengths**
3. **Visualizing Thales' and Pythagoras' theorems**
4. **Constructing equilateral triangles**
5. **Programming custom tools: perpendicular lines, parallel lines, equilateral triangles**
6. **Finding 8 centers of triangles – challenge: Euler's segment and the nine-point circle of a triangle**

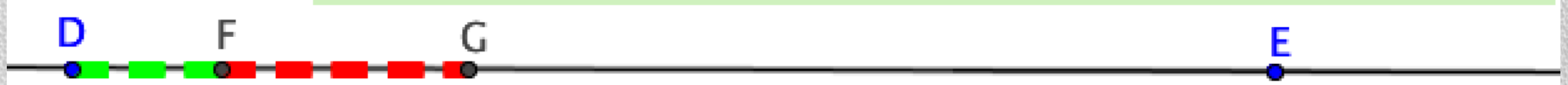
Move: Drag or select objects (Esc)

An example of what you will construct:



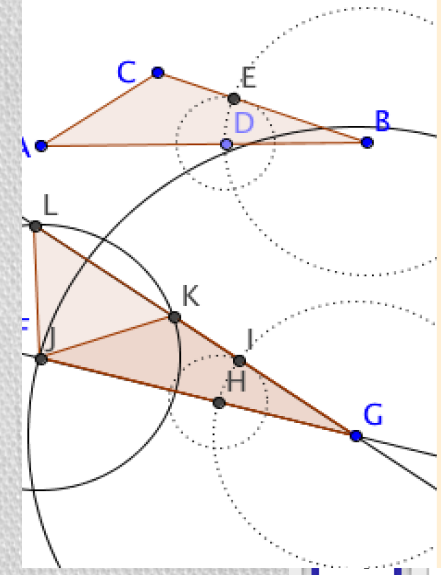
Construct a segment whose length = sum of two lengths

1. Construct a circle with center through a point, its radius and a chord.
(A radius is a segment from a circle's center to a point on its circumference--like AB--and a chord is a segment connecting two points on its circumference--like BC.)
2. Construct a line like DE and construct a segment along it, whose length is the sum of the lengths of your radius + chord.
3. Drag each point, segment or circle to make sure that the length of the segment changes dynamically correctly.



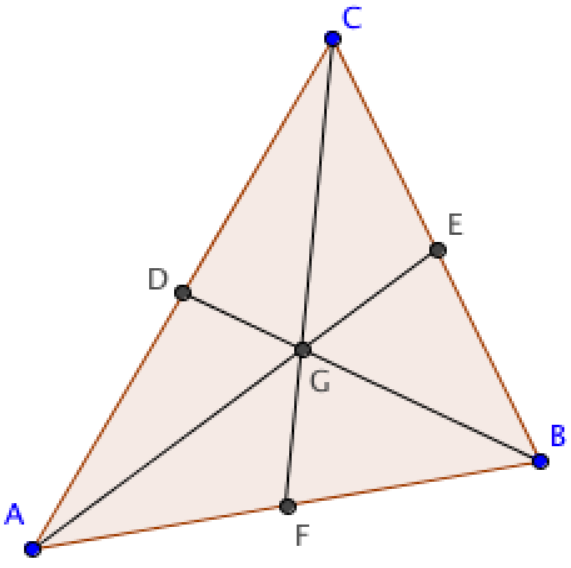
Intermediate topics

7. Rigid transformations, symmetry, proof of area of a triangle
8. Exploring angles of triangles and similar triangles
9. Visualizing congruent triangles
10. Solving typical geometry problems
11. Constructing inscribed triangles & polygons
12. Building a hierarchy of kinds of triangles



This is a tricky case.
Given triangle ABC, construct another triangle with an angle equal to ABC, a side along the angle equal to side AB, and a side opposite the angle equal to side AC.

1. Use the compass tool to copy angle ABC to angle HGI
2. Use the compass tool to copy side AB to side GJ and
3. to copy side AC to side JK.
4. Now drag point K to meet the side extending GI.
5. Notice that for some shapes of triangle ABC, there are two points that satisfy the constraint SSA, but that only one of them constructs a triangle congruent to ABC
6. Discuss this in the chat.



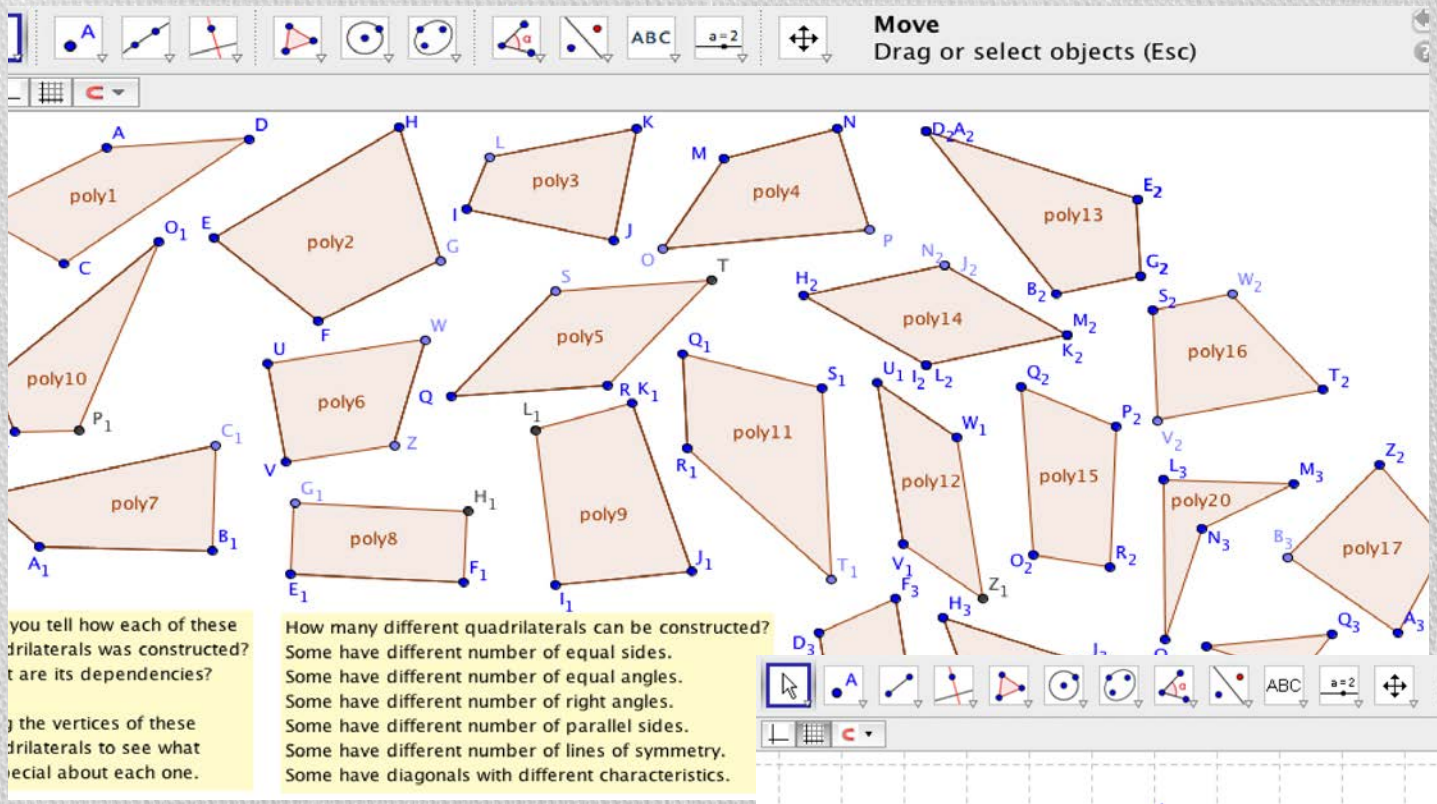
To construct the centroid of a triangle, construct the midpoints of the three sides (you can use the Midpoint tool for this).
Then construct Segments from the Midpoints to the opposite vertex.
Construct the Point where these Segments intersect.
(Note that all three Segments intersect at the same location, so you can use the intersection of any two Segments.)

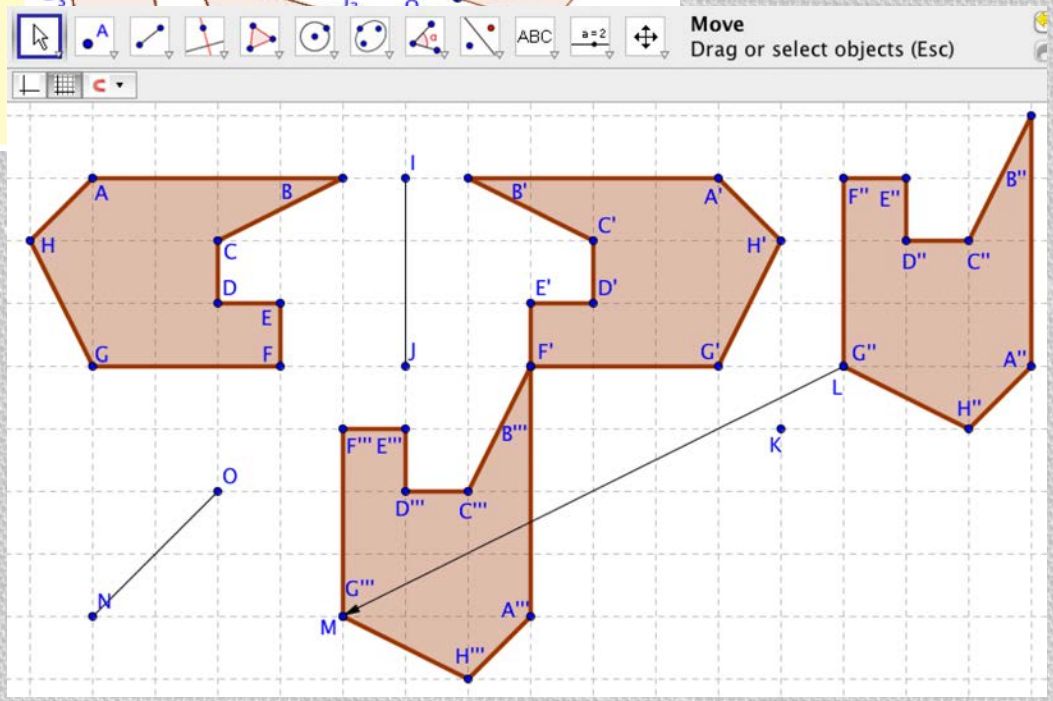
Now create a custom tool to automatically construct the centroid given the three vertices of a triangle.

Create some different triangles and their centroids.
Drag the vertices of the triangle and observe how the centroid behaves. Is it always inside the triangle?

Advanced topics

13. Constructing a square and other quadrilaterals
14. Building the hierarchy of quadrilaterals
15. Constructing a tangent geometrically and using the GeoGebra algebra interface
16. Proving incenter properties and Euler segment properties with constructed dependencies
17. Modeling a factory workflow with systems of rigid transformations
18. Exploring taxicab transformational geometry


Move
Drag or select objects (Esc)
poly1
poly2
poly3
poly4
poly5
poly6
poly7
poly8
poly9
poly10
poly11
poly12
poly13
poly14
poly15
poly16
poly17
poly20
you tell how each of these
drilaterals was constructed?
t are its dependencies?
g the vertices of these
drilaterals to see what
ecial about each one.
How many different quadrilaterals can be constructed?
Some have different number of equal sides.
Some have different number of equal angles.
Some have different number of right angles.
Some have different number of parallel sides.
Some have different number of lines of symmetry.
Some have diagonals with different characteristics.


Move
Drag or select objects (Esc)

11. Design-based research of human-centered geometry

Discover: The curricular topics guide student teams and teacher teams to discover dependencies in dynamic-geometric figures

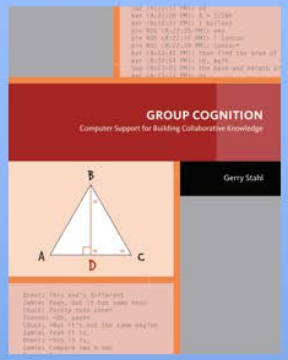
Create: The curricular topics guide student teams and teacher teams to creatively design and construct dependencies in dynamic-geometric figures

Understand, explain, prove: Students and teachers learn to view geometric truths in terms of constructed dependencies. They begin to see the causality of the world as human/social creative-discovery involving designed dependencies

The VMT Project evolves its pedagogical approach through iterative analysis of interactions among teams of researchers or teachers or students using the technology and resources. The research process is reflected in its publications and presentations.

The Virtual Math Teams Trilogy

Group Cognition (2006)



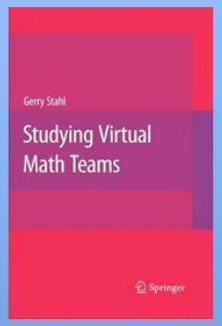
Computer Support for Building Collaborative Knowledge

MIT Press, 510 pages
Available for Kindle

The theory of group cognition emerges from several studies of CSCL and CSCW technologies. Analysis of interaction. Theory of CSCL.

www.GerryStahl.net/elibrary/gc

Studying Virtual Math Teams (2009)

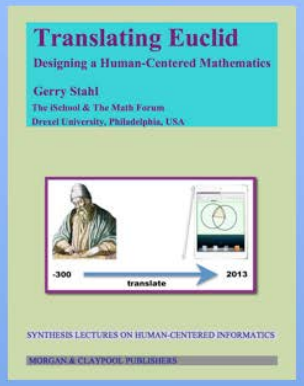


Springer Press, 626 pages
CSCL Book Series, paperback

Studies of the VMT Project technology, pedagogy, analysis, theory by team members and international collaborators

www.GerryStahl.net/elibrary/svmt

Translating Euclid (2013)



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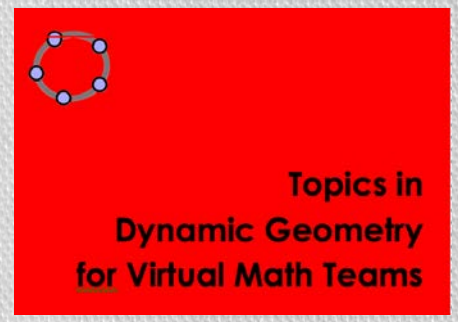

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